**LINEAR FUNCTIONS**

In 2004 in a city, there were 17,500 homes with internet connections. A service provider predicts that each year an additional 750 homes will get internet connections.

(a) Write a linear model for the number of homes, \( n \), with internet connections \( t \) years after 2004.

(b) Write down an assumption made by the model

(c) Comment on the validity of the model for large values of \( t \)

\[
\begin{align*}
\frac{n}{t} &= a + b \\
2004: &\\
\text{gradient:} &
\end{align*}
\]

(a) \( n = 750t + 17500 \)

(b) 750 extra homes get internet each year

(c) The model suggests that as \( t \to \infty \), \( n \to \infty \)

but you can't have infinitely many homes in one town.
A company uses a model to determine how the amount of fertiliser used, $f$ kg per hectare, affects the grain yield $g$, measured in tonnes per hectare. 

$$g = 6 + 0.03f - 0.00006f^2$$

(a) Interpret the value of the constant 6 in this model.

(b) A farmer uses 20 kg of fertiliser per hectare. How much more fertiliser would he need to increase his grain yield by 1 tonne per hectare?

$$g = 6 + 0.03(20) - 0.00006(20)^2$$

$$= 6 + 0.6 - 0.00006(400)$$

$$= 6.576$$

$$7.576 = 6 + 0.03f - 0.00006f^2$$

$$0.00006f^2 - 0.03f + 1.576 = 0$$

$$f = \frac{440.35}{8} \quad \text{or} \quad f = 59.65$$

$$59.65 - 20 = 39.65 \text{ kg per hectare}$$
SKETCHING FACTORISED POLYNOMIALS

Each factor gives a root. Put roots on x axis.

Determine start & end quadrants from basic shape

Classify root types & sketch

Odd power

Even power

Draw an inflection point

Add y axis to sketch

Add y intercept (sub in x=0)

\[ y = (4-x)(x-1)^3(2+x)^2 \]

-2 1 4

\[ (4-x) = 0 \quad (x-1) = 0 \quad (2+x) = 0 \]

\[ 4 = x \quad x = 1 \quad x = -2 \]

Single-slice \quad Triple-inflection \quad Double-touch
**Graph Transformations**

**Reflection**
- $f(-x)$
- $-f(x)$

**Translation**
- $f(x+a)$
- $f(x)+a$

**Stretches**
- $af(x)$

The journey of $x$
- $cf(ax+b)+d$

Sketch $y = 1 - f(3x-1)$

Graphs showing different transformations:
- Reflection
- Translation
- Stretching
Describe the transformation which maps \( f(x) = x^2 \) onto \( g(x) = 3x^2 + 6x - 1 \). What is the equation of \( g(x) \)?

\[ f(x) = \frac{1}{x^2} \text{ is translated by } (\frac{2}{3}, -3) \text{ then reflected over the } x \text{ axis to obtain the graph of } g(x). \]

**Journey**

**Parent:**

**Journey:**

- **Translate**
- **Reflect over**...
- **Stretch of**...
- **Over the**... axis about the... axis

\[ g(x) = \]
CUBIC/QUARTIC FUNCTIONS

What is the equation corresponding to this graph?

\[
y = A (x+3)^3 + C
\]

\[
y = A (x-2)^2 + 1
\]

When \( x=3 \), \( y=0 \):

\[
0 = A (3-2)^3 + 1
\]

\[
0 = A (1)^3 + 1
\]

\[
A = -1
\]

\[
y = -(x-2)^3 + 1
\]
Reciprocal Functions

Sketch: \( y = \frac{k}{x^2} - 1, \ k < 0, \) and \( y = 3x + 1 \) on the same axis.
State, with reason, the no. of solutions to the equation
\( x^2(3x+1) = k, \ k < 0, \) and whether the solution will be
a positive or a negative number.

**Asymptotes & asymptotic behaviour**

- \( y = \frac{1}{x} \)
- \( y = \frac{1}{x^2} \)
- \( y = \frac{1}{x+a} \)

**Aeroplane landing**

\( \frac{1}{(x+a)^2} \)

Note: \( k = x^2(3x+1), \ k < 0 \)

Sketch, reflect, translate.
**SKETCHING RATIONAL FUNCTIONS**

**VERTICAL ASYMPOTOTE**

The x which makes the denominator zero.

**HORIZONTAL ASYMPOTOTE**

Long term behaviour. Put in large x.

**Y INTERCEPT**

Sub in $x = 0$.

**X INTERCEPT**

Put $y = 0$ solve to get x (numerator = 0).

Think of an aeroplane landing

**NOT**: landed

**NOT**: fly by

**NOT**: crashing

**NOT**: taking off

Vertical asymptote: $x = -3$

Horizontal asymptote: $y = \frac{4(\text{big no.}) - 1}{(\text{big no.}) + 3} = 4$

Y intercept: $y = \frac{4(0) - 1}{(0) + 3} = -\frac{1}{3}$

X intercept: $0 = \frac{4x - 1}{x + 3} \Rightarrow 0 = 4x - 1 \Rightarrow x = \frac{1}{4}$
**Take this topic seriously!!**

**Linear Inequalities**
- Rearrange as if it was an = BUT...
- Flip sign if x or ÷ both sides by a negative no.
- Watch out for going too fast
  - $3 > 4 + n$
  - $n > 1$

**Quadratic Inequalities**
- Use a sketch.
  - $x < a$ or $x > b$
- Set notation
  - $\{x : a < x < b\}$

**THINGS I DID**

**SOLVING INEQUALITIES**

Find the set of values of $x$ which satisfy both $5x - 10 > 4x + 7$ and $2x^2 - 10x + 5x$

- $3 < x < \frac{5}{4} + \frac{1}{4}\sqrt{105}$
**SKETCHING INEQUALITIES**

**ALGEBRAIC TECHNIQUES**

**Take this topic seriously!!**

**Step 1**
Sketch the line or curve, with =
- Use a solid line for ≤ or ≥
- Use a dotted line for < or >

**Step 2**
Use a test point (usually x=0, y=0)
- Substitute test point coordinates into the inequality
- Is the resulting statement true?
  - YES: we want the side of the line or curve where the test point is
  - NO: we don't want the side with the test point

Draw and label 'R', the region which satisfies
\[ y \geq x^2 - x - 2 \quad \text{and} \quad y \leq 3 \quad \text{and} \quad x - y < 1 \]