OCR
Oxford Cambridge and RSA

Accredited

# A Level Mathematics A

**H240/03** Pure Mathematics and Mechanics
Sample Question Paper

## Date – Morning/Afternoon

Version 2

Time allowed: 2 hours

**You must have:**
- Printed Answer Booklet

**You may use:**
- a scientific or graphical calculator



**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g$ m s$^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

# Formulae
# A Level Mathematics A (H240)

## Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

## Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

## Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \ldots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \ldots\ + b^n \qquad (n \in \mathbb{N})$,

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \ldots\ + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ \ldots \quad \left(|x|<1,\ n \in \mathbb{R}\right)$$

## Differentiation

| $\mathrm{f}(x)$ | $\mathrm{f}'(x)$ |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\mathrm{cosec}^2 x$ |
| $\mathrm{cosec}\, x$ | $-\mathrm{cosec}\, x \cot x$ |

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

## Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

## Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

## Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1-\frac{1}{2}\theta^2, \tan\theta \approx \theta$ where $\theta$ is measured in radians

**Trigonometric identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$

**Numerical methods**

Trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

**Probability**

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B \mid A) = \mathrm{P}(B)\,\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

**Standard deviation**

$\sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

**The binomial distribution**

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of $X$ is $np$, variance of $X$ is $np(1 - p)$

**Hypothesis test for the mean of a normal distribution**

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

**Percentage points of the normal distribution**

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $p$, the table gives the value of $z$ such that $P(Z \leq z) = p$.

| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

**Kinematics**

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2} a t^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2} a t^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

## Section A: Pure Mathematics

Answer **all** the questions

**1** **(a)** If $|x| = 3$, find the possible values of $|2x - 1|$. **[3]**

**(b)** Find the set of values of $x$ for which $|2x - 1| > x + 1$.
Give your answer in set notation. **[4]**

**2** **(a)** Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,\mathrm{d}x$. **[3]**

**(b)** Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(a)**. **[1]**

**3** **In this question you must show detailed reasoning.**

Given that $5\sin 2x = 3\cos x$, where $0^\circ < x < 90^\circ$, find the exact value of $\sin x$. **[4]**

**4** For a small angle $\theta$, where $\theta$ is in radians, show that $1 + \cos\theta - 3\cos^2\theta \approx -1 + \frac{5}{2}\theta^2$. **[4]**

**5** **(a)** Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of $x$. **[3]**

**(b)** The expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is $1+x-\frac{2}{9}x^2+\ldots$.

Find the possible values of the constants $p$ and $q$. **[5]**

**6** A curve has equation $y=x^2+kx-4x^{-1}$ where $k$ is a constant.

Given that the curve has a minimum point when $x=-2$

- find the value of $k$
- show that the curve has a point of inflection which is not a stationary point. **[7]**

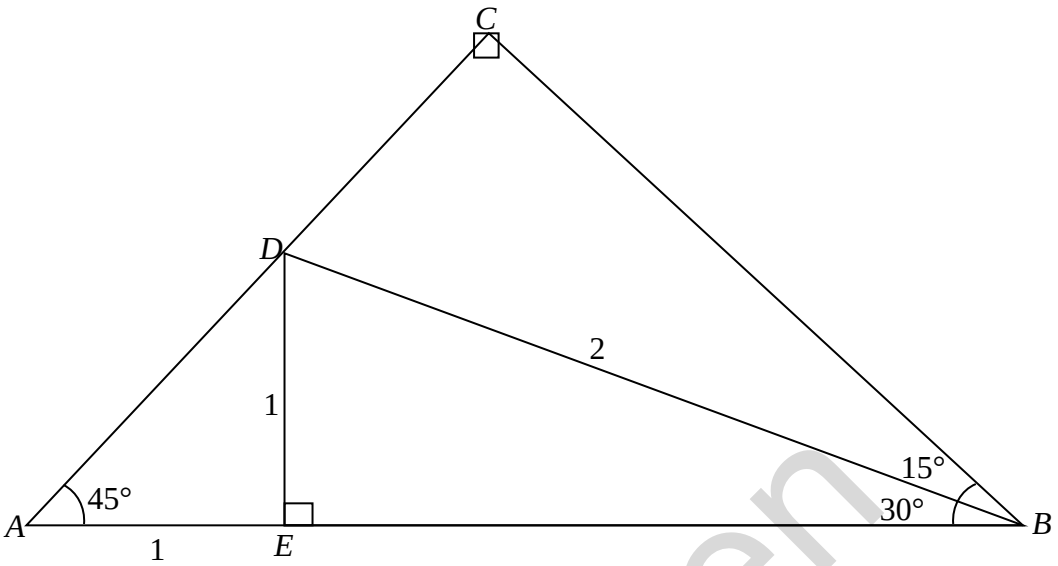
**7** **(a)** Find $\int 5x^3\sqrt{x^2+1}\,\mathrm{d}x$. **[5]**

**(b)** Find $\int \theta\tan^2\theta\,\mathrm{d}\theta$.

You may use the result $\int \tan\theta\,\mathrm{d}\theta=\ln|\sec\theta|+c$. **[5]**

**8 In this question you must show detailed reasoning.**

The diagram shows triangle *ABC*.




The angles *CAB* and *ABC* are each $45^\circ$, and angle $ACB = 90^\circ$.
The points *D* and *E* lie on *AC* and *AB* respectively. $AE = DE = 1$, $DB = 2$.
Angle $BED = 90^\circ$, angle $EBD = 30^\circ$ and angle $DBC = 15^\circ$.

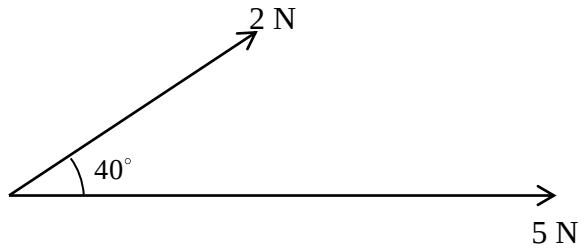
**(a)** Show that $BC = \dfrac{\sqrt{2}+\sqrt{6}}{2}$. **[3]**

**(b)** By considering triangle *BCD*, show that $\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. **[3]**

## Section B: Mechanics

Answer **all** the questions

**9** Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.




**(a)** Calculate the magnitude of the resultant force on the particle. **[3]**

**(b)** Calculate the angle between this resultant force and the force of magnitude 5 N. **[1]**

**10** A body of mass 20 kg is on a rough plane inclined at angle $\alpha$ to the horizontal.
The body is held at rest on the plane by the action of a force of magnitude $P$ N.
The force is acting up the plane in a direction parallel to a line of greatest slope of the plane.
The coefficient of friction between the body and the plane is $\mu$.

**(a)** When $P = 100$, the body is on the point of sliding down the plane.

Show that $g \sin\alpha = g\mu\cos\alpha + 5$. **[4]**

**(b)** When $P$ is increased to 150, the body is on the point of sliding up the plane.

Use this, and your answer to part **(a)**, to find an expression for $\alpha$ in terms of $g$. **[3]**

**11** In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector **r** metres at time *t* seconds is given by $\mathbf{r} = 2t^3\mathbf{i} + \left(5t^2 - 4t\right)\mathbf{j}$.

**(a)** Show that when $t = 0.7$ the bearing on which the particle is moving is approximately $044^\circ$. **[3]**

**(b)** Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**

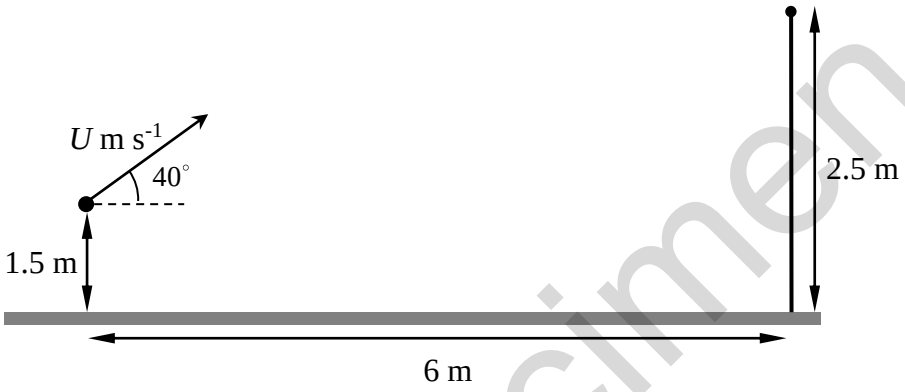
**(c)** Determine the times at which the particle is moving on a bearing of $045^\circ$. **[2]**

**12** A girl is practising netball.
She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop.
The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.

- The initial velocity of the ball has magnitude $U$ m s$^{-1}$.
- The angle of projection is $40^\circ$.
- The ball is modelled as a particle.
- The hoop is modelled as a point.

This is shown on the diagram below.




**(a)** For $U = 10$, find

**(i)** the greatest height above the ground reached by the ball **[5]**

**(ii)** the distance between the ball and the hoop when the ball is vertically above the hoop. **[4]**

**(b)** Calculate the value of $U$ which allows her to hit the hoop. **[3]**

**(c)** How appropriate is this model for predicting the path of the ball when it is thrown by the girl? **[1]**

**(d)** Suggest one improvement that might be made to this model. **[1]**

**13** Particle $A$, of mass $m$ kg, lies on the plane $\Pi_1$ inclined at an angle of $\tan^{-1}\frac{3}{4}$ to the horizontal.

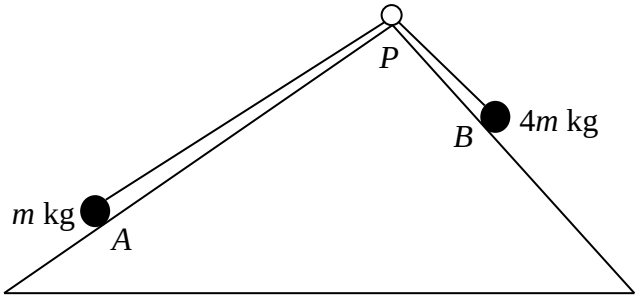
Particle $B$, of $4m$ kg, lies on the plane $\Pi_2$ inclined at an angle of $\tan^{-1}\frac{4}{3}$ to the horizontal.

The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at $P$.

The coefficient of friction between particle $A$ and $\Pi_1$ is $\frac{1}{3}$ and plane $\Pi_2$ is smooth.

Particle $A$ is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.




**(a)** Show that when $A$ is released it accelerates towards the pulley at $\frac{7g}{15}$ m s$^{-2}$. **[6]**

**(b)** Assuming that $A$ does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its speed is $\sqrt{\frac{7g}{30}}$ m s$^{-1}$. **[2]**

**14** A uniform ladder $AB$ of mass 35 kg and length 7 m rests with its end $A$ on rough horizontal ground and its end $B$ against a rough vertical wall.

The ladder is inclined at an angle of $45^\circ$ to the horizontal.

A man of mass 70 kg is standing on the ladder at a point $C$, which is $x$ metres from $A$.

The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$.

The system is in limiting equilibrium.

Find $x$. **[8]**

**END OF QUESTION PAPER**

**BLANK PAGE**

OCR
Oxford Cambridge and RSA

# …day June 20XX – Morning/Afternoon

**A Level Mathematics A**

**H240/03 Pure Mathematics and Mechanics**

**SAMPLE MARK SCHEME**

**Duration:** 2 hours

**MAXIMUM MARK 100**



**This document consists of 20 pages**

## Text Instructions

### 1. Annotations and abbreviations

| Annotation in scoris | Meaning |
|---|---|
| ✓and ✗ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| **Other abbreviations in mark scheme** | **Meaning** |
| E1 | Mark for explaining a result or establishing a given result |
| dep* | Mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This question included the instruction: In this question you must show detailed reasoning. |

## 2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

**M**
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**
Mark for a correct result or statement independent of Method marks.

**E**
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 1 | (a) | | 5<br>Substituting $x=-3$ into $\lvert 2x-1\rvert$<br>7 | **B1**<br>**M1**<br>**A1**<br>**[3]** | **1.1**<br>**1.1a**<br>**1.1** | | |
| 1 | (b) | | $2x-1>x+1$ therefore $x>2$<br>$-(2x-1)>x+1$ (Allow ± in bracket)<br>$x<0$<br>$\{x:x<0\}\cup\{x:x>2\}$ | **B1**<br>**M1**<br>**A1**<br>**A1**<br>**[4]** | **1.1**<br>**3.1a**<br>**1.1**<br>**2.5** | **OR**<br>**B1** for a sketch of $y=\lvert 2x-1\rvert$ and $y=x+1$ on the same axes<br>**M1** attempt to find the points of intersection<br>**A1** obtain $x>2$ and $x<0$<br>**A1** $\{x:x<0\}\cup\{x:x>2\}$ | **OR**<br>**B1** $(2x-1)^2>(x+1)^2$ seen<br>**M1** attempt to multiply out and simplify, then solve quadratic<br>**A1** obtain $x>2$ and $x<0$<br>**A1** $\{x:x<0\}\cup\{x:x>2\}$ |
| 2 | (a) | | $\frac{0.25}{2}\left(1+0.7071+2\left(0.970+0.8944+0.8\right)\right)$<br>0.880 | **B1**<br>**M1**<br>**A1**<br>**[3]** | **1.1**<br>**1.1a**<br>**1.1** | Obtain all five ordinates and no others:<br>0.7071, 0.8944, 1, 0.8, 0.970<br>Use correct structure for trapezium rule with $h=0.25$<br>0.880 or better (0.87953077) | Accept exact values: $1, \frac{4}{\sqrt{17}}, \frac{2}{\sqrt{5}}, \frac{4}{5}, \frac{1}{\sqrt{2}}$<br>$x$-coordinates used **M0**.<br>Omission of large brackets unless implied by correct answer **M0**<br>Accept 0.88 (0.87953077) |
| 2 | (b) | | "Use smaller intervals" or " use more trapezia" | **B1**<br>**[1]** | **2.4** | | |

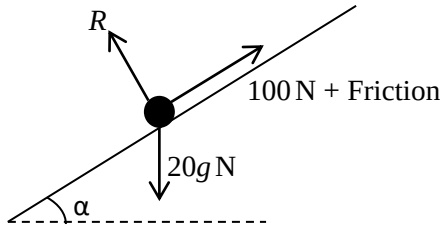
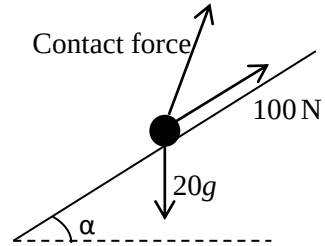
| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 3 | | | **DR**<br>$5\sin 2x = 3\cos x \Rightarrow 10\sin x\cos x = 3\cos x$ | **B1** | **1.1** | Use $\sin 2x = 2\sin x\cos x$ to obtain correct identity | **SC2** For use of identity followed by cancelling $\cos x$, leading to $\sin x = \frac{3}{10}$. |
| | | | $\cos x(10\sin x - 3) = 0$ | **M1** | **1.1a** | Attempt to factorise | |
| | | | $\cos x \neq 0$ for $0^\circ < x < 90^\circ$ | **E1** | **2.1** | | |
| | | | so $\sin x = \frac{3}{10}$ | **A1** | **1.1** | | |
| | | | | **[4]** | | | |
| 4 | | | When $\theta$ is small<br>$1 + \cos\theta - 3\cos^2\theta$<br>$\approx 1 + \left(1 - \frac{1}{2}\theta^2\right) - 3\left(1 - \frac{1}{2}\theta^2\right)^2$ | **M1** | **1.1a** | Attempt to use $\cos\theta \approx 1 - \frac{1}{2}\theta^2$<br>or<br>$= 1 + \left(1 - \frac{1}{2}\theta^2 + ...\right) - 3\left(1 - \frac{1}{2}\theta^2 + ...\right)^2$ | **OR**<br>**M1** Attempt to use $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ |
| | | | $= 1 + \left(1 - \frac{1}{2}\theta^2\right) - 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$<br>$= 1 + 1 - \frac{1}{2}\theta^2 - 3 + 3\theta^2 - \frac{3}{4}\theta^4$ | **M1** | **1.1** | Multiply out | **M1** use trigonometric identity<br>$1 + \cos\theta - 3\cos^2\theta$<br>$= 1 + \cos\theta - \frac{3}{2} - \frac{3}{2}\cos 2\theta$ |
| | | | Since $\theta$ is small, we can neglect the higher order terms | **E1** | **2.5** | For explanation of loss of $\theta^4$ term and consistent use of notation throughout (Working need not be fully correct) | **E1** For showing clearly which identity has been used and consistent use of notation throughout |
| | | | so $1 + \cos\theta - 3\cos^2\theta \approx -1 + \frac{5}{2}\theta^2$ as required | **E1** | **2.1** | AG Clearly obtained www<br>Condone $\theta^4$ term missing without explanation and inconsistent notation | **E1** AG Clearly obtained www<br>Condone inconsistent notation |
| | | | | **[4]** | | | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 5 | (a) | | Obtain $1+\frac{1}{3}px$ | B1 | 1.1 | | |
| | | | $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)(px)^2$ | M1 | 1.1 | | Attempt the $x^2$ term at least in the form ${}^6C_2kx^2$ |
| | | | Obtain $1+\frac{1}{3}px-\frac{1}{9}p^2x^2$ | A1 | 1.1 | Must be simplified | |
| | | | | [3] | | | |
| 5 | (b) | | $(1+qx)\left(1+\frac{1}{3}px-\frac{1}{9}p^2x^2\right)$ | M1 | 3.1a | | Expand $(1+qx)$ and their $1+\frac{1}{3}px-\frac{1}{9}p^2x^2$ and compare coefficients |
| | | | $=1+\left(\frac{1}{3}p+q\right)x+\left(\frac{1}{3}pq-\frac{1}{9}p^2\right)x^2$ | | | | |
| | | | $\frac{1}{3}p+q=1$ (*) | M1 | 3.1a | Obtain two equations in $p$ and $q$ and show evidence of substitution for $p$ or $q$ to obtain an equation in one variable | |
| | | | $\frac{1}{3}pq-\frac{1}{9}p^2=-\frac{2}{9}$ | | | | |
| | | | $2p^2-3p-2=0$ | M1 | 1.1 | Solve a 3 term quadratic equation in a single variable. | Or $18q^2-27q+7=0$ Solve their quadratic |
| | | | $p=2$ or $-\frac{1}{2}$ | A1 | 1.1 | Obtain any two values | |
| | | | $q=\frac{1}{3}$ or $\frac{7}{6}$ | A1FT | 1.1 | Obtain all 4 values , or FT their $p$ and (*) | with indication of correct pairings |
| | | | | [5] | | | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 6 | | | $\frac{dy}{dx} = 2x + k + 4x^{-2}$ | M1 | 1.1a | Attempt to differentiate | Power decreases by 1 for at least 2 terms |
| | | | $2(-2) + k + 4(-2)^{-2} = 0$ | M1 | 3.1a | Substitute $x = -2$, equate to 0 and attempt to solve | |
| | | | $k = 3$ | A1 | 1.1 | | |
| | | | $\frac{d^2y}{dx^2} = 2 - 8x^{-3}$ | | | | |
| | | | $2 - 8x^{-3} = 0$ | M1 | 3.1a | Equate second derivative to 0 and attempt to solve | |
| | | | $x = 4^{\frac{1}{3}}$ | A1 | 1.1 | | |
| | | | for $x < 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} < 0$<br>for $x > 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} > 0$ | E1 | 2.1 | Consider convex/concave either side of $x = 4^{\frac{1}{3}}$ and conclude | |
| | | | When $x = 4^{\frac{1}{3}}, \frac{dy}{dx} \neq 0$ hence not a stationary point | E1 | 2.1 | Consider gradient at $x = 4^{\frac{1}{3}}$, or justify that $x = -2$ is the only stationary point | |
| | | | | [7] | | | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 7 | (a) | | $u = x^2 + 1$<br>$\mathrm{d}u = 2x\mathrm{d}x$ | M1 | 1.1a | Attempt a substitution of $x$ and $\mathrm{d}x$ | **M0** for $\mathrm{d}u = \mathrm{d}x$ |
| | | | $\frac{5}{2}\int (u-1)u^{\frac{1}{2}}\mathrm{d}u$ | M1 | 1.1 | Replace as far as $k\int (u-1)u^{\frac{1}{2}}du$ | |
| | | | $\frac{5}{2}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u$ | A1 | 1.1 | | |
| | | | $u^{\frac{5}{2}} - \frac{5}{3}u^{\frac{3}{2}} + c$ | M1 | 1.1 | Integrate their integral if in $u$ | |
| | | | $\left(x^2+1\right)^{\frac{5}{2}} - \frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}} + c$ | A1 | 1.1 | Do not condone missing $+c$ in both **(a)** and **(b)** | |
| | | | | [5] | | | |
| 7 | (b) | | $\int \tan^2\theta\,\mathrm{d}\theta = \int \left(\sec^2\theta - 1\right)\mathrm{d}\theta$ | M1 | 1.1 | Award for sight of the intermediate result | **OR** **M1** $\int \theta\tan^2\theta\,\mathrm{d}\theta = \int \theta\left(\sec^2\theta - 1\right)\mathrm{d}\theta$ |
| | | | $= \tan\theta - \theta$ | A1 | 1.1 | | **A1** $= \int \theta\,\sec^2\theta\,\mathrm{d}\theta - \int \theta\,\mathrm{d}\theta$ |
| | | | $u = \theta, \mathrm{d}v = \tan^2\theta$ | M1 | 3.1a | Recognise integration by parts with appropriate choice of $u$ and $\mathrm{d}v$ | **M1** $u = \theta,\ dv = \sec^2\theta$ |
| | | | So $\int \theta\tan^2\theta\,\mathrm{d}\theta = \theta(\tan\theta - \theta) - \int (\tan\theta - \theta)\,\mathrm{d}\theta$ | A1 | 1.1 | Obtain correct intermediate result | **A1** So $\int \theta\tan^2\theta\,\mathrm{d}\theta$ $= \theta\tan\theta - \int \tan\theta\,\mathrm{d}\theta - \frac{1}{2}\theta^2$ |
| | | | $-\frac{1}{2}\theta^2 + \theta\tan\theta - \ln\lvert\sec\theta\rvert + c$ | A1 | 1.1 | | **A1** $= -\frac{1}{2}\theta^2 + \theta\tan\theta - \ln\lvert\sec\theta\rvert + c$ |
| | | | | [5] | | | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 8 | (a) | | **DR**<br>$BE = \sqrt{3}$ from the standard triangle $BDE$<br>$BC = AB\cos 45$<br>$BC = \frac{1+\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2}$ | **B1**<br>**M1**<br>**E1**<br>**[3]** | **2.2a**<br>**2.1**<br>**2.2a** | Or $AB = 1+\sqrt{3}$ seen<br>oe or Pythagoras' theorem<br>AG | **B0** for decimal<br>Must be seen<br>$\frac{1+\sqrt{3}}{\sqrt{2}}$ must be seen |
| 8 | (b) | | **DR**<br>Triangle $ABC$ is isosceles so $BC = AC$ but $AC = CD + \sqrt{2}$<br>so $CD = \frac{\sqrt{2}+\sqrt{6}}{2} - \sqrt{2}$<br>$= \frac{\sqrt{6}-\sqrt{2}}{2}$<br>$\sin 15 = \frac{CD}{BD} = \frac{\sqrt{6}-\sqrt{2}}{2} \div 2 = \frac{\sqrt{6}-\sqrt{2}}{4}$ | **B1**<br>**M1**<br>**A1**<br>**[3]** | **2.4**<br>**2.1**<br>**2.2a** | State or imply that $BC = AC$ and state $AC = CD + \sqrt{2}$<br>Obtain expression for $CD$, may be unsimplified<br>Obtain expression for $\sin 15$ and simplify to answer given | <br>**M0** if decimals seen<br>**SC1** for showing using addition formula |
| 9 | (a) | | Attempt resolution of forces<br>Horizontal component $= 5 + 2\cos 40\ (= 6.5321)$<br>Vertical component $= 2\sin 40\ (= 1.2856)$<br>$\sqrt{6.5321^2 + 1.2856^2} = 6.66\,\text{N}$ | **M1**<br>**A1**<br>**A1**<br>**[3]** | **1.1a**<br>**1.1**<br>**1.1** | Allow sin/cos confusion<br>Allow for either the horizontal or vertical component correct<br>Use correct method for magnitude | **OR**<br>**M1** Form triangle of forces<br>**A1** Use cosine rule with $140^\circ$<br>**A1** Obtain $6.66\,\text{N}$ |
| 9 | (b) | | $\tan^{-1}\left(\frac{2\sin 40}{5+2\cos 40}\right) = 11.1^\circ$ | **B1FT**<br>**[1]** | **1.1** | **FT** their components from part (i) | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 10 | (a) | |  | B1 | 2.1 | Any equivalent which makes clear the relationships between:<br><br>Reaction, 100 N force, friction acting upwards, weight of $20g$ N<br><br>A diagram is not *necessary* provided that sufficient explanation is given. | **OR**<br>  |
| | | | Resolve parallel to the slope:<br>$100 + F - 20g\sin\alpha = 0$ (*) | M1 | 3.3 | | |
| | | | Resolve perpendicular to the slope and friction force is maximum:<br>$R = 20g\cos\alpha$ and $F = \mu R$ | M1 | 3.3 | | |
| | | | Substitute and obtain<br>$20g\sin\alpha = 20g\mu\cos\alpha + 100$ | E1<br><br>[4] | 1.1 | AG | |
| 10 | (b) | | All forces shown on diagram of inclined plane | | | Reaction, 150 N force, friction acting downwards, weight of $20g$ N | |
| | | | Resolve parallel to the slope:<br>$150 - F - 20g\sin\alpha = 0$ (**) | B1 | 3.3 | | |
| | | | From * and **<br>$250 - 40g\sin\alpha = 0$ | M1 | 3.4 | Eliminate $\mu$ and attempt to solve for $\alpha$. | One valid step after elimination required |
| | | | $\alpha = \sin^{-1}\frac{25}{4g}$ | A1<br><br>[3] | 1.1 | | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 11 | (a) | | $\mathbf{v} = 6t^2\mathbf{i} + (10t - 4)\mathbf{j}$<br>$\mathbf{v} = 2.94\mathbf{i} + 3\mathbf{j}$<br>$90 - \tan^{-1}\left(\frac{2.94}{3}\right)$<br>$= 044^\circ$ | B1<br><br>M1<br><br>A1<br>[3] | 1.1<br><br>3.1a<br><br>1.1 | At least one term reduces in power by 1<br>Substitution of $t = 0.7$, use $\tan^{-1}\left(\frac{y}{x}\right)$ and obtain $90 - 45.578 = 44.4^\circ$ to give a 3 figure bearing | For a complete method to find a bearing |
| 11 | (b) | | $\mathbf{a} = 12t\mathbf{i} + 10\mathbf{j}$<br>$\mathbf{a} = 8.4\mathbf{i} + 10\mathbf{j}$<br>Use $\mathbf{F} = m\mathbf{a}$ and use Pythagoras<br>Obtain 1.57 N | M1<br>A1<br>M1<br>A1FT<br>[4] | 1.1<br>1.1<br>3.3<br>3.4 | Attempt differentiation of **v**<br>Substitute $t = 0.7$<br><br>FT their **a** at $t$ = 0.7 | |
| 11 | (c) | | $6t^2 = 10t - 4$<br>$6t^2 - 10t + 4 = 0$ so $t = 1$ or $\frac{2}{3}$<br>E.g. **i** component always positive so both values are valid | M1<br><br>E1<br>[2] | 2.2a<br><br>2.3 | Equate **i** and **j** components and solve FT their **v** from part (i) if it leads to a quadratic<br>BC<br>Must include comment on why equating components is sufficient in this case. | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 12 | (a) | (i) | Vertical component of $U = 10\sin 40$ | B1 | 1.1 | | |
| | | | Vertical component of velocity = $10\sin 40 - gt = 0$ | M1 | 3.3 | Use $v = u - gt$ with $v = 0$<br>Allow sign error or sin/cos confusion | |
| | | | Obtain $t = 0.656$ | A1 | 1.1 | | 0.6559057242… |
| | | | Vertical displacement = $10\sin 40t - \frac{1}{2}gt^2\ (+c)$ | M1 | 3.4 | Use $s = ut + \frac{1}{2}gt^2$ or $s = \int v\,dt$ | Allow if initial height not seen<br>**M1** may be awarded if seen in part **(a)(ii)** |
| | | | Obtain 2.11 + 1.5 = 3.61 m | A1FT<br>[5] | 1.1 | FT their "2.11" + 1.5 | 3.608040363… |
| 12 | (a) | (ii) | Horizontal component of $U = 10\cos 40$ | B1 | 1.1 | Use the horizontal component of $U$ | Allow $10\sin 40$ if $10\cos 40$ given in part (i) |
| | | | $6 = 10\cos 40t$ | M1 | 3.3 | Attempt horizontal resolution equated to 6<br>Allow sin/cos error | |
| | | | $t = 0.783$ | A1 | 1.1 | | 0.7832443736… |
| | | | $(2.028586218 + 1.5) - 2.5 = 1.03$ m | A1 | 3.4 | Substitute $t$ in $10\sin 40t - \frac{1}{2}gt^2\ (+1.5)$ and subtract 2.5 | |
| | | | | [4] | | | |
| 12 | (b) | | Use $1 = 6\tan 40 - \frac{(9.8)6^2\sec^2 40}{2U^2}$ | M1 | 3.1b | Use $y = x\tan\theta - \frac{gx^2\sec^2\theta}{2U^2}$ with $x = 6$ and $\theta = 40$ | Allow $y = 2.5$ for **M1** |
| | | | $U^2 = 74.5....$ | M1 | 1.1 | Attempt to make $U$ the subject | **OR** BC |
| | | | Obtain $U = 8.63$ | A1 | 1.1 | BC | 8.631677404… |
| | | | | [3] | | | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 12 | (c) | | E.g. Not very appropriate since it relies on throwing at a very precise angle and velocity.<br>E.g. Not very appropriate since it does not take into account air resistance which will cause the ball to fall short<br>E.g. Not very appropriate since the target she is aiming at is actually a ring, so she has some flexibility | **E1**<br><br>**[1]** | **3.5a** | **E1** for one valid statement | |
| 12 | (d) | | E.g. The ball could not be modelled as a particle so that air resistance is included.<br>E.g. The angle could be a variable.<br>E.g. Angles and velocities could be given as ranges.<br>E.g. The hoop could be modelled as a line of points. | **E1**<br><br>**[1]** | **3.5c** | **E1** for one valid improvement | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 13 | (a) | | Resolving vertically to the plane for Particle A<br>$R = mg\cos\alpha = \frac{4}{5}mg$ | B1 | 1.1 | Obtain $\frac{4}{5}mg$ | |
| | | | Since A is in motion, $F_s = \mu R = \frac{1}{3}\left(\frac{4}{5}\right)mg = \frac{4}{15}mg$ | B1 | 2.2a | Obtain $\frac{4}{15}mg$ | |
| | | | Resolving horizontally to the plane for both particles:<br>$T - \frac{13mg}{15} = ma$<br>$-T + \frac{16mg}{5} = 4ma$ | M1<br>A1 | 3.1b<br>2.1 | Must obtain two equations in $T$ and $a$<br>Particle A:<br>Attempt resolution as far as stating $T - F_s - mg\sin\alpha = ma$<br>Particle B:<br>Attempt resolution as far as stating $-T + 4mg\sin\beta = 4ma$ | |
| | | | $a = \frac{7g}{15}$ | M1<br>E1<br>[6] | 1.1<br>2.4 | Solve their simultaneous equations to find $a$ in terms of $g$.<br>AG Solution must include clear diagrams or explanation for $F_s$ and for horizontal resolutions. | |
| 13 | (b) | | $\frac{7g}{30} = 2\times\frac{7g}{15}\times s$<br>$s = \frac{1}{4}$ | M1<br>E1<br>[2] | 1.1<br>2.1 | Use $v^2 = 0^2 + 2as$<br>AG Must include sufficient working to justify the given answer from the constant acceleration formula | |

| Question | | | Answer | Marks | AO | Guidance | |
|---|---|---|---|---|---|---|---|
| 14 | | | Let $F_G$ be the frictional force at ground level and $R_G$ the reaction<br>Let $F_W$ be the frictional force at the wall and $R_W$ the reaction<br>Let $x$ be the distance the man can ascend before the ladder slips | **B1** | **2.1** | Either on a diagram or in words, B1 is awarded for a clear definition of the force variables used | |
| | | | $F_G = \frac{1}{2}R_G$ and $F_W = \frac{1}{3}R_W$ | **B1** | **3.3** | Both statements required | |
| | | | Resolve horizontally and vertically:<br>$F_G = R_W$<br>$R_G + F_W = 105g$ | **B1** | **3.1b** | Both resolutions required<br>Accept numerical value of $g$ used | |
| | | | | **M1** | **1.1** | Attempt to solve the 4 equations simultaneously to obtain at least two numerical values for the variables.<br>May be implied by later working | |
| | | | $F_W = 15g$<br>$R_W = 45g = F_G$<br>$R_G = 90g$ | **B1** | **3.2a** | **B1** for either $F_W$ and $R_W$ or $F_G$ and $R_G$ | |
| | | | Moments about the foot of the ladder:<br>$35g(3.5\cos 45) + (70g\cos 45)x = 45g(7\cos 45)$<br>$+15g(7\sin 45)$ | **M1**<br>**A1** | **3.3**<br>**3.4** | Allow sign errors and sin/cos confusion<br>Correct statement | Or similarly about the top of the ladder |
| | | | $x = 4.25$ | **A1**<br>**[8]** | **1.1** | cao | |

**Assessment Objectives (AO) Grid**

| Question | AO1 | AO2 | AO3 (PS) | AO3 (M) | Total |
|---|---|---|---|---|---|
| **1a** | 3 | | | | **3** |
| **1b** | 2 | 1 | 1 | | **4** |
| **2a** | 3 | | | | **3** |
| **2b** | | 1 | | | **1** |
| **3** | 3 | 1 | | | **4** |
| **4** | 2 | 2 | | | **4** |
| **5a** | 3 | | | | **3** |
| **5b** | 3 | | 2 | | **5** |
| **6** | 3 | 2 | 2 | | **7** |
| **7a** | 3 | | | | **5** |
| **7b** | 5 | | 1 | | **5** |
| **8a** | 4 | 3 | | | **3** |
| **8b** | | 3 | | | **3** |
| **9a** | 3 | | | | **3** |
| **9b** | 1 | | | | **1** |
| **10a** | 1 | 1 | | 2 | **4** |
| **10b** | 1 | | | 2 | **3** |
| **11a** | 2 | | 1 | | **3** |
| **11b** | 2 | | | 2 | **4** |
| **11c** | | 2 | | | **2** |
| **12ai** | 3 | | | 2 | **5** |
| **12aii** | 2 | | | 2 | **4** |
| **12b** | 2 | | 1 | | **3** |
| **12c** | | | | 1 | **1** |
| **12d** | | | | 1 | **1** |
| **13a** | 2 | 3 | 1 | | **6** |
| **13b** | 1 | 1 | | | **2** |
| **14** | 2 | 1 | 2 | 3 | **8** |
| **Totals** | **53** | **21** | **11** | **15** | **100** |



PS = Problem Solving
M = Modelling

**BLANK PAGE**

**BLANK PAGE**

# Summary of Updates

| Date | Version | Change |
|---|---|---|
| October 2018 | 2 | We've reviewed the look and feel of our papers through text, tone, language, images and formatting. For more information please see our assessment principles in our "Exploring our question papers" brochures on our website. |

OCR
Oxford Cambridge and RSA

Accredited

# A Level Mathematics A

**H240/03** Pure Mathematics and Mechanics
Printed Answer Booklet

# Date – Morning/Afternoon

## Time allowed: 2 hours

**You must have:**
- Question Paper H240/03 (inserted)

**You may use:**
- a scientific or graphical calculator



| | |
|---|---|
| **First name** | |
| **Last name** | |
| **Centre number** | |
| **Candidate number** | |

**INSTRUCTIONS**
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION**
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

## Section A: Pure Mathematics

**1(a)**

**1(b)**

**2(a)**

**2(b)**

**3**

| | |
|---|---|
| 4 | |



**DO NOT WRITE IN THIS SPACE**

**5(a)**

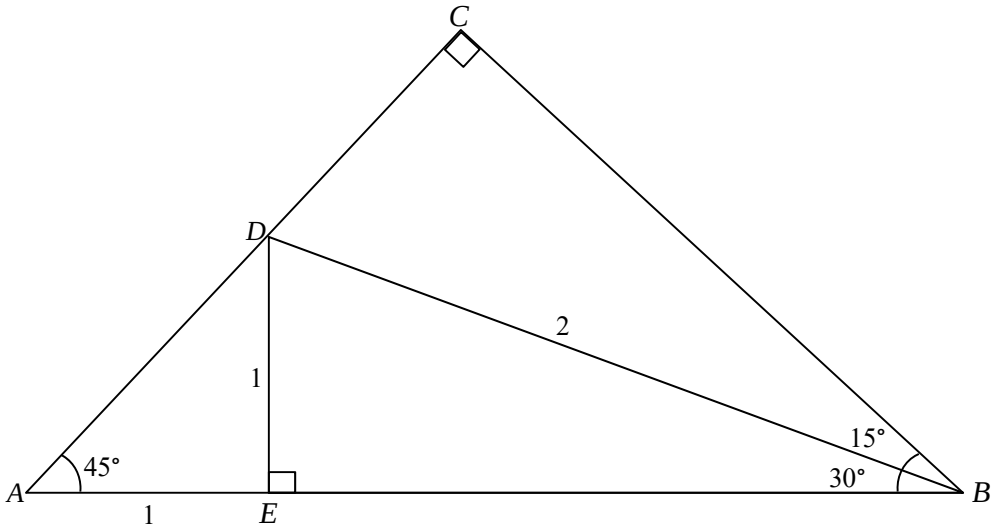
**5(b)**

6

**7(a)**

**7(b)**

**8(a)**




**8(b)**

**Section B: Mechanics**

| | |
|---|---|
| **9(a)** | |
| **9(b)** | |



**DO NOT WRITE IN THIS SPACE**

**10(a)**

**10(b)**

**11(a)**

**11(b)**

**11(c)**

| | |
|---|---|
| **12(a)(i)** | |

| | |
|---|---|
| **12(a)(ii)** | |
| **12(b)** | |
| **12(c)** | |
| **12(d)** | |

**13(a)**

**13(b)**

14

**DO NOT WRITE ON THIS PAGE**