## Potential Energy Curves

Q 1. If the potential energy $(U)$ of a particle is given by $U=1 / 2 k x^{2}$, where $k$ is the spring constant and x is the displacement from equilibrium, what does the force ( F ) acting on the particle look like?
A) $F=k x$
B) $F=-k x$
C) $F=k / x$
D) $F=-k / x$

Q 2. The potential energy curve for a particle is given by $U=m g x$, where $m$ is the mass, $g$ is the gravitational acceleration, and x is the height above the Earth. What is the force acting on the particle due to gravity?
A) $F=m g$
B) $F=-m g$
C) $F=g / x$
D) $F=-g / x$

Q3. In a potential energy graph, what does the point where the slope ( $\mathrm{dU} / \mathrm{dx}$ ) is zero indicate about the force acting on the particle at that point?
A) The force is maximum.
B) The force is zero.
C) The force is increasing.
D) The force is negative.

Q4. If a particle is at a position where the kinetic energy ( $K$ ) is zero, and the potential energy $(\mathrm{U})$ is equal to the total mechanical energy $(\mathrm{E})$, what is significant about this point?
A) It is a point of unstable equilibrium.
B) It is a turning point where the particle changes direction.
C) It indicates maximum potential energy.
D) It is a point of maximum force.

Q5. A particle moves in a potential energy landscape described by $\mathrm{U}(\mathrm{x})$. If the particle is at a point where the potential energy is a local minimum, what can be said about the equilibrium at this point?
A) It is stable equilibrium.
B) It is unstable equilibrium.
C) It is neutral equilibrium.
D) It is non-equilibrium.

Q6. A particle's potential energy is described by $\mathrm{U}=-\mathrm{ax}+\mathrm{bx}$, where a and b are constants. At what value of $x$ is the force on the particle zero?
A) $x=b / 2 a$
B) $x=-b / a$
C) $x=b / a$
D) $x=0$

Q7. Considering the potential energy function $\mathrm{U}=\sin (\mathrm{x})$, where x is in radians, what is the force on the particle at $x=\pi / 2$ radians?
A) $F=0$
B) $F=1$
C) $F=-1$
D) $\mathrm{F}=\cos (\pi / 2)$

Q8. A particle's potential energy is described by $\mathrm{U}=\mathrm{A}(\mathrm{x}-\mathrm{h})^{2}$, where A and h are constants. If $\mathrm{h}=3$ and $A>0$, what is the nature of the force experienced by the particle at $x=3$ ?
A) The force is zero.
B) The force is maximum.
C) The force is positive.
D) The force is negative.

Q9. Which of the following statements is true when a particle moves from a region of high potential energy to a region of low potential energy under the influence of a conservative force?
A) The force does positive work and increases the kinetic energy of the particle.
B) The force does negative work and decreases the kinetic energy of the particle.
C) The force does positive work and decreases the potential energy of the particle.
D) The force does negative work and increases the potential energy of the particle.

Q 10. If a potential energy function $U(x)$ has a sharp peak at $x=0$, what does this imply about the forces acting on a particle near $\mathrm{x}=0$ ?
A) The forces are zero near $\mathrm{x}=0$.
B) The forces are large and positive near $x=0$.
C) The forces are large and negative near $x=0$.
D) The forces change direction at $x=0$.

## Answers UnCubed

A1: B) $F=-k x$
The derivative of $U=1 / 2 k x^{2}$ with respect to $x$ is $d U / d x=k x$. According to the relation $F=-$ $\mathrm{dU} / \mathrm{dx}$, the force is $\mathrm{F}=-\mathrm{kx}$, which aligns with Hooke's Law, indicating the force exerted by a spring.

A2: B) $F=-m g$
Differentiating $U=m g x$ with respect to $x$ gives $d U / d x=m g$. Since $F=-d U / d x$, the force is $F=$ -mg , which is the gravitational force acting upward against the direction of increasing $x$ (height).

A3: B) The force is zero.
At points where the slope of the potential energy curve, $\mathrm{dU} / \mathrm{dx}$, is zero, the force, given by F $=-d U / d x$, is also zero. This indicates a point where no net force acts on the particle, potentially an equilibrium point.

A4: B) It is a turning point where the particle changes direction.
At points where $\mathrm{K}=0$ and $\mathrm{E}=\mathrm{U}$, the particle has zero kinetic energy, meaning it stops momentarily. Such points are turning points where the direction of motion reverses.

A5: A) It is stable equilibrium.
At a local minimum of potential energy, any small displacement leads to a restoring force that pushes the particle back toward the minimum, characterizing stable equilibrium.

A6: C) $x=b / a$
To find where the force is zero, differentiate $U=-a x^{2}+b x$ to get $d U / d x=-2 a x+b$. Set $d U / d x$ $=0$ for the force to be zero:
$-2 a x+b=0$
$x=b /(2 a)$
A7: A) $F=0$
The force F is given by $\mathrm{F}=-\mathrm{dU} / \mathrm{dx}$. Differentiating $\mathrm{U}=\sin (\mathrm{x})$, we get $\mathrm{dU} / \mathrm{dx}=\cos (\mathrm{x})$. At $\mathrm{x}=$ $\pi / 2, \cos (\pi / 2)=0$. Thus, $F=0$

A8: C) The force is zero.
For the potential energy function $\mathrm{U}=\mathrm{A}(\mathrm{x}-\mathrm{h})^{2}$, differentiate to find the force:
$d U / d x=2 A(x-h)$
At $x=3$ (and given $h=3$ ), $d U / d x=2 A(3-3)=0$.
Thus, the force $F=-d U / d x=0$. This indicates that at $x=3$, the particle experiences no force, as it is at an equilibrium position.

A9: A) The force does positive work and increases the kinetic energy of the particle.
When a particle moves to a lower potential energy under a conservative force, the potential energy decrease translates into an increase in kinetic energy, meaning the work done by the force is positive.

A10: D) The forces change direction at $\mathrm{x}=0$.
A sharp peak in the potential energy function implies a rapid change in the slope of $U(x)$ at $x$ $=0$. This rapid change in slope indicates that the direction and magnitude of the force ( $\mathrm{F}=-$ $\mathrm{dU} / \mathrm{dx}$ ) also change significantly at this point.

