

Quiz1 - Complex Numbers

[total marks on quiz: 40 marks]

No GDC allowed on the quiz.

1. Given that $\frac{2}{x+iy} + \frac{1}{1-2i} = \frac{2}{5} + i$ where x and y are real, find the value of x and the value of y .

[6 marks]

2. Find the three cube roots $-2+2i$ and express them in exponential form, $re^{i\theta}$.

[9 marks]

3. Consider the following two complex numbers

$$z = \frac{3+3i}{1-i} \quad \text{and} \quad w = \frac{4}{1+i\sqrt{3}}$$

- (a) Write each in modulus-argument form, $r \operatorname{cis} \theta$.

[6 marks]

- (b) Hence, find simplified expressions for zw and $\frac{z}{w}$ in modulus-argument form, $r \operatorname{cis} \theta$.

[6 marks]

4. (a) Find all roots for the equation $x^4 + 16 = 0$ given that $x \in \mathbb{C}$.

[8 marks]

- (b) Hence, express $x^4 + 16$ as the product of two quadratic polynomials with real coefficients.

[5 marks]

Bonus: Show that $\frac{\cos 2\theta + i \sin 2\theta}{\cos 3\theta + i \sin 3\theta} = \cos \theta - i \sin \theta$

[+4 marks]