You must write all of the detail of the proof to get full marks!

Find a counter-example to disprove each of the following:
(a) \( \sqrt{x^2 + y^2} < x + y \)  
(c) If \( x, y \) are consecutive odd numbers then one of \( x \) or \( y \) must be prime.

Find numbers which satisfy the conditions stated, and show the statement is false.

Then...

Explain in detail that your numbers satisfy the stated conditions and show the statement is false.

Therefore...

Conclude that the statement is not true.

Don't be put off by having to try quite a few numbers. 
PTO for a list of tricky counter examples &c.

A means \( x \in \mathbb{R} \) 
\( x \in \mathbb{Z} \) 
\( \text{prime} \) means
**PROOF BY EXHAUSTION**

**PROOF**

Prove that 343 is the only pandigital 3-digit cube number.

- $4^3 = 64$
- $5^3 = 125$
- $6^3 = 216$
- $7^3 = 343$
- $8^3 = 512$
- $9^3 = 729$
- $10^3 = 1000$

Prove that for any integer $n$, $n^2 - n - 1$ is always odd.

- If $n$ is odd, then $n^2$ is odd (since the square of an odd number is odd).
  - $n^2$ is odd
  - $n^2 - n - 1$ is even

- If $n$ is even, then $n^2$ is even (since the square of an even number is even).
  - $n^2$ is even
  - $n^2 - n - 1$ is odd


**STRUCTURE OF PROOF**

List all the possibilities/possible cases.

Explain how you know that your list covers all possibilities.

Test each possibility in turn.

Conclude that the statement is true.
**PROOF**

**EVEN/ODD & CONSECUTIVE
NUMBER PROOFS**

**Odd Number**

\[ 2k \pm 1 \quad k \in \mathbb{Z} \]

**Even Number**

\[ 2k \quad k \in \mathbb{Z} \]

**Consecutive Integers**

\[ n-1, n, n+1 \]
\[ \text{or} \]
\[ n, n+1, n+2 \]
\[ \text{or} \]
\[ 2n, 2n+1, 2n+2 \]

**Consecutive Odd Numbers**

\[ 2k+1, 2k+3 \quad k \in \mathbb{Z} \]

**Consecutive Even Numbers**

\[ 2k, 2k+2 \quad k \in \mathbb{Z} \]

**Proof that the product of 2 odd numbers is odd**

**Proof that the sum of the squares of 2 consecutive odd positive integers is always even and positive.**
RATIONAL NUMBERS

A number is rational if (and only if) it can be expressed as a quotient of two integers.

\[
\frac{m}{n}, \quad m, n \in \mathbb{Z}
\]

IRRATIONAL NUMBERS

Any real number which cannot be expressed as a quotient of two integers is irrational.

(e.g., \( \pi, \sqrt{2} \))