Chapter 5. Applications of Integration

5.1 Areas Between Curves

(1) Integrating with respect to x

$$Area = Base \times height$$

① Area when $f(x) \geq g(x)$

The area A of the region S bounded by $y = f(x)$, $y = g(x)$ on $[a\,,b]$, where $f(x)$ and $g(x)$ are continuous and $f(x) \geq g(x)$ for all x in $[a\,,b]$

$$S = \lim_{n\to\infty} \sum_{i=1}^{n} \{f(x_i) - g(x_i)\}\Delta x = \int_a^b [f(x) - g(x)]dx$$

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$ex)$ $Find\ the\ area\ of\ the\ region\ bounded\ above\ y = x^2 + 1\ ,bounded\ below$
$y = x\ and\ bounded\ on\ the\ sides\ by\ x = 0\ and\ x = 1$

$ex)$ $Find\ the\ area\ enclosed\ by\ y = x^2\ \ and\ y = 2x - x^2$

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$ex)\ Find\ Area\ of\ the\ regions\ bounded\ by\ curves\ y = \frac{x}{\sqrt{x^2+1}}\ \ and\ y = x^4 - x$

② General Area

The Area between the curves $y = f(x)\ \ and\ y = g(x)$ and between $x = a\ and\ x = b$ is

$$A = \int_a^b |f(x) - g(x)|dx$$

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$ex)$ *Find the area bounded by* $y = \sin x\ , y = \cos x\ , x = 0\ , x = \frac{\pi}{2}$

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(2) Integrating with respect to y

The Area between the curves $x = f(y)\ \ and\ x = g(y)$ and between $y = c\ and\ y = d$ is

$$A = \int_c^d |f(y) - g(y)| dy$$

$ex)$ $Find\ the\ Area\ enclosed\ by\ \ y = x - 1\ , y^2 = 2x + 6$

$ex)$ $Find\ the\ Area\ of\ the\ region\ enclosed\ by\ x + 2y = 3\ , y = x\ and\ y = \frac{1}{4}x\ (x \geq 0)$

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