Sum & Product of the Roots of a Polynomial Equation

If the **quadratic equation** $ax^2+bx+c=0,\ a\neq 0$ has roots α and β, then

the **sum of the roots,** $\alpha+\beta=-\frac{b}{a}$ and the **product of the roots**, $\alpha\beta=\frac{c}{a}$

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sum & product of the roots of any **polynnomial equation**

For the polynomial equation of degree n given by $P(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0=0,\ a_n\neq 0$

the **sum of the roots** is $-\frac{a_{n-1}}{a_n}$ and the **product of the roots** is $\frac{(-1)^n a_0}{a_n}$

Exercises – **No** calculator on all questions [**worked solutions** included]

1. The equation $x^2-5x-2=0$ has roots α and β.

(a) Write down the value $\alpha+\beta$ and the value of $\alpha\beta$.

(b) Find the value of $\alpha^2\beta+\alpha\beta^2$.

(c) Find a quadratic equation which has roots $\alpha^2\beta$ and $\alpha\beta^2$.

2. If α and β are the roots of the equation $2x^2+3x-7=0$ has roots, find the quadratic equation with integral coefficients whose roots are:

(a) $2\alpha,\ 2\beta$ (b) $\frac{2}{\alpha},\ \frac{2}{\beta}$

3. Consider the polynomial $f(x)=2x^3+3x^2-6x-18,\ x\in\mathbb{R}$.

(a) For the polynomial equation $f(x)=0$, state

(i) the sum of the roots;

(ii) the product of the roots.

A new polynomial equation is defined to be $g(x)=f(x-5)$

(b) Find the sum of the roots of the equation $g(x)=0$.

4. Consider the equation $2x^4-13x^3+27x^2-13x-15=0$. Given that one of the zeros of the equation is $x_1=2-\mathrm{i}$, find the other three zeros x_2, x_3 and x_4.