ROTATIONAL DYNAMICS

Rotational Dynamics

1. If an object or system is not rotating, which of the following must be true?
   - A The object or system has a high rotational inertia
   - B There is no torque acting on the object or system
   - C There are an even number of torques acting on the object or system
   - D The net torque on the object or system is zero

2. What is the SI unit for rotational inertia?
   - A kg
   - B $kg \cdot m^2$
   - C kg/m
   - D kg·m

3. A pulley is spinning about an axle through its center with a constant angular velocity. Which of the following is required for the pulley to continue spinning with the same angular velocity?
   - A A single torque acting in the same direction as the angular velocity
   - B A net torque acting in the opposite direction as the angular velocity
   - C A net torque acting in the same direction as the angular velocity
   - D None of the above

4. The rotational inertia of an object is...
   - A the mass of the object
   - B the resistance of the object to a linear acceleration
   - C the resistance of the object to an angular acceleration
   - D the radius of the object

5. A person pushes on the handle of a door, producing a torque with a magnitude of 5 N·m about the door hinge. A second person pushes on the handle on the opposite side of the door, producing a torque with a magnitude of 8 N·m about the door hinge. What is the magnitude of the net torque about the door hinge?
   - A 13 N·m
   - B 3 N·m
   - C 8 N·m
   - D 5 N·m

6. A person places a spinning top on a table and gives it an initial angular velocity. After some time the top slows down and eventually stops spinning. Which of the following explains why the top slows down and stops?
   - A There is a net torque acting on the top in the opposite direction as its angular velocity
   - B There is no net torque acting on the top to keep it spinning
   - C All rotating object tend to slow down and stop on their own
   - D None of the above

7. If the rotational inertia of an object remains the same but the net torque acting on the object decreases…
   - A the angular speed of the object or system decreases
   - B the angular acceleration of the object or system increases
   - C the angular speed of the object or system increases
   - D the angular acceleration of the object or system decreases

8. If an object or system is in rotational equilibrium, which of the following is true? (Select all that apply)
   - A There are no torques acting on the object or system
   - B The angular acceleration of the object or system is zero
   - C The net torque on the object or system is zero
   - D There are no forces acting on the object or system

9. A net torque is applied to wheel A which gives it an angular acceleration α_A. If the same net torque is applied to wheel B which has a rotational inertia that is three times the rotational inertia of wheel A, what is the angular acceleration of wheel B in terms of α_A?
   - A $\alpha_A /3$
   - B α_A
   - C $3\alpha_A$
   - D $9\alpha_A$

10. 4 blocks are placed on a beam resting on a pivot point. If the system of the blocks and the beam is balanced on the pivot point, which of the following is true? (Select all that apply)
   - A There must be 2 blocks on each side of the pivot point
   - B The angular acceleration of the system is zero
   - C The system is in rotational equilibrium
   - D The sum of the torques produced by the blocks and the beam around the pivot point is zero

11. A tennis ball can be modeled as a spherical shell with a mass of m and a radius of R. If a net torque of τ is applied to a tennis ball, what is its angular acceleration?
   - A $\frac{\tau}{m}$
   - B $\frac{3\tau}{2mR^2}$
   - C $\frac{\tau}{mR^2}$
   - D $\frac{2\tau}{3mR^2}$

12. A system consists of several blocks on a long beam, which is resting on a pivot point. Which of the following would increase the system's rotational inertia about the pivot point? (Select all that apply)
   - A Move one of the blocks farther from the pivot point
   - B Increase the mass of one of the blocks
   - C Move one of the blocks closer to the pivot point
   - D Increase the mass of the beam

13. Two blocks are placed on a beam with negligible mass which is resting on a pivot point in rotational equilibrium. Which of the following must be true? (Select all that apply)

A If one block has more mass it must be farther from the pivot point than the block with less mass

B The blocks have the same mass

C If one block has more mass it must be closer to the pivot point than the block with less mass

D The blocks are the same distance from the pivot point

14. Two small spheres are connected by a rod with a length of L and negligible mass which rotates about its center. One sphere has a mass of m_1 and the other sphere has a mass of m_2. What is the rotational inertia of the system of the two spheres and the rod?

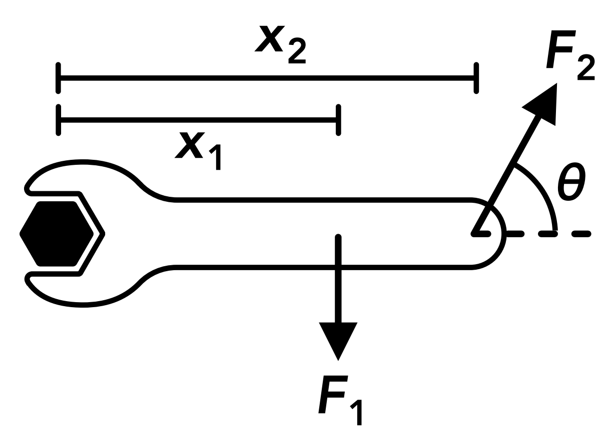
A $m_1 + m_2$

B $m_1L^2 + m_2L^2$

C $\frac{m_1L^2}{2} + \frac{m_2L^2}{2}$

D $\frac{m_1L^2}{4} + \frac{m_2L^2}{4}$

15. Two forces are exerted on a wrench as shown on the right. Which of the following represents the net torque on the wrench about the center axis of the bolt on the left?



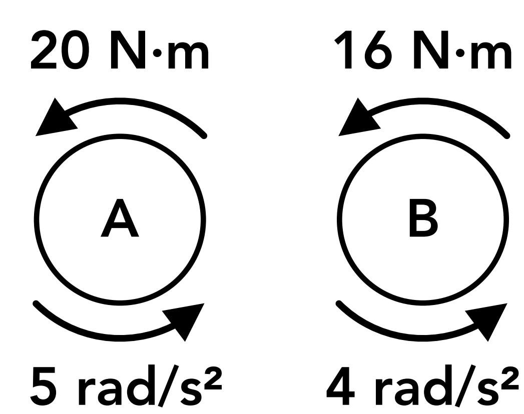
A $x_2F_2\sin(\theta) - x_1F_1$

B $x_2F_2 - x_1F_1$

C $x_2F_2\cos(\theta) - x_1F_1$

D $x_2F_2\sin(\theta) + x_1F_1$

16. The net torques and angular accelerations for two objects are shown on the right. How do the rotational inertias of each object compare?



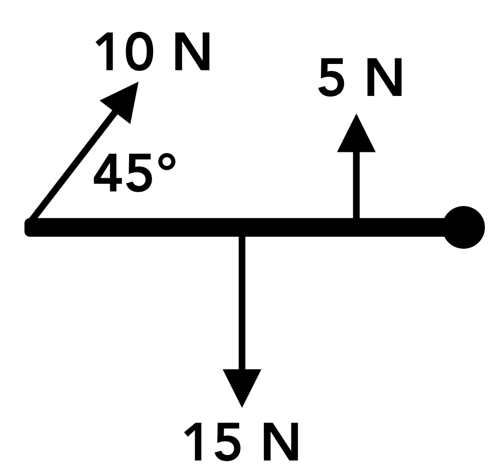
A $I_A < I_B$

B $I_A = I_B$

C $I_A > I_B$

D Cannot be determined

17. Three forces are exerted on a door as shown on the right which can rotate about its hinge on the right. The 5 N force is 10 cm away from the hinge, the 15 N force is 20 cm away from the hinge, and the 10 N force is 40 cm away from the hinge. Which direction will the door accelerate?



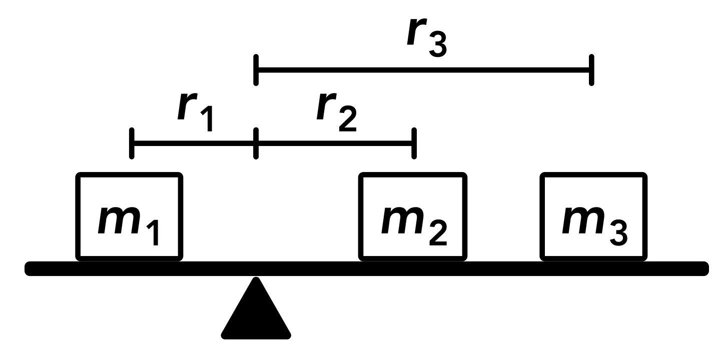
A Clockwise

B Counterclockwise

C The door does not accelerate

D Cannot be determined

18. Three blocks are on a beam with negligible mass which is balanced on a pivot point as shown on the right. What is the mass m_3 in terms of the other variables?



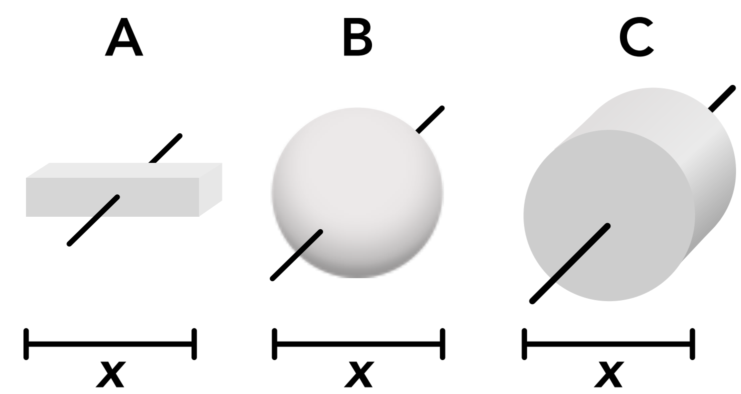
A $\frac{-r_1 m_1 - r_2 m_2}{r_3}$

B $m_1 - m_2$

C $\frac{r_1 m_1 - r_2 m_2}{r_3}$

D $m_1 + m_2$

19. Three solid objects are shown on the right. The objects have the same mass and the same width (or diameter), and each one rotates about the line passing through its center. Rank the objects in terms of their rotational inertias.



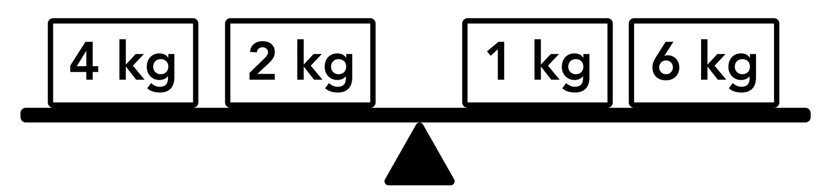
A A < C < B

B B < C < A

C B < A < C

D A < B < C

20. Four blocks are placed on a beam with negligible mass which is on a pivot point as shown on the right. The 4 kg block is centered 50 cm from the pivot, the 2 kg block is centered 30 cm from the pivot, the 1 kg block is centered 20 cm from the pivot, and the 6 kg block is centered 40 cm from the pivot (the figure is not drawn to scale). Which way will the beam rotate when released from rest?



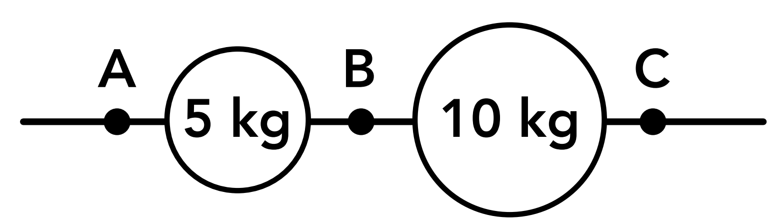
A Clockwise

B Counterclockwise

C It will not rotate

D Cannot be determined

21. Two spheres are connected by a rod with negligible mass as shown on the right. If the system of the spheres and rod were to rotate about any of the labeled points, which point of rotation would result in the maximum rotational inertia?



A Point A

B Point B

C Point C

D The rotational inertia is the same for each point

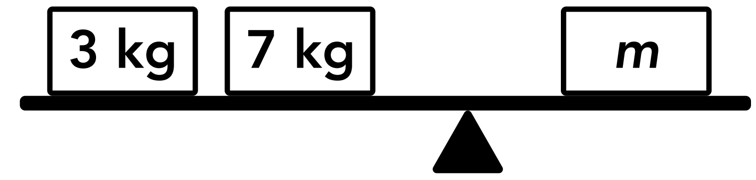
22. A wheel with a rotational inertia of 50 $kg \cdot m^2$ is experiencing an angular acceleration of 4 rad/s^2. What is the net torque acting on the wheel?

23. Two forces are applied perpendicularly to a door. A 20 N force is applied 60 cm from the hinge which would rotate the door clockwise on its own, and a 50 N force is applied 70 cm from the hinge which would rotate the door counterclockwise on its own. If the door experiences an angular acceleration of 2 rad/s^2 due to the two forces, what is the rotational inertia of the door?

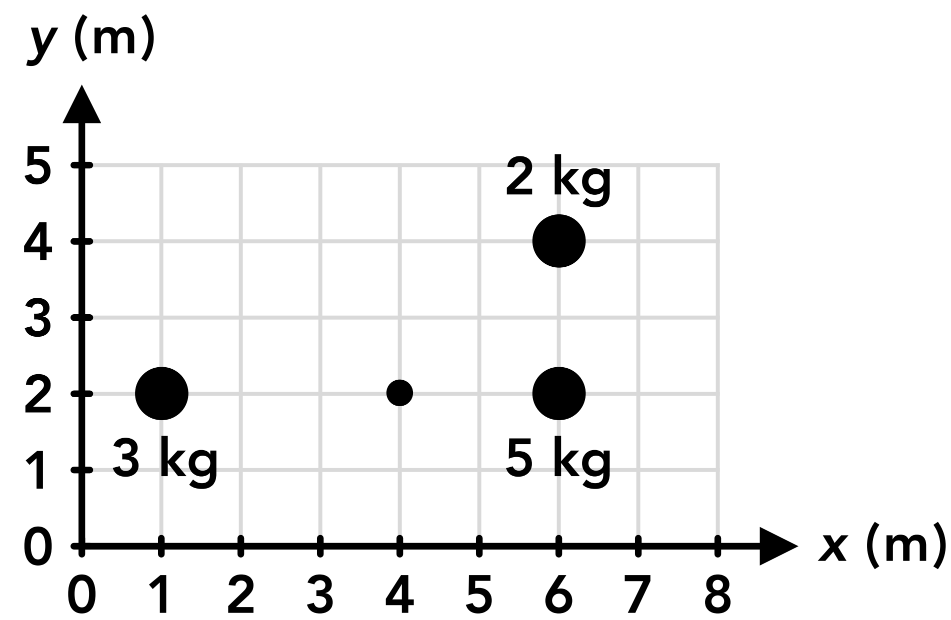
24. A person is trying to loosen a stuck bolt with a wrench. They apply a 30 N force at an angle of 50° to the wrench handle, 20 cm from the center of the bolt. If the bolt remains stuck and the wrench does not rotate, what must be the torque exerted on the bolt due to the friction forces between the bolt and the material it's contacting?

25. A person pushes on a door with a force of 25 N at an angle of 70° to the door, 0.8 m from the door hinge. Assume the door can be modeled as a solid rod with a mass of 40 kg and a width of 1 m. If no other forces are acting on the door which affect its rotation, what will be the angular acceleration of the door?

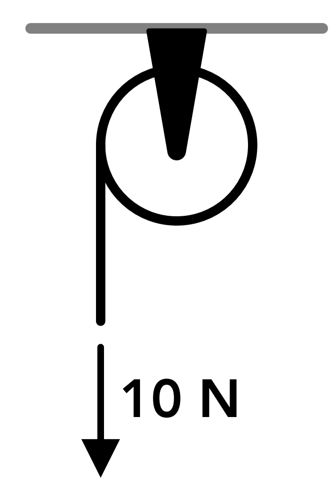
26. Three blocks are sitting on a beam with negligible mass as shown on the right. The 3 kg block is centered 80 cm from the pivot, the 7 kg block is centered 30 cm from the pivot, and the block with mass ***m*** is centered 40 cm from the pivot (the figure is not drawn to scale). If the beam is balanced, what is the mass ***m***?



27. Three spherical masses are at the positions shown on the right. If the system of three masses were to rotate about the point shown at coordinates (4, 2) and we treat the spheres as point masses with negligible diameters, what is the rotational inertia of the system?



28. A rope is wrapped around the outside of a pulley with a radius of 30 cm and a mass of 12 kg. A constant 10 N force is applied to the rope as shown on the right. Assume the pulley can be modeled as a solid cylinder and the pulley is frictionless. If the pulley starts at rest, what is the angular speed of the pulley after 3 seconds?



Answers

1. D	11. B	21. A
2. B	12. A, C, D	22. 200 N·m
3. D	13. C	23. 11.5 kg·m²
4. C	14. D	24. 4.6 N·m
5. B	15. A	25. 1.4 rad/s²
6. A	16. B	26. 11.25 kg
7. D	17. A	27. 63 kg·m²
8. B, C	18. C	28. 16.7 rad/s
9. A	19. D	
10. B, C, D	20. C	

Answers - Rotational Dynamics

1. **Answer: D**
   If we apply Newton's 1st law to rotational motion, an object that is not rotating (it's angular velocity is zero) will remain at rest unless there is a net torque acting on the object (the total sum of the torques is not zero). There may be torques acting on the object but they "cancel out" in opposite directions.

2. **Answer: B**
   The SI unit for rotational inertia or moment of inertia ***I*** is kg·m².

3. **Answer: D**
   If we apply Newton's 1st law to rotational motion, an object will maintain its angular velocity unless there is a net torque acting on it. No torque is required for the pulley to continue spinning at the same angular velocity.

4. **Answer: C**
   An object's rotational inertia or moment of inertia is how much an object resists an angular acceleration, and it depends on how the mass of the object is distributed from its axis of rotation.

5. **Answer: B**
   The two people are both pushing on the door from opposite sides so the forces applied to the door and the torques produced act in opposite directions. One torque is positive and the other torque is negative so the net torque is the difference between the two torques, either positive or negative 3 N·m. The question asks for the magnitude of the net torque which is the absolute value of the net torque (positive).

6. **Answer: A**
   If we apply Newton's laws of motion to the rotational motion of the top, the top experiences an angular acceleration (a change in angular velocity, from some initial velocity to zero velocity). The angular acceleration is caused by a net torque acting on the top. The torques acting on the top could be from a friction force from the table or from the drag force (air resistance) from the air surrounding the top.

7. **Answer: D**
   From Newton's 2nd law applied to rotational motion, the net torque on an object is equal to the rotational inertia times the angular acceleration, $\tau_{net} = I\alpha$. If the rotational inertia is the same and the net torque decreases, the angular acceleration must also decrease.

8. **Answer: B, C**
   An object or system is in rotational equilibrium if the net torque and the angular acceleration are zero.

9. **Answer: A**
The net torque is equal to the rotational inertia multiplied by the angular acceleration, $\tau_{net} = I\alpha$. If the net torque is the same and the rotational inertia is multiplied by 3, the angular acceleration must be divided by 3.

10. **Answer: B, C, D**
If the system is balanced (it's not rotating) then it's in rotational equilibrium and the net torque and the angular acceleration about the pivot point are zero. There does not have to be 2 blocks on each side for it to be balanced.

11. **Answer: B**
The net torque is equal to the rotational inertia times the angular acceleration, $\tau_{net} = I\alpha$. The rotational inertia for a spherical shell is $\frac{2}{3}mR^2$.

$$\tau_{net} = I\alpha \qquad \alpha = \frac{\tau}{I} = \frac{\tau}{2/3\,mR^2} = \frac{3\tau}{2mR^2}$$

12. **Answer: A, C, D**
A system's rotational inertia depends on the masses in the system and how far that mass is distributed from the axis of rotation (the pivot point in this example). Increasing the mass of any object in the system and moving any object farther from the axis of rotation will increase the system's rotational inertia.

13. **Answer: C**
If the system is in rotational equilibrium, the net torque about the pivot point is zero so the two blocks must produce torques with equal magnitudes and opposite directions. The torque produced by each block is equal to its distance from the pivot point and the perpendicular force it exerts on the beam which is its weight force, $\tau = rF_{\perp} = rmg$. The blocks can have different masses and be different distances from the pivot point, but if one block has more mass it must be closer to the pivot point to produce a torque with the same magnitude as the block with less mass that is farther from the pivot point.

14. **Answer: D**
The system can be treated as a two point masses which are each a distance of $L/2$ from the axis of rotation.

$$I = m_1r_1^2 + m_2r_2^2 = m_1\left(\frac{L}{2}\right)^2 + m_2\left(\frac{L}{2}\right)^2 = \frac{m_1L^2}{4} + \frac{m_2L^2}{4}$$

15. **Answer: A**
The net torque is the sum of the torques produced by each force, and counterclockwise is conventionally the positive direction.
$$\tau_{net} = \sum\tau = \tau_2 - \tau_1 = r_2F_{2\perp} - r_1F_{1\perp} = x_2F_2\sin(\theta) - x_1F_1$$

16. **Answer: B**
The net torque acting on an object is equal to its rotational inertia times its angular acceleration.
Object A: $\tau_{net} = I\alpha \qquad (20\ \text{N·m}) = I\,(5\ \text{rad/s}^2) \qquad I = 4\ \text{kg·m}^2$
Object B: $\tau_{net} = I\alpha \qquad (16\ \text{N·m}) = I\,(4\ \text{rad/s}^2) \qquad I = 4\ \text{kg·m}^2$

17. **Answer: A**
The angular acceleration is in the same direction as the net torque acting on the door about the hinge. Using counterclockwise as the positive direction, the direction of the net torque is negative which is clockwise.
$$\sum\tau = \tau_{15\,N} - \tau_{10\,N} - \tau_{5\,N} = (0.2\ \text{m})(15\ \text{N}) - (0.4\ \text{m})(10\ \text{N})\sin(45°) - (0.1\ \text{m})(5\ \text{N}) = -0.3\ \text{N·m}$$

18. **Answer: C**
The beam is balanced on the pivot point so the net torque about the pivot point is zero. The torques produced by each block are equal to their distance from the pivot point times their weight force. Using counterclockwise as positive:

$$\sum\tau = \tau_1 - \tau_2 - \tau_3 = r_1m_1g - r_2m_2g - r_3m_3g = 0 \qquad r_3m_3 = r_1m_1 - r_2m_2 \qquad m_3 = \frac{r_1m_1 - r_2m_2}{r_3}$$

19. **Answer: D**
Object A is a solid rod, object B is a solid sphere and object C is a solid cylinder. The objects have the same mass, the radius of the sphere and cylinder is $x/2$ and the length of the rod is x.

Object A: $I = \frac{1}{12}mL^2 = \frac{1}{12}mx^2$

Object B: $I = \frac{2}{5}mR^2 = \frac{2}{5}m\left(\frac{x}{2}\right)^2 = \frac{1}{10}mx^2$

Object C: $I = \frac{1}{2}mR^2 = \frac{1}{2}m\left(\frac{x}{2}\right)^2 = \frac{1}{8}mx^2$

20. **Answer: C**
The beam will rotate in the direction of the net torque acting on it (the direction of the resulting angular acceleration). The torques on the beam are produced by the weight force of each block. The net torque is zero so the beam does not rotate (it's balanced). Using counterclockwise as positive:
$\sum\tau = \tau_4 + \tau_2 - \tau_1 - \tau_6 = r_4F_4 + r_2F_2 - r_1F_1 - r_6F_6$
$= (0.5\text{ m})(4\text{ kg})g + (0.3\text{ m})(2\text{ kg})g - (0.2\text{ m})(1\text{ kg})g - (0.4\text{ m})(6\text{ kg})g = 0\text{ N·m}$

21. **Answer: A**
The greater the mass and the farther the mass is from the point of rotation, the higher the system's rotational inertia. If the spheres and rod rotate about point A, more mass is farther from the point of rotation (the 10 kg mass is farther away) so the rotational inertia is highest. The rotational inertia would be lowest if the system rotated about point B.

22. **Answer: 200 N·m**
$\tau_{net} = I\alpha \qquad \tau_{net} = (50\text{ kg·m}^2)(4\text{ rad/s}^2) = 200\text{ N·m}$

23. **Answer: 11.5 kg·m²**
$\tau_{net} = I\alpha \qquad (0.7\text{ m})(50\text{ N}) - (0.6\text{ m})(20\text{ N}) = I(2\text{ rad/s}^2) \qquad I = 11.5\text{ kg·m}^2$

24. **Answer: 4.6 N·m**
If the wrench does not rotate, the angular acceleration is zero and the net torque on the bolt is zero. The torque exerted on the bolt due to friction must be equal and opposite to the torque produced by the person.
$\tau_{net} = I\alpha \qquad \tau_{person} - \tau_{friction} = I(0) \qquad \tau_{friction} = \tau_{person} = rF_\perp = (0.2\text{ m})(30\text{ N})\sin(50°) = 4.6\text{ N·m}$

25. **Answer: 1.4 rad/s²**
$\tau_{net} = I\alpha \qquad rF_\perp = \frac{1}{3}mL^2\alpha \qquad (0.8\text{ m})(25\text{ N})\sin(70°) = \frac{1}{3}(40\text{ kg})(1\text{ m})^2\alpha \qquad \alpha = 1.4\text{ rad/s}^2$

26. **Answer: 11.25 kg**
The beam is balanced so the net torque acting on the beam about the pivot point is zero. The torques acting on the beam are produced by the weight forces of each block. Using counterclockwise as positive:
$\tau_{net} = I\alpha \qquad \tau_3 + \tau_7 - \tau_m = I(0) \qquad r_3F_3 + r_7F_7 - r_mF_m = 0$
$(0.8\text{ m})(3\text{ kg})g + (0.3\text{ m})(7\text{ kg})g - (0.4\text{ m})mg = 0 \qquad m = 11.25\text{ kg}$

27. **Answer: 63 kg·m²**
The rotational inertia for a system of point masses (we assume all of the mass of one object is located at its center) is the sum of each mass multiplied by its distance from the point of rotation squared. The pythagorean theorem must be used to find the distance between the 2 kg mass and the point of rotation.
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 = (3\ \text{kg})(3\ \text{m})^2 + (5\ \text{kg})(2\ \text{m})^2 + (2\ \text{kg})\left(\sqrt{(2\ \text{m})^2 + (2\ \text{m})^2}\right)^2 = 63\ \text{kg·m}^2$$

28. **Answer: 16.7 rad/s**
We can find the angular acceleration of the pulley and then use that to find the angular speed after 3 seconds.
$$\tau_{\text{net}} = I\alpha \qquad rF_\perp = \frac{1}{2}mR^2\alpha \qquad (0.3\ \text{m})(10\ \text{N}) = \frac{1}{2}(12\ \text{kg})(0.3\ \text{m})^2\alpha \qquad \alpha = 5.56\ \text{rad/s}^2$$
$$\alpha = \frac{\Delta\omega}{\Delta t} \qquad (5.56\ \text{rad/s}^2) = \frac{\omega_f - (0\ \text{rad/s})}{(3\ \text{s})} \qquad \omega_f = 16.7\ \text{rad/s}$$