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Class 11/12| AP Physics | IIT JEE | NEET

## Conical Pendulum

Key Idea:
The conical pendulum is a unique variation from the regular pendulum, where the ball traces a horizontal circle, forming a conical shape. This lesson explores the forces and motion in a three-dimensional plane.


Step 1: Identifying the Forces on the Pendulum Ball

1. Weight (w): Acting vertically downward, w = mg (where m is mass and g is gravity).
2. Tension (F): In the string. We'll use the symbol $F$ for tension to avoid confusion with the time period T .


Step 2: Decomposing the Tension into Components

1. Vertical Component: $F \operatorname{Cos} \beta$
2. Horizontal Component: F Sin $\beta$


The horizontal component provides the centripetal force for the circular motion.

Step 3: Writing the Force Equations

1. Vertical (YY axis): Vertical motion is absent, hence $a=0$. Using the Equation
$F=m a$
$F \cos \beta-m g=0$
(YY Direction)
$F \operatorname{Cos} \beta=m g$
2. Horizontal:
$\mathrm{F} \operatorname{Sin} \beta=m a_{\text {radial }}$
(2) (XX Direction)
$\mathrm{F} \sin \beta=\frac{m v^{2}}{r}$
(where $v$ is speed, $r=I \operatorname{Sin} \beta$ )

Step 4: Relating Centripetal Acceleration to Angle Beta
Centripetal Acceleration Equation

$$
\begin{gather*}
a_{\text {radial }}=(\mathrm{F} \sin \beta) / \mathrm{m} \\
\text { Substituting } \quad \mathrm{F}=\mathrm{mg} / \operatorname{Cos} \beta \\
a_{\text {radial }}=\mathrm{g} \operatorname{Tan} \beta \tag{3}
\end{gather*}
$$

Step 5: Determining the Time Period T of Oscillation
$T=$ Circumference/ Speed $=2 \pi R / v \quad$ (time for one revolution)

$$
\begin{array}{ll}
\mathrm{v}=\sqrt{ }(\operatorname{gr} \operatorname{Tan} \beta) & \text { (using } a_{\text {radial }}=g \operatorname{Tan} \beta=\frac{v^{2}}{r} \text { ) } \\
\mathrm{T}=2 \pi \sqrt{ }(\mathrm{~L} \operatorname{Cos} \beta / \mathrm{g}) & \text { (substitute } v \text { from above and } \mathrm{r}=\mathrm{I} \operatorname{\operatorname {Sin}\beta )}
\end{array}
$$

Observations and Implications

- With fixed L , as $\beta$ increases, $\operatorname{Cos} \beta$ decreases, shortening T.
- Tension $\mathrm{F}=\mathrm{mg} / \operatorname{Cos} \beta$ increases with $\beta$.
- A 90-degree swing is impossible (T would be zero, $F$ and $v$ infinite).

