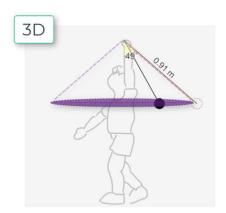


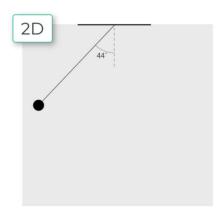


Conical Pendulum

Key Idea:

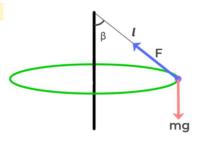
The conical pendulum is a unique variation from the regular pendulum, where the ball traces a horizontal circle, forming a conical shape. This lesson explores the forces and motion in a three-dimensional plane.





Step 1: Identifying the Forces on the Pendulum Ball

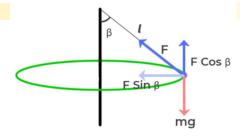
- Weight (w): Acting vertically downward, w = mg (where m is mass and g is gravity).
- 2. **Tension (F)**: In the string. We'll use the symbol F for tension to avoid confusion with the time period T.



Step 2: Decomposing the Tension into Components

1. **Vertical Component**: F Cos β

2. **Horizontal Component**: F Sin β



The horizontal component provides the centripetal force for the circular motion.



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Step 3: Writing the Force Equations

1. **Vertical (YY axis)**: Vertical motion is absent, hence a = 0. Using the Equation

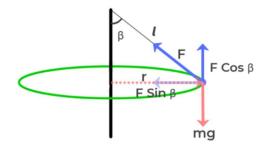
F = ma

F Cos β - mg = 0

(YY Direction)

F Cos β = mg

(1)



2. Horizontal:

F Sin β = $m \, a_{radial}$

(2) (XX Direction)

$$\mathsf{F}\,\mathsf{Sin}\,\beta = \frac{mv^2}{r}$$

(where v is speed, $r = I \sin \beta$)

Step 4: Relating Centripetal Acceleration to Angle Beta

Centripetal Acceleration Equation

$$a_{radial} = (F \sin \beta)/m$$

from (2)

Substituting $F = mg/Cos \beta$

from (1)

$$a_{radial}$$
 = g Tan β

(3)

Step 5: Determining the Time Period T of Oscillation

T = Circumference/ Speed = $2 \pi R / v$

(time for one revolution)

$$v = \sqrt{(gr Tan \beta)}$$

(using
$$a_{radial} = g \operatorname{Tan} \beta = \frac{v^2}{r}$$
)

$$T = 2 \pi \sqrt{(L \cos \beta/g)}$$

(substitute v from above and $r = I \sin \beta$)



Observations and Implications

- With fixed L, as β increases, Cos β decreases, shortening T.
- Tension $F = mg/Cos \beta$ increases with β .
- A 90-degree swing is impossible (T would be zero, F and v infinite).