Vector Basics

Introduction

Scalar –

Vector -

<u>Notation</u>

Let's say we have a scalar k:

- if $k \neq 0$

- if $k > 0$

- if $k < 0$

- if $k = 0$

Two vectors are ________________________ if and only if they're nonzero multiples of each other

Adding & Subtracting

We have two, non-parallel vectors $\overline{AB}$ and $\overline{AC}$:

Let $\bar{a} =< a_1, a_2 >$ and $\bar{b} =< b_1, b_2 >$

- addition:

- subtraction:

- scalar multiplication:

Example Set 1

1. Find $3\bar{a}$, $\bar{a}+\bar{b}$, $\bar{a}-\bar{b}$
   a. $\bar{a} =< 4, 0 >$ $\bar{b} =< 0, -5 >$

   b. $\bar{a} =< 1, 1 >$ $\bar{b} =< 3,\ 2 >$

   c. $\bar{a} =< 1, 3 >$ $\bar{b} = -6\bar{a}$

2. Find $4\bar{a} - 2\bar{b}$ if $\bar{a} =< 3, 1 > + < -1, 2 >$, $\bar{b} =< 6, 5 > - < 1, 2 >$

Position Vector

- Vector that starts at the ____________________ and ends at the point $P(x, y)$ is called the

  ______________________________________ of point P

Finding the vector between 2 non-origin points:

Example Set #2

1. Graph the position vectors for each point. Then, find and graph the vector between the two points.
   a. $A(\ 3, 2\ ),\ B(\ 5, 7)$

   b. $A(-3, -2\ ),\ B(\ 4, -5)$

Magnitude

Unit Vectors

Cartesian Vectors

Using Angles to Find Vector Components

Properties of Vectors

- Commutative Law:
- Associative Law:
- Additive Identity:
- Additive Inverse:
- k is scalar
- k_1, k_2 are scalars
- Zero Vector:

Example Set #3

1. Find the magnitude of $\|\bar{a}+\bar{b}\|$ and $\|\bar{a}-\bar{b}\|$.
   a. $\bar{a} =< 7, 10 >$, $\bar{b} =< 1, 2 >$

   b. $\bar{a} = 2\hat{\imath} + 4\hat{\jmath}$, $\bar{b} = -\hat{\imath} + 4\hat{\jmath}$

2. Find $-3\bar{a} - 5\bar{b}$ if $\bar{a} = \hat{\imath} + \hat{\jmath}$ and $\bar{b} = 3\hat{\imath} - 2\hat{\jmath}$

3. Find the vector $\overline{AB}$ and use Cartesian form. Graph $\overline{AB}$. Also, find $||\overline{AB}||$ and $\hat{u}_{AB}$.
   a. $A\ (0, 3)$, $B(2, 0)$

b. $A\ (2,1),\ \ B(-4,5)$

4. Determine if the vectors are parallel.
   a. $\bar{a} = 4\hat{\imath} + 6\hat{\jmath}, \quad \bar{b} = -4\hat{\imath} - 6\hat{\jmath}$

   b. $\bar{a} = 4\hat{\imath} + 6\hat{\jmath}, \quad \bar{b} = 10\hat{\imath} + 15\hat{\jmath}$

   c. $\bar{a} = 4\hat{\imath} + 6\hat{\jmath}, \quad \bar{b} = (5\hat{\imath} + \hat{\jmath}) - (7\hat{\imath} + 4\hat{\jmath})$

5. Find a vector $\bar{b}$ that's parallel to $\bar{a} = 3\hat{\imath} + 7\hat{\jmath}$ and has the magnitude $\|\bar{b}\| = 2$.

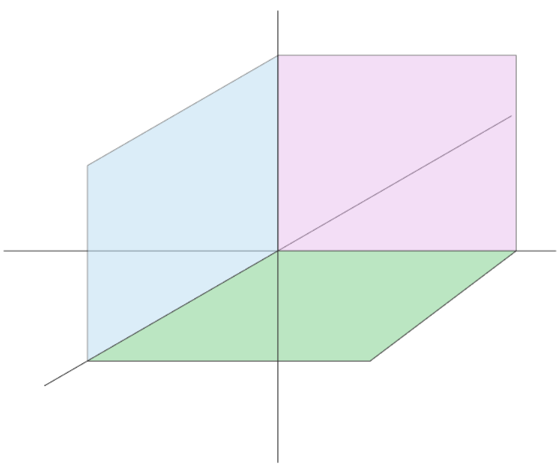
6. Find the vector components of $\bar{a}, \bar{b}, \bar{c}, \bar{d}$.

3D Vectors in Cartesian Systems

Cartesian systems are made up of x, y, z axes.

- Location of the axes is determined by the ____
- To find the z axis:

Each pair of coordinate axes determines a _______________________________________.



Distance Formula for 3D Systems

Find the distance between 2 points $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$

Unit Vector for 3D Systems

Finding Unit Vector from Vectors and Angles

Example Set #4

1. Consider the point $P(-2, 5, 4)$. Graph the point. If the lines are drawn from P perpendicular to the coordinate planes, what are the coordinates of the point at the base of each perpendicular.

2. Graph the points and then find the distance between them and the unit vector of $\overline{BA}$.
   $A(3, 4, -5), B(2, -2, 5)$

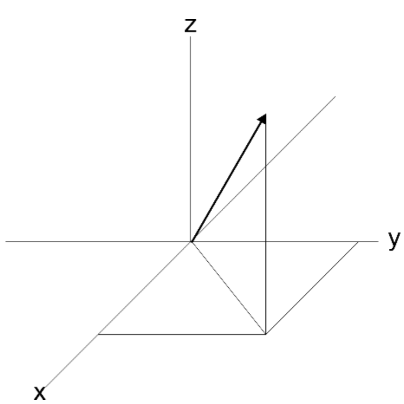
3. Solve for the unknown x if $A(x, 2, 3), B(2, 1, 1)$, and $\|\overline{AB}\| = \sqrt{21}$.

4. Find a unit vector in the opposite direction of $\bar{a} =< 10, -5, 10 >$.

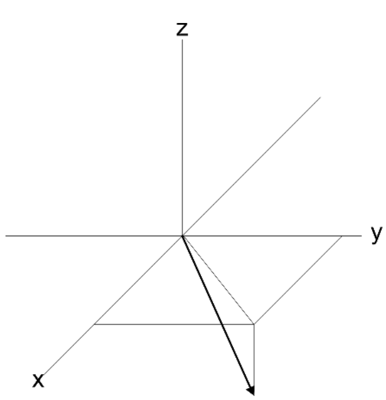
5. Find a vector $\bar{b}$ for which $\|\bar{b}\| = \frac{1}{2}$ that is parallel to $\bar{a} = -6\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ but has the opposite direction.

6. Find the position vector $\bar{r}$.

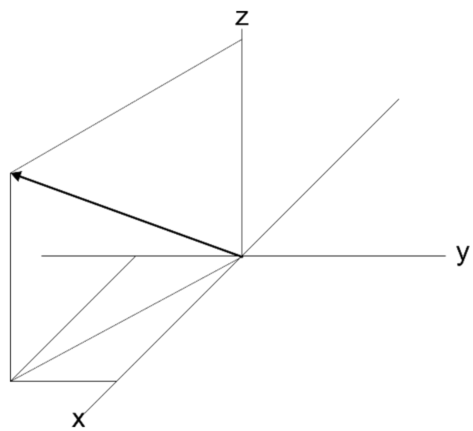
a.



b.



c.



Dot Product (aka Inner Product or Scalar Product)

Dot product defined as

Orthogonal Vectors

Let's look at the sign of $\bar{a} \cdot \bar{b}$

Finding the Angle Between Vectors

Direction Cosines

Projection of $\bar{a}$ onto $\bar{b}$

Example Set #5

1. Let $\bar{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ $\quad \bar{b} = \hat{\imath} + 5\hat{\jmath} + 6\hat{k}$ $\quad \bar{c} = 7\hat{\imath} + 10\hat{\jmath} + 9\hat{k}$
   Find:
   a) $\bar{a} \cdot \bar{c}$

   b) $2\bar{b} \cdot \bar{c}$

   c) $(\bar{c} \cdot \bar{a})\bar{b}$

2. What's $\bar{a} \cdot \bar{b}$ if $\|\bar{a}\| = 10$ $\quad \|\bar{b}\| = 5$ $\quad \theta = 45°$

3. Are the vectors orthogonal?
   a) $< 2, 0, 1 >$ $\quad < -4, 3, 8 >$

   b) $< 2, 0, 1 >$ $\quad \hat{\imath} - 4\hat{\jmath} + 6\hat{k}$

   c) $3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ $\quad < 1, -1, 1 >$

4. What value of c makes the vectors perpendicular? $< 2, -c, 3 >$ $< 3, 2, 4 >$

5. Find the angle θ between the vectors.
   a) $< 3, 0, -1 >$ $< 2, 0, 2 >$

   b) $\bar{a} = 2\hat{\imath} + 4\hat{\jmath}$ $\bar{b} = -\hat{\imath} - \hat{\jmath} + 4\hat{k}$

6. Find the direction cosine angles of $\bar{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

7. Find the projection of $\bar{a}$ onto $\bar{b}$.

a) $\bar{a} =< -5, 5 >$ $\bar{b} =< -3, 4 >$

b) $\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ $\bar{b} = -2\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Properties of Cross Product

Scalar Triple Product

Example Set #6

1. Find $\bar{a} \times \bar{b}$.

   a) $\bar{a} = \hat{\imath} \cdot \hat{\jmath} \quad \bar{b} = 2\hat{\jmath} + 5\hat{k}$

   b) $\bar{a} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k} \quad \bar{b} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$

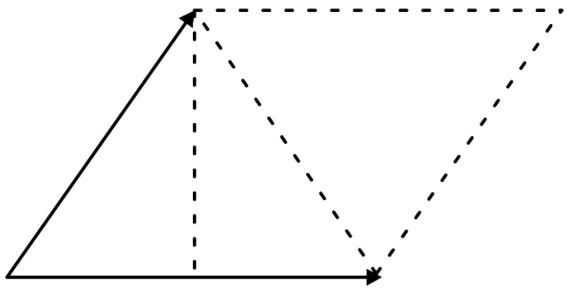
2. Find a vector that's perpendicular to both $\bar{a}$ and $\bar{b}$: $\bar{a} = 2\hat{\imath} + 7\hat{\jmath} - 4\hat{k}$, $\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

3. Find $\|4\hat{\jmath} - 5(\hat{\imath} \times \hat{\jmath})\|$.

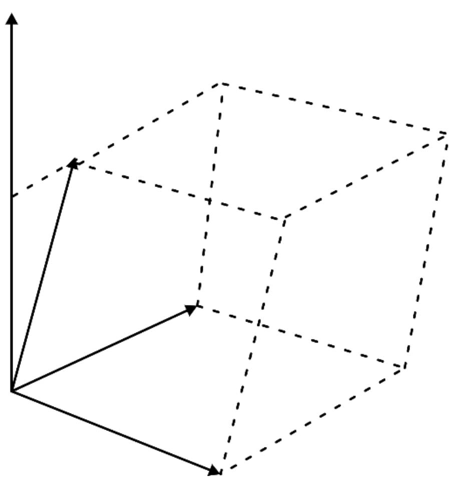
4. Find $2\hat{\jmath} \cdot \left[\hat{\imath} \times \left(\hat{\jmath} - 3\hat{k}\right)\right]$.

5. Find $\bar{a} \cdot \left(\bar{b} \times \bar{c}\right)$ if $\bar{a} \times \bar{b} = 4\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$, $\bar{c} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$.

Areas



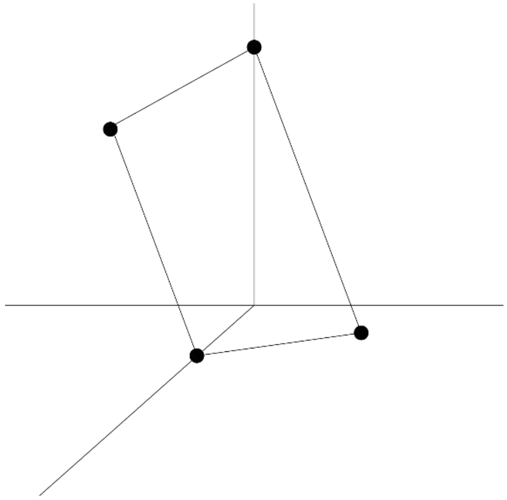
Volume of Parallelepiped



Coplanar Vectors

Example Set #7

1. Verify the quadrilateral is a parallelogram and find the area.



2. Find the area of the triangle defined by points:

   a) $A(1,1,1)$ $B(1,2,1)$ $C(1,1,2)$

   b) $A(1,2,4)$ $B(1,-1,3)$ $C(-1,-1,2)$

3. Find the volume of the parallelepiped defined by $\bar{a} = \hat{\imath} + \hat{\jmath}$ $\bar{b} = -\hat{\imath} + 4\hat{\jmath}$ $\bar{c} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

CHERISH QUALLS, PH.D.

4. Are the following vectors coplanar? $\bar{a} = 4\hat{\imath} + 6\hat{\jmath}$ $\bar{b} = -2\hat{\imath} + 6\hat{\jmath} - 6\hat{k}$ $\bar{c} = 2.5\hat{\imath} + 3\hat{\jmath} + 0.5\hat{k}$