Determining Potential Energy Values

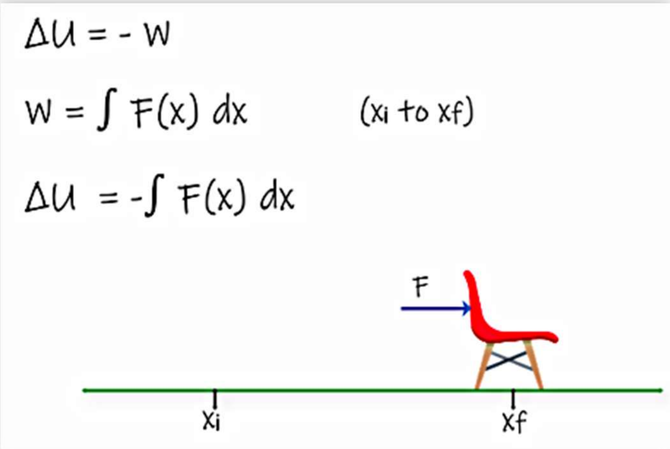
Key Idea

When a conservative force acts on an object, causing it to move, the work done by the force leads to a change in the object's potential energy. This change is quantified by the equation:

$$\Delta U = -W \qquad (1)$$

ΔU = change in PE

W = work done by the force



If the force changes with position, the work done can be expressed as:

$$W = \int F(x)\,dx \qquad (2)$$

as the object moves from x_i to x_f

Therefore, *if we can find the work done by the force, we can find the change in potential energy using equation (1)*

Gravitational Potential Energy

Gravitational PE is calculated considering the objects position *relative* to the Earth.

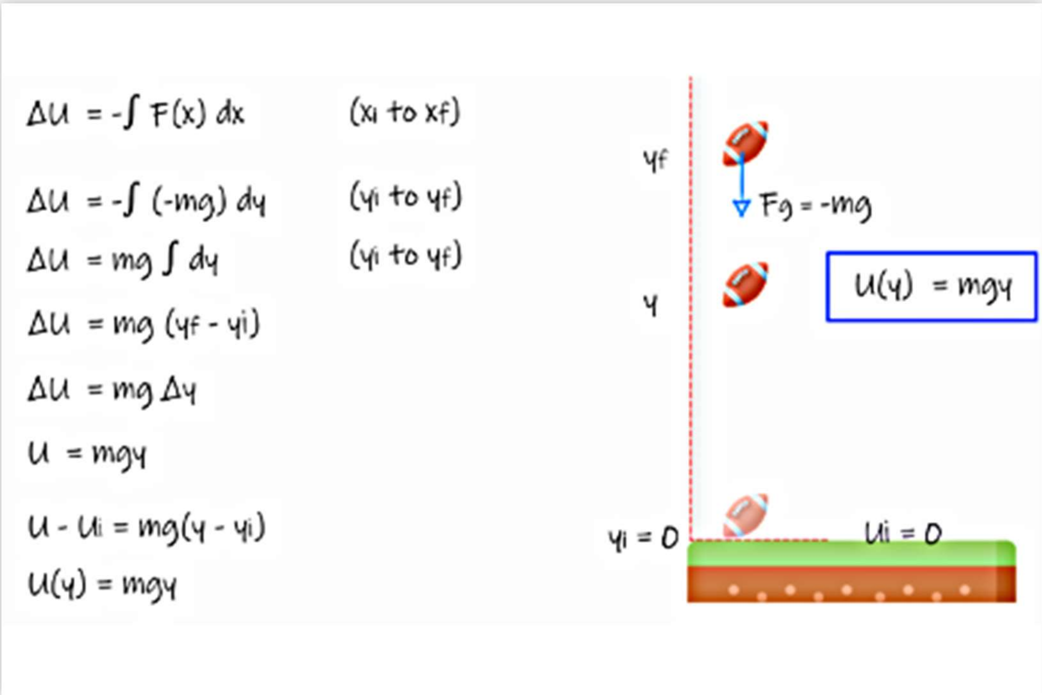
Consider an object of mass m moving vertically with a gravitational force -mg. The change in PE as the object moves from height y_i to y_f is:

$\Delta U = -\int (-mg)\, dy$ (Limits change from y_i to y_f)

$= mg(y_f - y_i)$

$= mg\, \Delta y$

$$\Delta U = = mg\, \Delta y$$



This equation shows change in gravitational PE depends on the change in height, Δy

To find PE at any height y, set the reference potential energy (U_i) to zero at a certain height (typically the Earth's surface, $y_i = 0$), leading to:

$$U = mgy$$

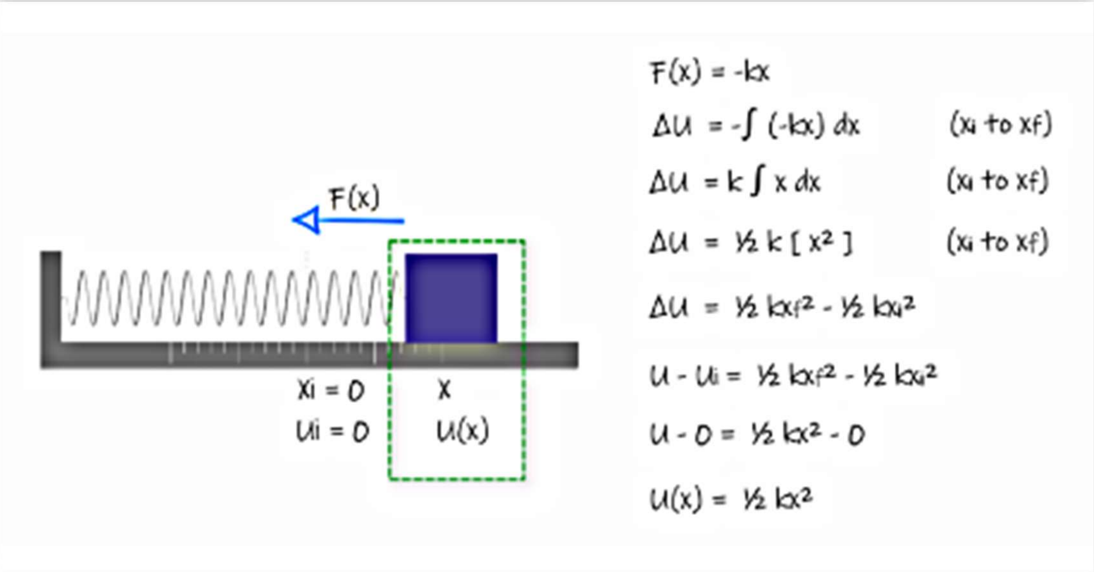
This formula indicates its dependence solely on the object's vertical position.

Elastic Potential Energy

Elastic PE is stored in a spring when it is compressed or stretched. For a spring with a force constant k, the change in elastic PE as the spring moves from position x_i to x_f is given by:

$\Delta U = -\int (-kx)\,dx = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$ (Using F = -kx)

$$\Delta U = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$$



By setting the reference potential energy (U_i) to zero when the spring is at its relaxed length ($x_i = 0$), we obtain the formula for the elastic potential energy at any position x:

$$U(x) = \frac{1}{2} kx^2$$

From the formula you can see, the elastic potential energy is proportional to the square of the spring's displacement from its relaxed length.

Summary of formulas and equations

S.N	Formula	When to Use	Common Mistakes to Avoid
1.	$\Delta U = -W$	General formula to calculate the change in PE resulting from work done by or against a force	Ignoring the sign of work (W); forgetting that a negative work value indicates an increase in potential energy
2.	$W = \int F(x)\,dx$	General formula to find the work done by a variable force over a displacement.	Not properly setting the limits of integration
3.	$\Delta U = -\int F(x)\,dx$	General formula to relate the change in PE to the work done by a variable force.	Confusing the signs; forgetting that ΔU is the negative of the work done.
4.	$\Delta U = mg\,(y_f - y_i)$ $= mg\,\Delta y$	To determine the change in gravitational PE as an object moves vertically.	Misidentifying the initial and final positions (y_i and y_f); neglecting the direction of gravity.
5.	$U = mgy$	To calculate the gravitational PE at a specific height above a reference level.	Choosing an inconsistent reference point for PE, leading to errors in U.
6.	$\Delta U = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$	To find the change in elastic PE of a spring from one position to another.	Confusing initial and final positions of the spring; not squaring the displacement properly.
7.	$U(x) = \frac{1}{2} kx^2$	To calculate the elastic PE of a spring at a specific displacement from its relaxed position.	Forgetting to use the spring's displacement from its relaxed length

Key Points to Remember:

a. Accuracy in Calculations: Pay careful attention to the details of each formula, such as the direction of forces, the signs of quantities, and the specific conditions under which the formula is applied.
b. Consistency in Reference Points: Ensure that reference points for potential energy calculations (like the ground level for gravitational potential energy or the relaxed position for a spring) are consistently applied throughout a problem.
c. Understanding of Physical Principles: Grasp the physical concepts behind each formula. This understanding helps in correctly applying the formulas and in intuitively checking if your results make sense.