STUDY GUIDE

ENERGY, WORK & POWER

The Law of Conservation of Energy

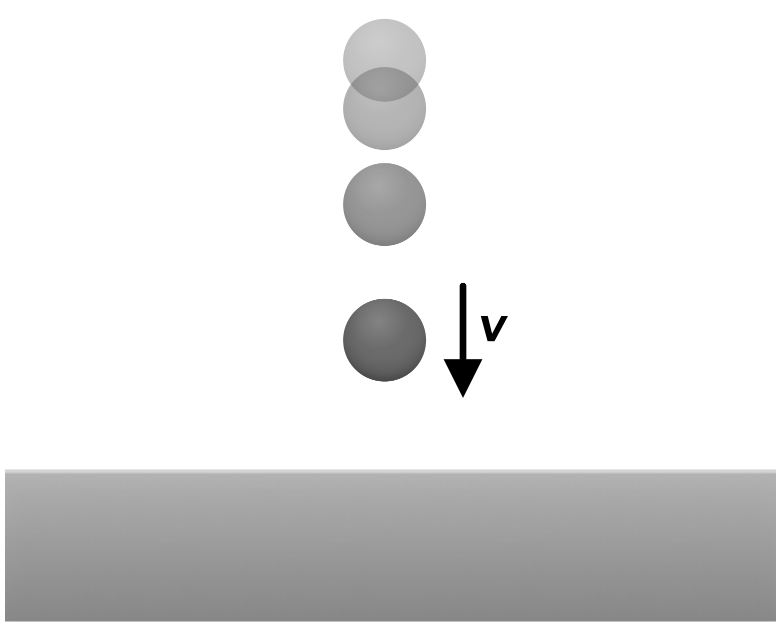
- **The law of conservation of energy**: the total amount of energy in the universe or an isolated system is conserved (it's constant and doesn't change over time).
- There are many different types of energy (kinetic energy, gravitational potential energy, etc.) and energy can be converted or transformed between those different types, but it cannot be created or destroyed.

Variables		SI Unit
E	energy	J
K	kinetic energy	J
U_g	gravitational potential energy	J
U_{sp}	spring potential energy	J

- Energy can be converted or transformed from one type of energy to any other type of energy. In the real world there are some conversions which are more common and less likely to happen in the reverse direction.

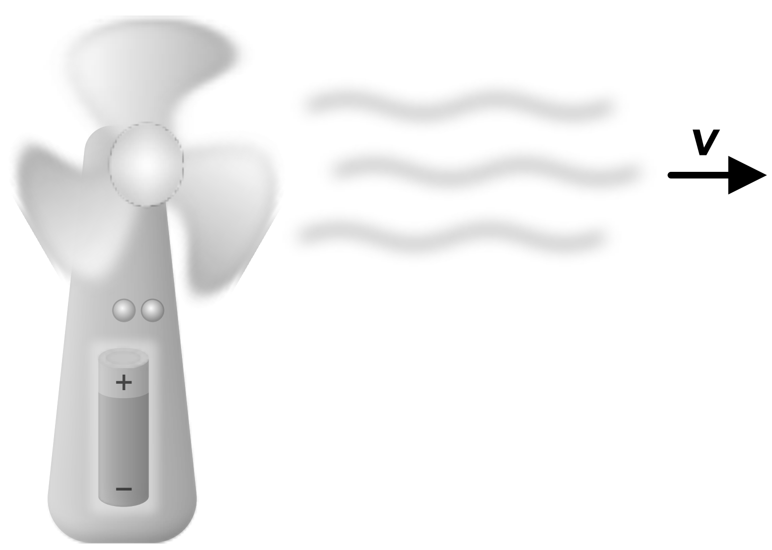
Gravitational potential energy is converted into kinetic energy as a ball falls towards the earth

$$U_g \rightarrow K$$

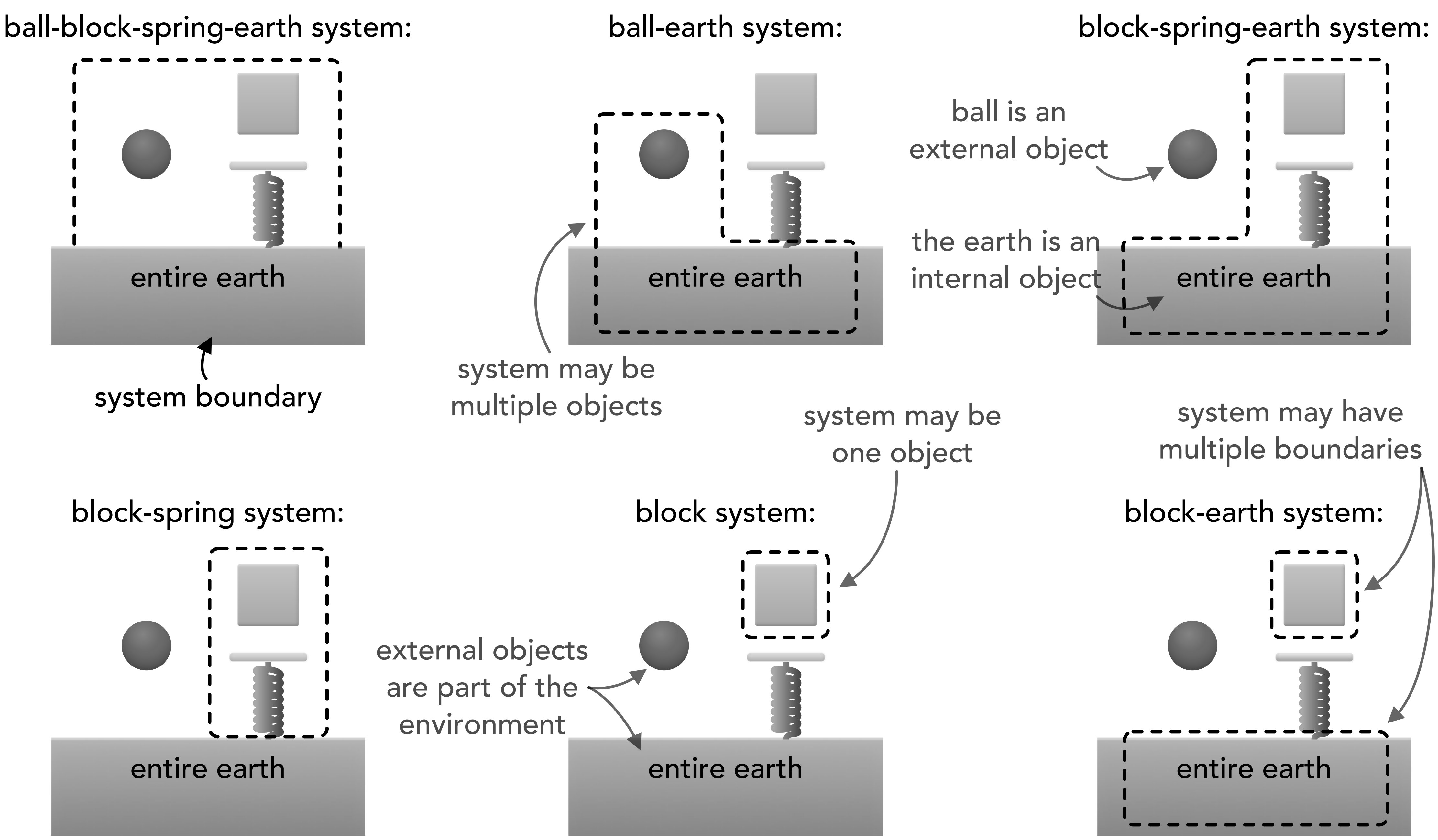


Chemical energy in a fan battery is converted into electrical energy, which is converted into rotational kinetic energy, translational kinetic energy, sound energy, light energy and thermal energy

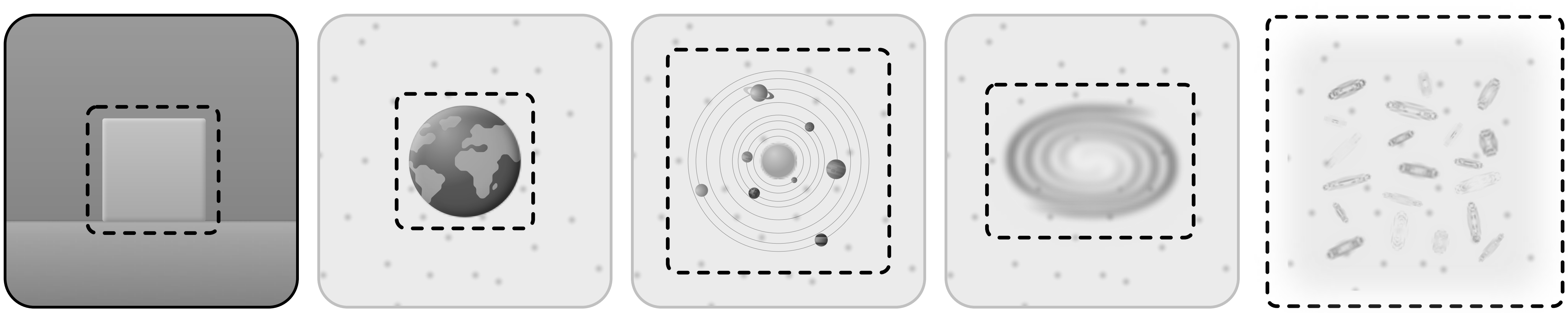
$$E_{chem} \rightarrow E_{elec} \rightarrow K_{rot} + K + E_{sound} + E_{light} + E_{therm}$$



- A **system** can be thought of as a selected group of objects which is separated from its **environment** by a chosen **boundary line**. There are no predefined or existing systems, a system is just what we choose it to be based on the objects that we're studying or the problem we're solving.
- Once a boundary line is drawn and a system is chosen, everything in the universe (energy, forces, objects) are considered either inside the system (internal) or outside the system (external) at any one moment in time. Energy and objects may move into or out of the system depending on the situation.
- Objects do not have to be in contact with each other to both be in the system. Mutliple separate boundary lines may be drawn to define the system so that only specific objects are included.



A system can be any size, from one object to the entire universe



- Energy and objects may move into or out of a system across the system boundary line.
- An **isolated system** is defined as a system where **energy does not enter or leave the system**. This is just a definition. If energy does enter or leave the system, it is not an isolated system.
- According to the law of conservation of energy, **the total amount of energy within an isolated system is conserved over time**. Energy within the system can be converted back and forth between different types, but the **total** amount of energy stays the same in an isolated system.

Conservation of energy (universe and isolated systems)

$$\Delta E_{total} = 0 \quad , \quad E_{total\ i} = E_{total\ f}$$

Isolated system: no energy moves into or out of the system, no work is done on or by the system, and no net external forces are acting on the system

If no energy enters or leaves a system (to or from the environment) that system is an "isolated system"

isolated system

$$E_{total} = K + U_g + U_{sp} + E_{therm} + E_{light} + E_{sound} + E_{chem} + E_{elec} + \ldots$$

$$\Delta E_{total} = 0$$

$$K \leftrightarrow U_g \leftrightarrow U_{sp} \leftrightarrow E_{therm}$$
$$E_{light} \leftrightarrow E_{sound} \leftrightarrow E_{chem} \leftrightarrow E_{elec}$$

environment

$$E_{total} = K + U_g + U_{sp} + E_{therm} + E_{light} + E_{sound} + E_{chem} + E_{elec} + \ldots$$

$$\Delta E_{total} = 0$$

$$K \leftrightarrow U_g \leftrightarrow U_{sp} \leftrightarrow E_{therm}$$
$$E_{light} \leftrightarrow E_{sound} \leftrightarrow E_{chem} \leftrightarrow E_{elec}$$

- The universe is considered an isolated system because there is nothing "outside" the universe and no external environment for energy to be transferred to or from.
- If you add up the total amount of each type of energy in the universe at one moment in time, that total will be the same at a different moment in time. The amounts of each type of energy may change, but the total amount of energy stays the same.

The total amount of energy in the universe stays the same over time

the universe at time t_i

$$E_{total\ i} = K + U_g + U_{sp} + E_{therm} + E_{light} + E_{sound} + E_{chem} + E_{elec} + \ldots$$

the universe at time t_f

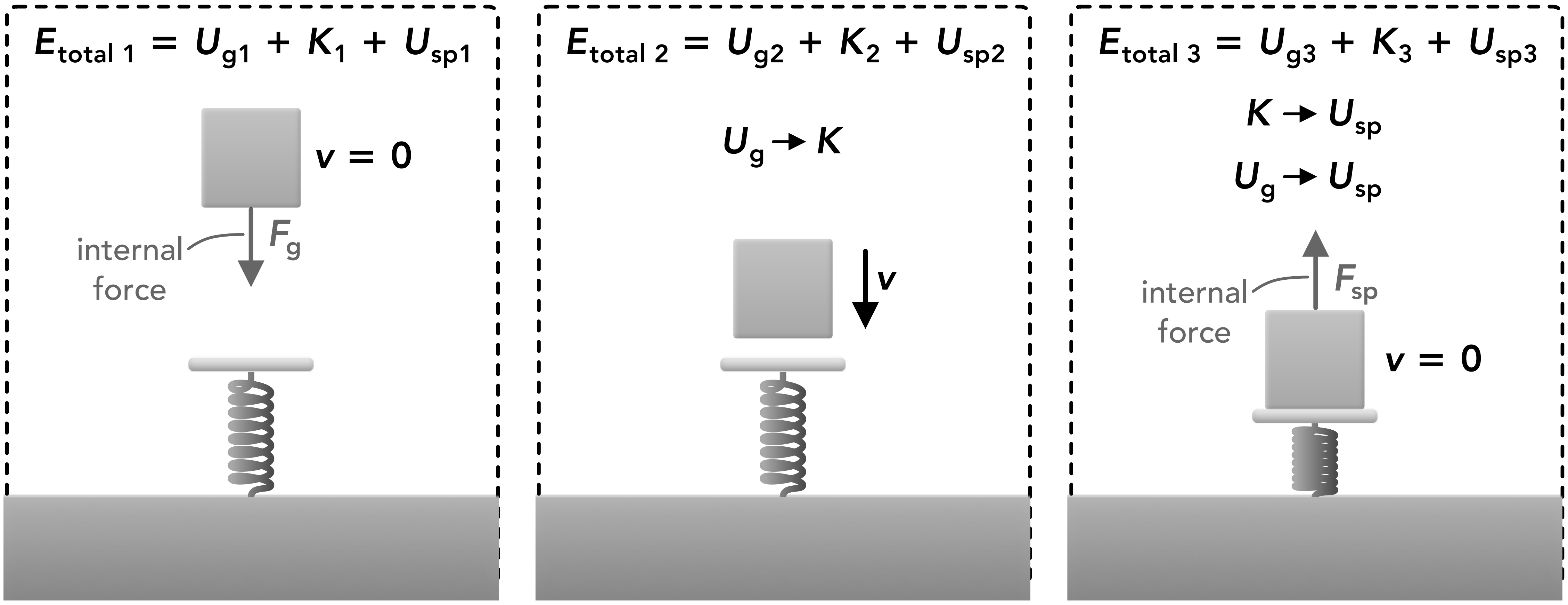
$$E_{total\ f} = K + U_g + U_{sp} + E_{therm} + E_{light} + E_{sound} + E_{chem} + E_{elec} + \ldots$$

$$E_{total\ i} = E_{total\ f}$$

The total amount of energy in an isolated system stays the same over time

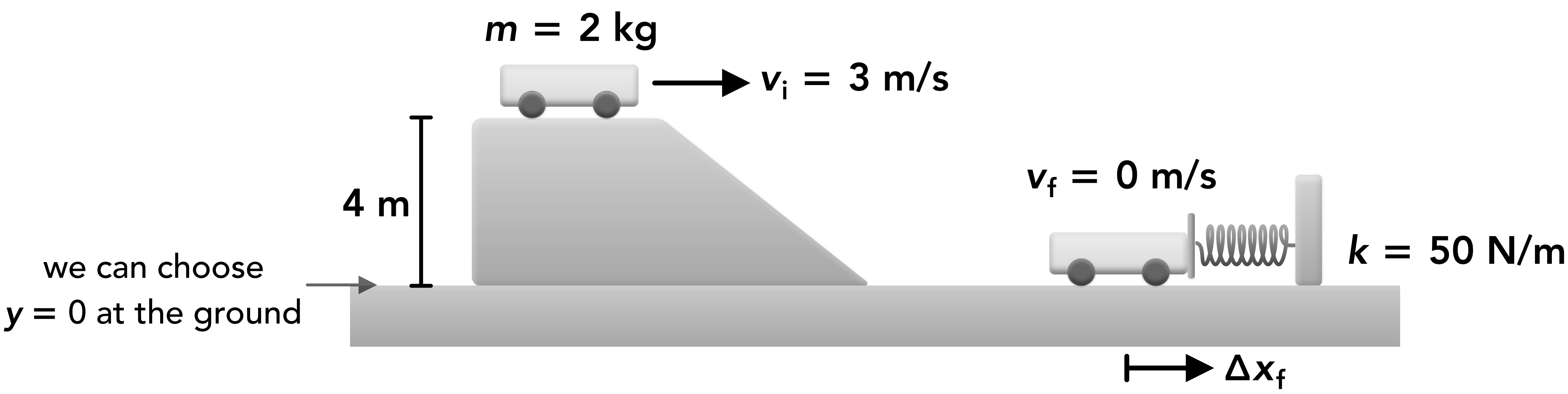
In the isolated system below, the block, the spring and the earth are all internal objects. The gravitational force between the block and the earth is an internal force which converts energy within the system. The spring force between the block and the spring is also an internal force which converts energy within the sytem. The gravitational force and the spring force are NOT external forces and do NOT do work on the system.

block-spring-earth system at time t_1	block-spring-earth system at time t_2	block-spring-earth system at time t_3



t_1	t_2	t_3
$U_{g1} = 10$ J	$U_{g2} = 5$ J	$U_{g3} = 3$ J
$K_1 = 0$ J	$K_2 = 5$ J	$K_3 = 0$ J
$U_{sp1} = 0$ J	$U_{sp2} = 0$ J	$U_{sp3} = 7$ J
$E_{total\ 1} = 10$ J →	$E_{total\ 2} = 10$ J →	$E_{total\ 3} = 10$ J

Example: A 2 kg cart is 4 m above the ground with a speed of 3 m/s. It rolls down a ramp and contacts a spring with a spring constant of 50 N/m. The cart compresses the spring and momentarily comes to a stop. At that moment, how much is the spring compressed? Assume there is negligible friction.



*We're assuming that the cart-spring-earth system is an isolated system so the total energy in the system is conserved over time. The final total energy is equal to the initial total energy:

there is kinetic energy, gravitational potential energy and spring potential energy involved in this scenario

$$E_{total\ i} = E_{total\ f}$$

$$K_i + U_{g\ i} + U_{sp\ i} = K_f + U_{g\ f} + U_{sp\ f}$$

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}k\Delta x_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}k\Delta x_f^2$$

$K = \frac{1}{2}mv^2$ — $\frac{1}{2}(2\text{ kg})(3\text{ m/s})^2$ — $\frac{1}{2}(2\text{ kg})(0\text{ m/s})^2$

$U_g = mgy$ — $+ (2\text{ kg})(9.8\text{ m/s}^2)(4\text{ m})$ — $+ (2\text{ kg})(9.8\text{ m/s}^2)(0\text{ m})$

$U_{sp} = \frac{1}{2}k\Delta x^2$ — $+ \frac{1}{2}(50\text{ N/m})(0\text{ m})^2$ — $+ \frac{1}{2}(50\text{ N/m})\Delta x_f^2$

$$87.4 = 25\Delta x_f^2$$

$$1.87\text{ m} = \Delta x_f$$

Work

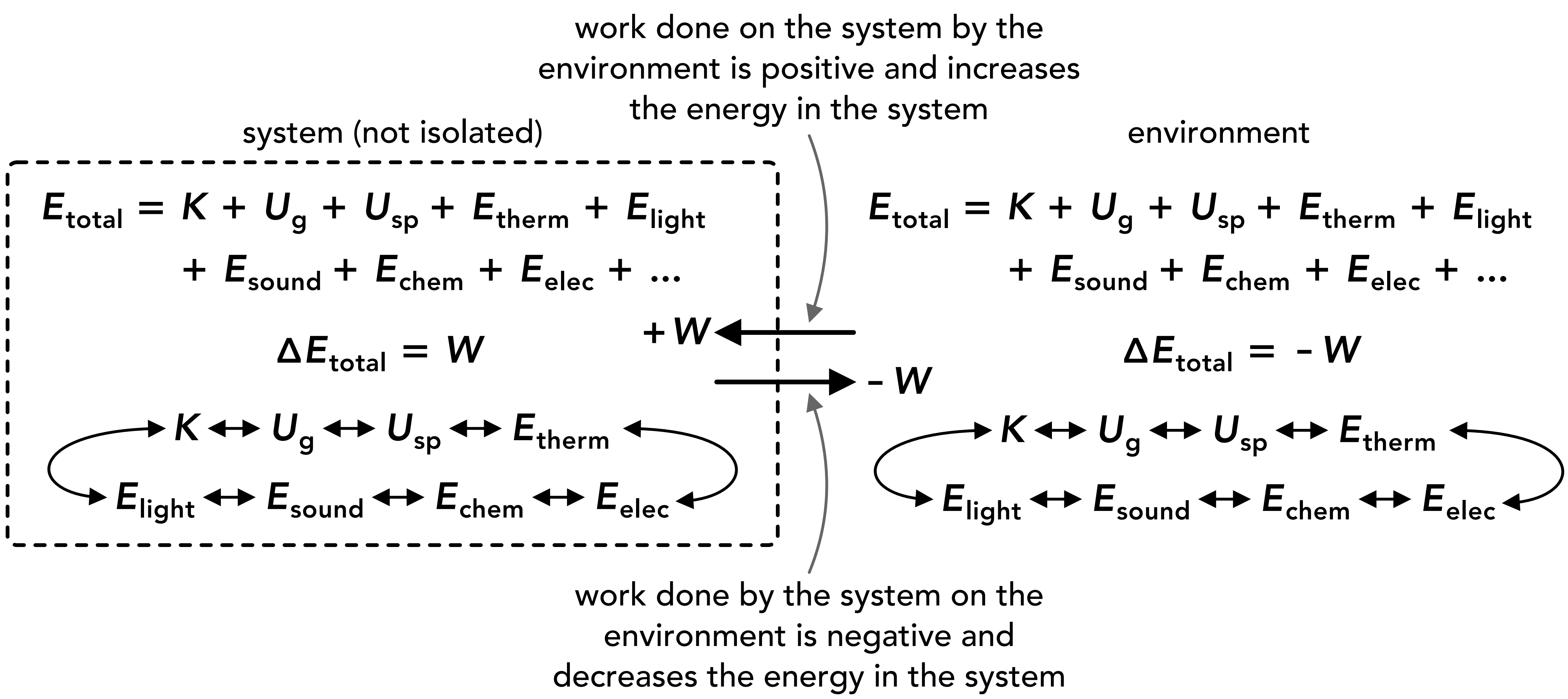
Variables		SI Unit
W	work	$J = N \cdot m$
E	energy	J
F	force	N
d	displacement	m

- The law of conservation of energy says the total amount of energy within an isolated system stays the same over time. But if the system is not isolated energy can move into or out of the sytem, so the energy within the system is not conserved (but the total amount of energy in the universe is conserved).
- **Work** is the transfer of energy into or out of a system. This happens when external forces are applied over some displacement.
- The SI unit of work is a joule (J), the unit of energy. 1 J = 1 N·m.

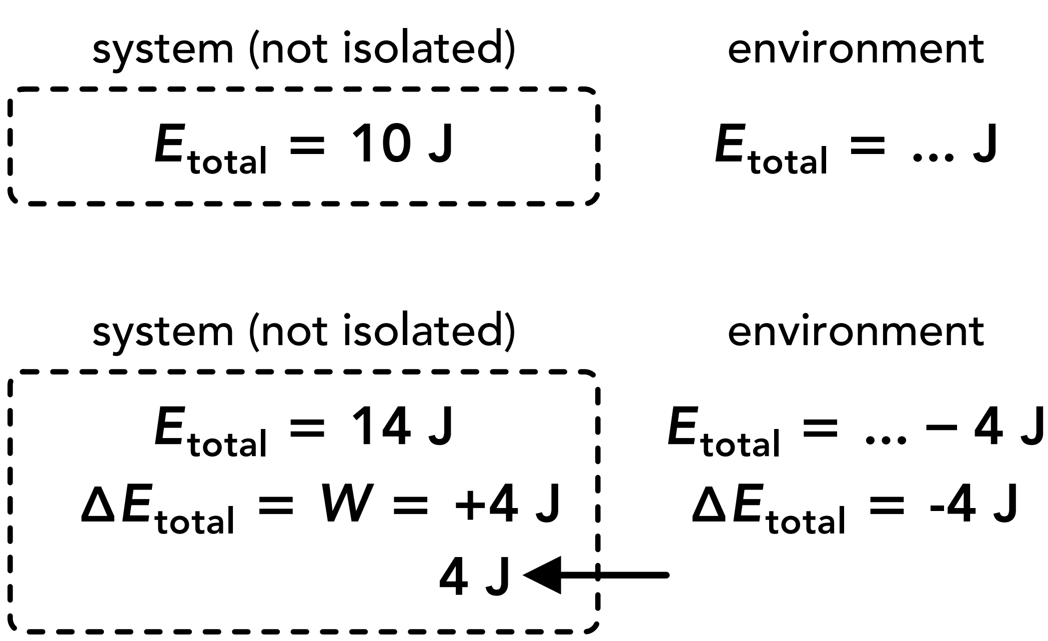
Work

$$\Delta E_{system} = W$$

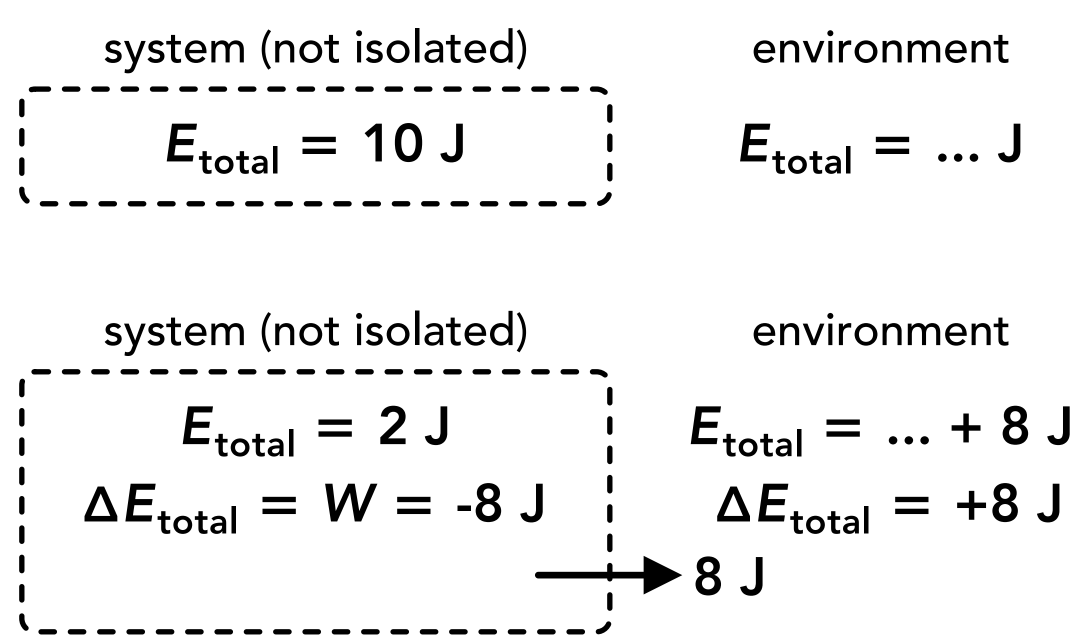
- If energy is transferred **into** the system (from the environment) we say work is done "on" the system. The total amount of energy inside the system **increases**. The total amount of energy in the environment decreases.
- If energy is transferred **out of** the system (to the environment) we say work is done "by" the system. The total amount of energy inside the system **decreases**. The total amount of energy in the environment increases.



Work done on the system is positive and increases the system's energy



Work done by the system is negative and decreases the system's energy

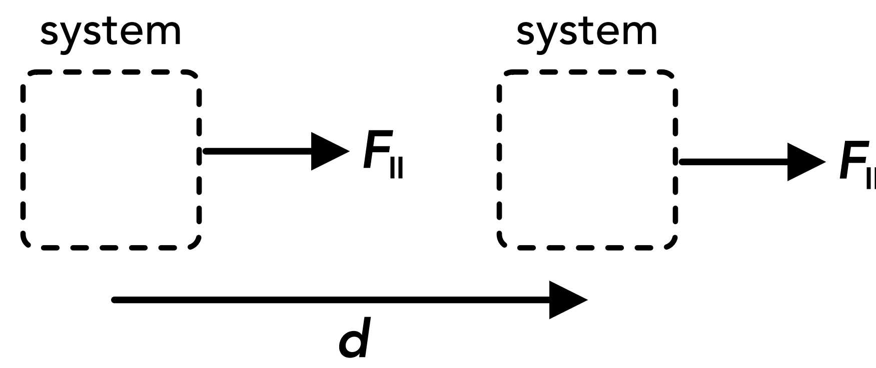


- The result of work is a change in the amount of energy within a system. But work is done on or by a system due to external forces when the system moves, and work can also be calculated as the displacement of the system multiplied by the component of the external force that is parallel to that displacement.

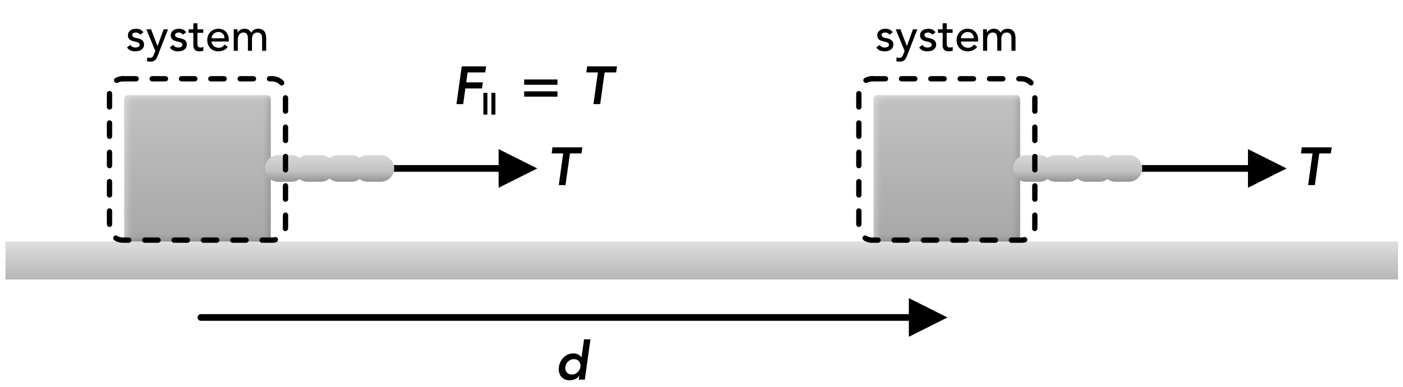
Work

$$W = F_{\parallel} d$$

$F_{\parallel}$: component of force parallel to d
*F is an external force
d : displacement of the system

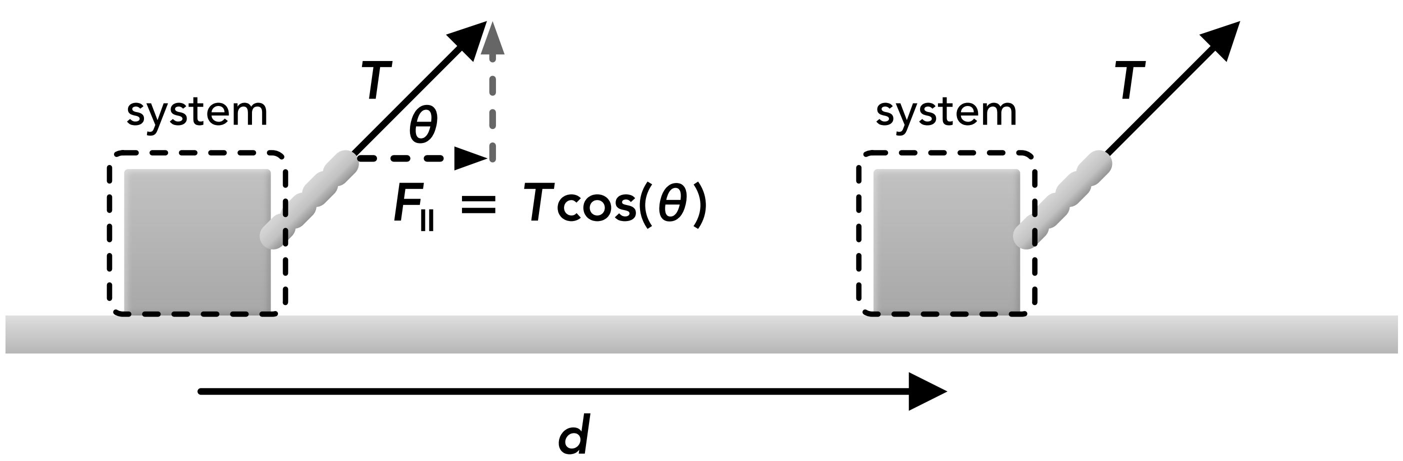


Only the force component parallel to the displacement does work on the system, and the sign of the work (positive or negative) depends on if the force component and displacement are in the same direction or opposite directions



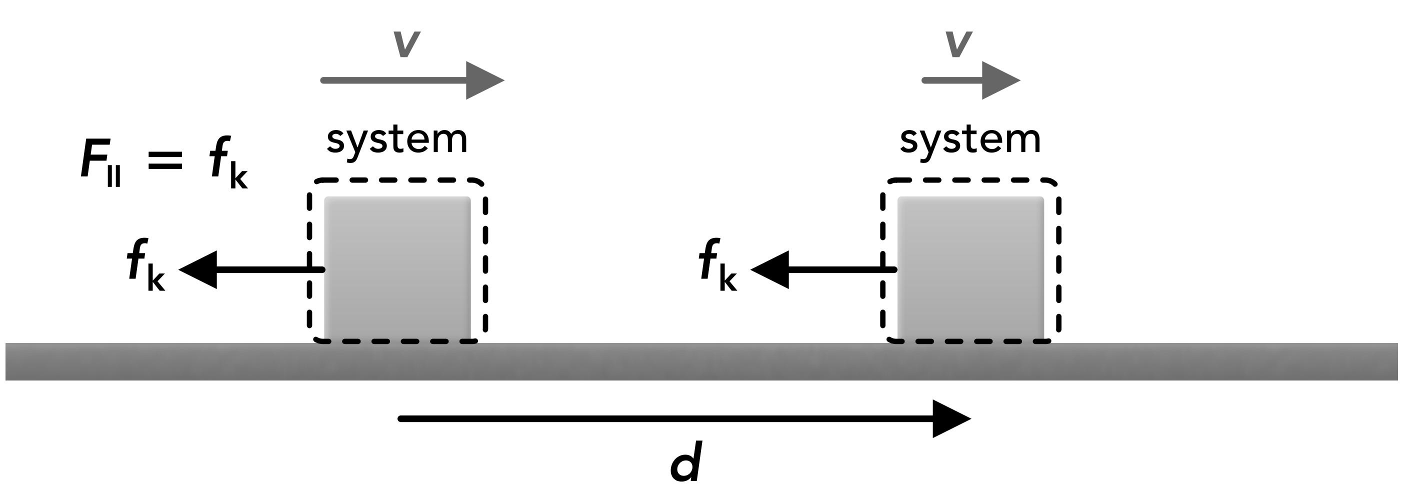
$F_{\parallel}$ and d are in the same direction, work is positive

$$W = F_{\parallel} d = Td$$



$F_{\parallel}$ and d are in the same direction, work is positive

$$W = F_{\parallel} d = T\cos(\theta) d$$



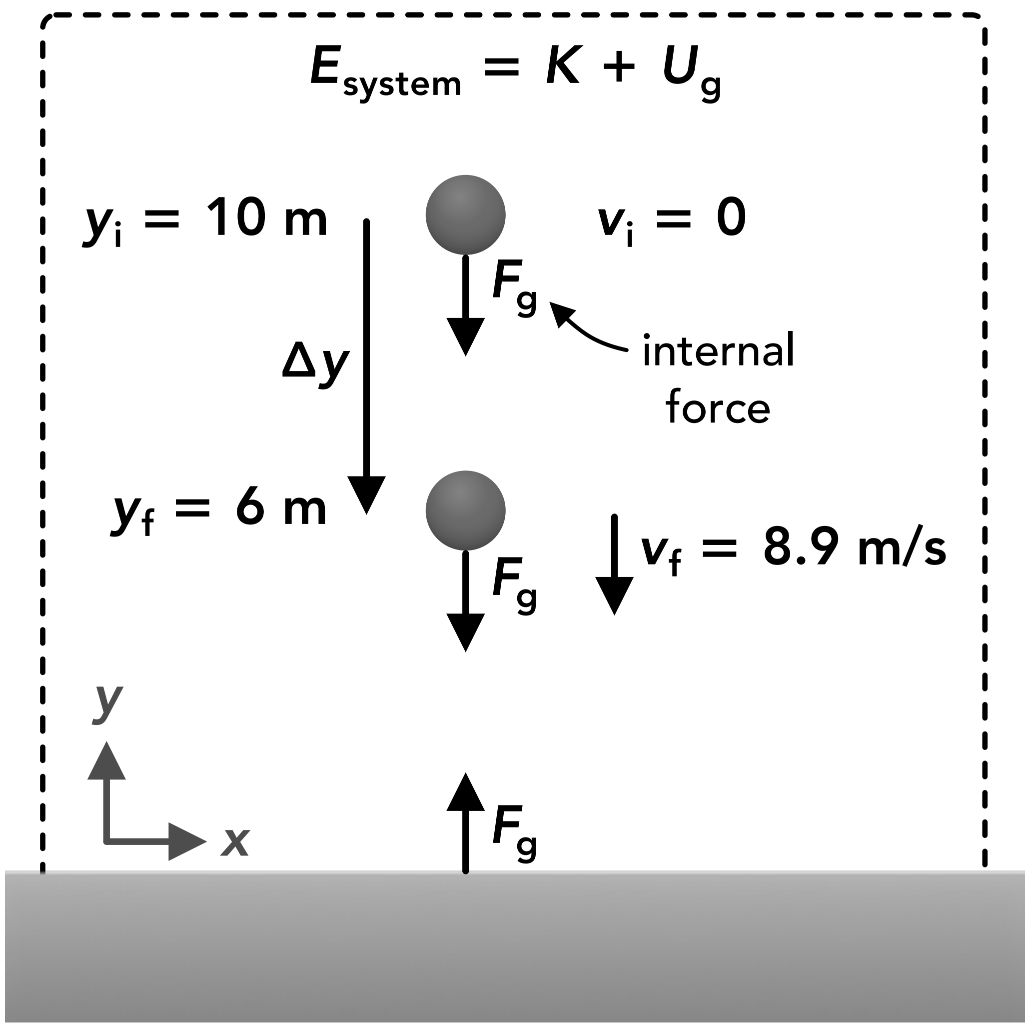
$F_{\parallel}$ and d are in opposite directions, work is negative

$$W = F_{\parallel} d = -f_k d$$

- **Internal forces** are any forces acting between objects within a system (internal objects). Internal forces can convert energy between different types within the system, but they do not change the total energy in the system.
- **External forces** are any forces acting between an object outside of a system (external object) and an object inside the system (internal object).
- **Work is done on or by a system due to external forces**. Internal forces do not do work on a system, they only only convert energy between different types within a system.

If the earth is included in the system, the gravitational force between the ball and the earth is an internal force which converts gravitational potential energy into kinetic energy within the system but it does not do work on the ball.

ball-earth system (isolated)

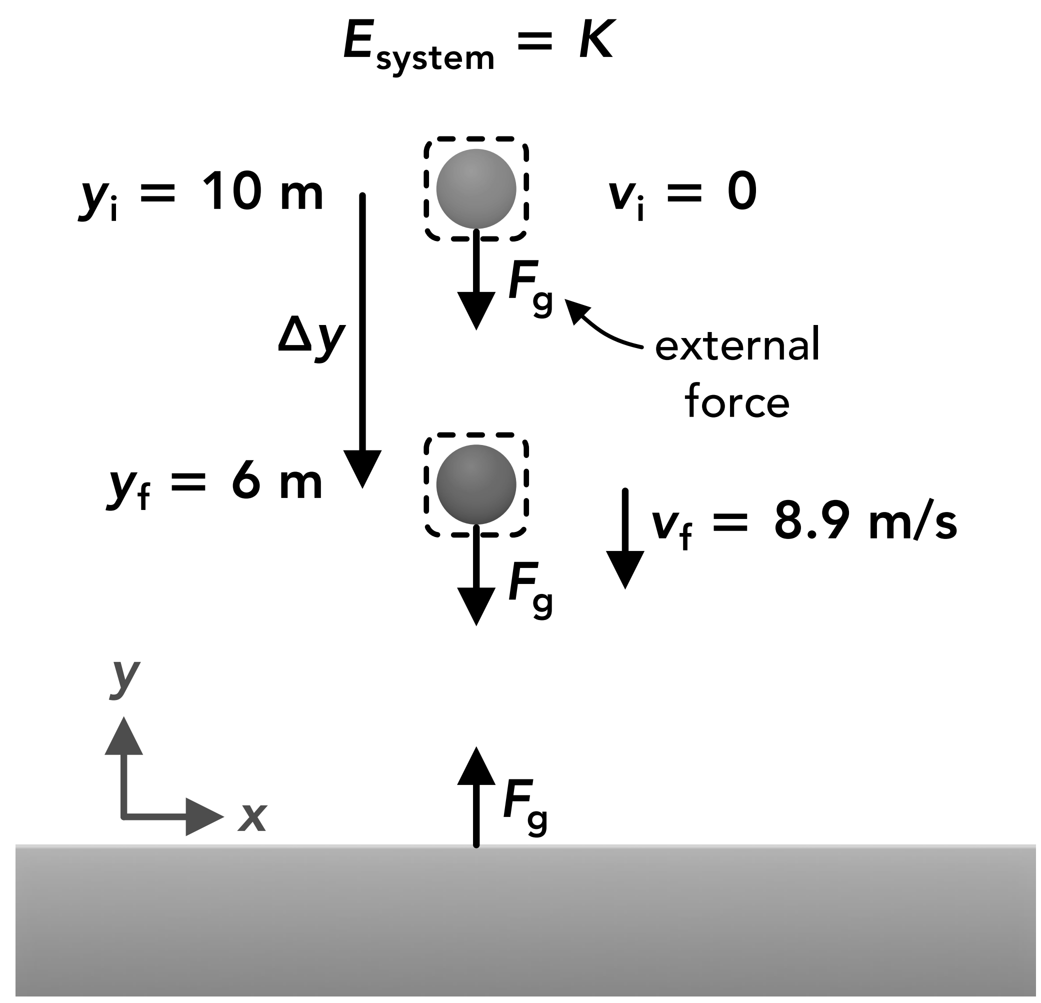


$$\Delta E_{system} = 0$$
$$E_{system\ i} = E_{system\ f}$$
$$K_i + U_{gi} = K_f + U_{gf}$$
$$K_f - K_i = -(U_{gf} - U_{gi})$$
$$\Delta K = -\Delta U_g$$
$$\Delta K = -(mg(6\text{ m} - 10\text{ m}))$$
$$\Delta K = mg(4\text{ m})$$

the change in the kinetic energy of the ball-earth system is equal to the negative of the change in gravitational potential energy

If the earth is not included in the system, the gravitational force acting on the ball is an external force which does work on the ball system, changing its kinetic energy. The ball system does not have gravitational potential energy because the earth is not included in the system.

ball system (not isolated)



$$\Delta E_{system} = W$$
$$\Delta K = F_{\parallel} d$$
$$\Delta K = (-mg)(6\text{ m} - 10\text{ m})$$
$$\Delta K = mg(4\text{ m})$$

the change in the kinetic energy of the ball system is equal to the work done on the system

Power

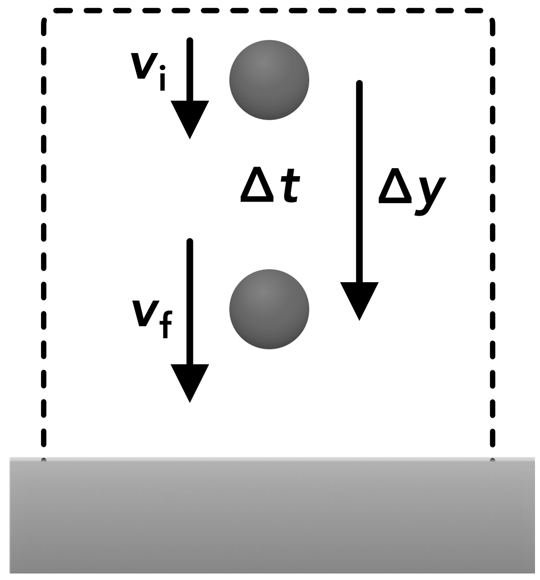
Variables		SI Unit
P	power	$W = \frac{J}{s}$
E	energy	J
W	work	J
F	force	N
v	velocity	$\frac{m}{s}$

- **Power** is the rate of energy converted or transferred over time.
- The SI unit for power is a watt (W) which should not be confused with the variable for work (***W***). 1 watt is equal to 1 joule/second (J/s).
- There are several equations used to calculate power. In each case, "power" means an amount of energy per unit of time.

If energy is being converted from one type of energy to another, the power is the change in one type of energy divided by the period of time

Power

$$P = \frac{\Delta E}{\Delta t}$$



Example:

$$P = \frac{\Delta K}{\Delta t} = \frac{-\Delta U_g}{\Delta t}$$

If energy is transferred into or out of a system through work, the power is the amount of work done divided by the period of time, which is also equal to the parallel force component multiplied by the velocity of the system

$$P = \frac{W}{\Delta t} = \frac{F_{\parallel} d}{\Delta t} = F_{\parallel} v$$

Power

$$P = \frac{W}{\Delta t} = F_{\parallel} v$$

$F_{\parallel}$: component of force parallel to v
*F is an external force
v : velocity of the system