Advanced RC Structures

8

Slender Column 1

- Buckling of Columns
- Compression Plus Bending
- Alignment Chart
- Moment Magnified Method

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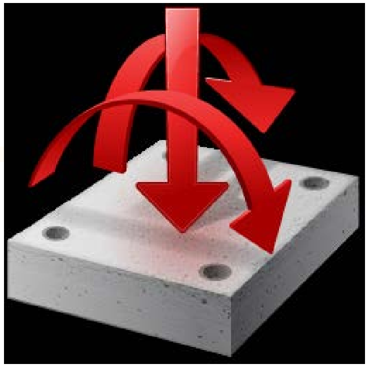
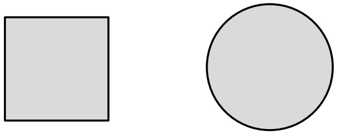
Types of Column Design

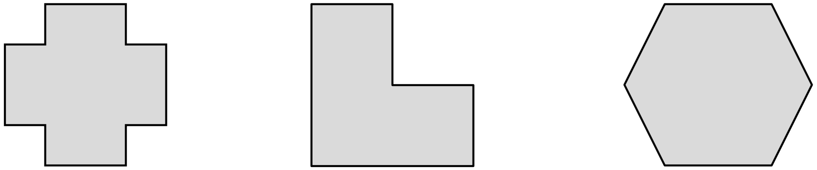


เสาสั้น (Short Column)
หน้าตัดสี่เหลี่ยมและวงกลม
รับแรงตามแนวแกน



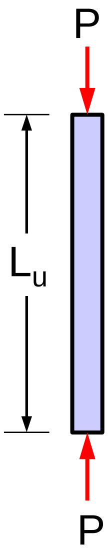
- ➡ รับแรงตามแนวแกน + โมเมนต์ดัด 2 แกน

- ➡ เสายาว (Slender Column)
- ➡ หน้าตัดซับซ้อน

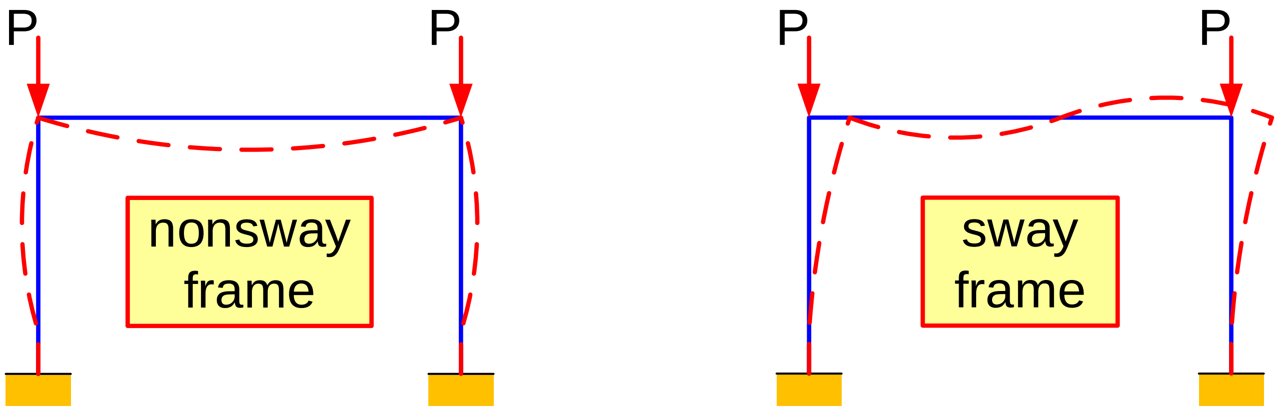


Slender Columns

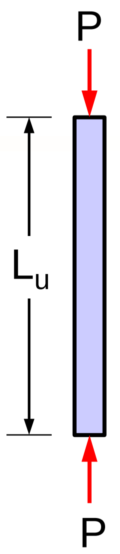
Slender Column = Column with a significant reduction in axial load capacity due to moments resulting from lateral deflections of the column (ACI Code: significant reduction ≥ 5%)



→ Less than 10% of columns in “braced” or “nonsway” frames and less than half of columns in “unbraced” or “sway” frames would be classified as “slender” following ACI Code.



Slenderness Ratio



$$\text{Slenderness ratio} = \frac{kL_u}{r}$$

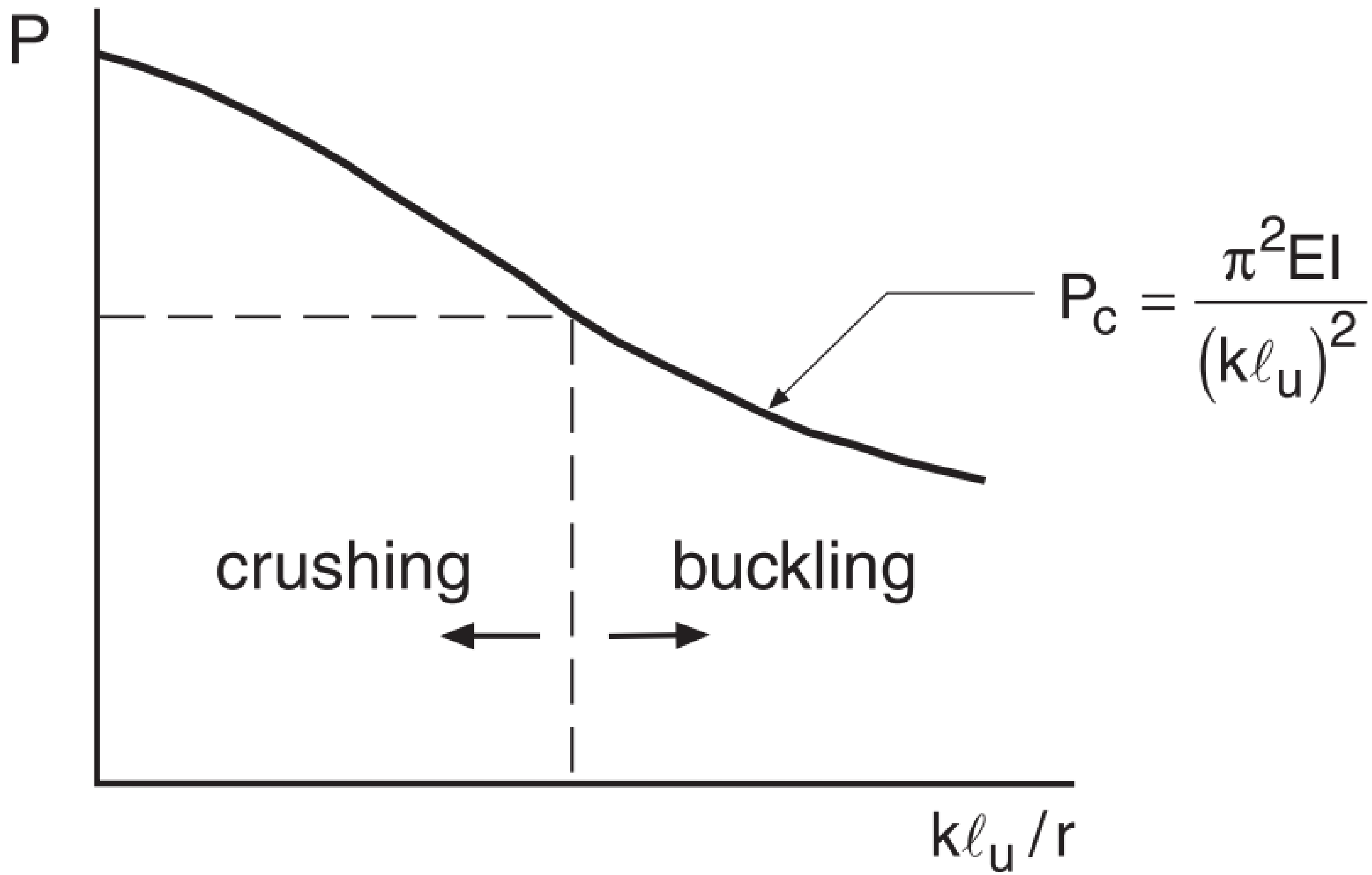
where **k** = effective length factor (depend on rotational and lateral restraints at the ends of the column)

L_u = unsupported column length

r = radius of gyration of the column cross-section

- A column is **slender** if its cross-section is small in comparison to its length.
- **Short column** has strength equal to that computed for its section.
- **Slender column** has strength reduced by second-order deformation.

Slenderness Effect on Column Failure

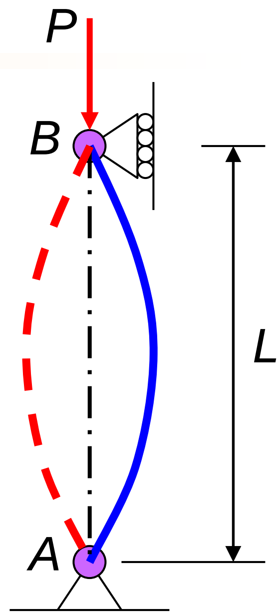


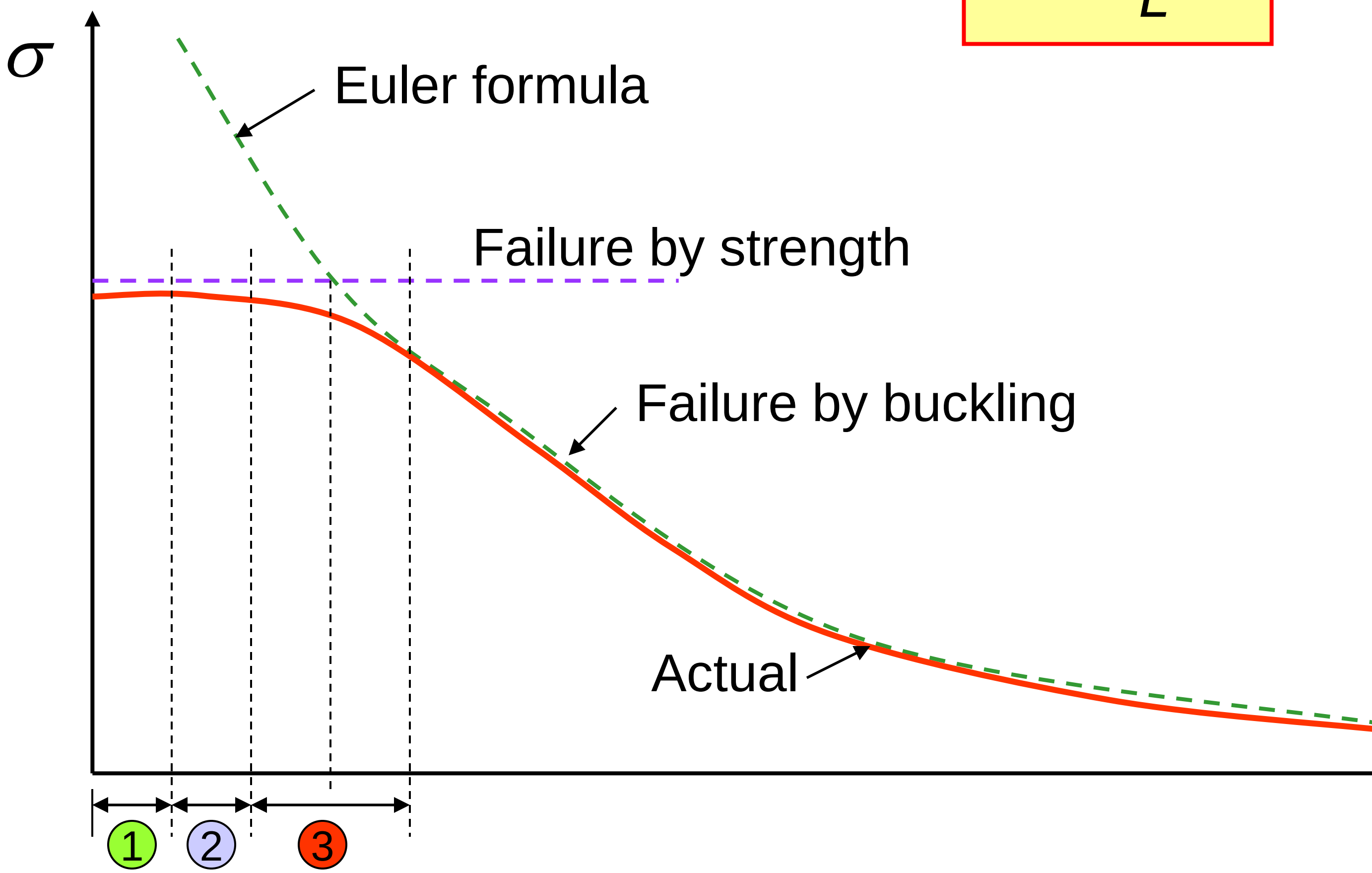
- Short column failure occurs by crushing (material failure)

- Slender column failure occur by buckling (stability failure)

Buckling Strength of Column

$$P = \frac{\pi^2 E I}{L^2}$$





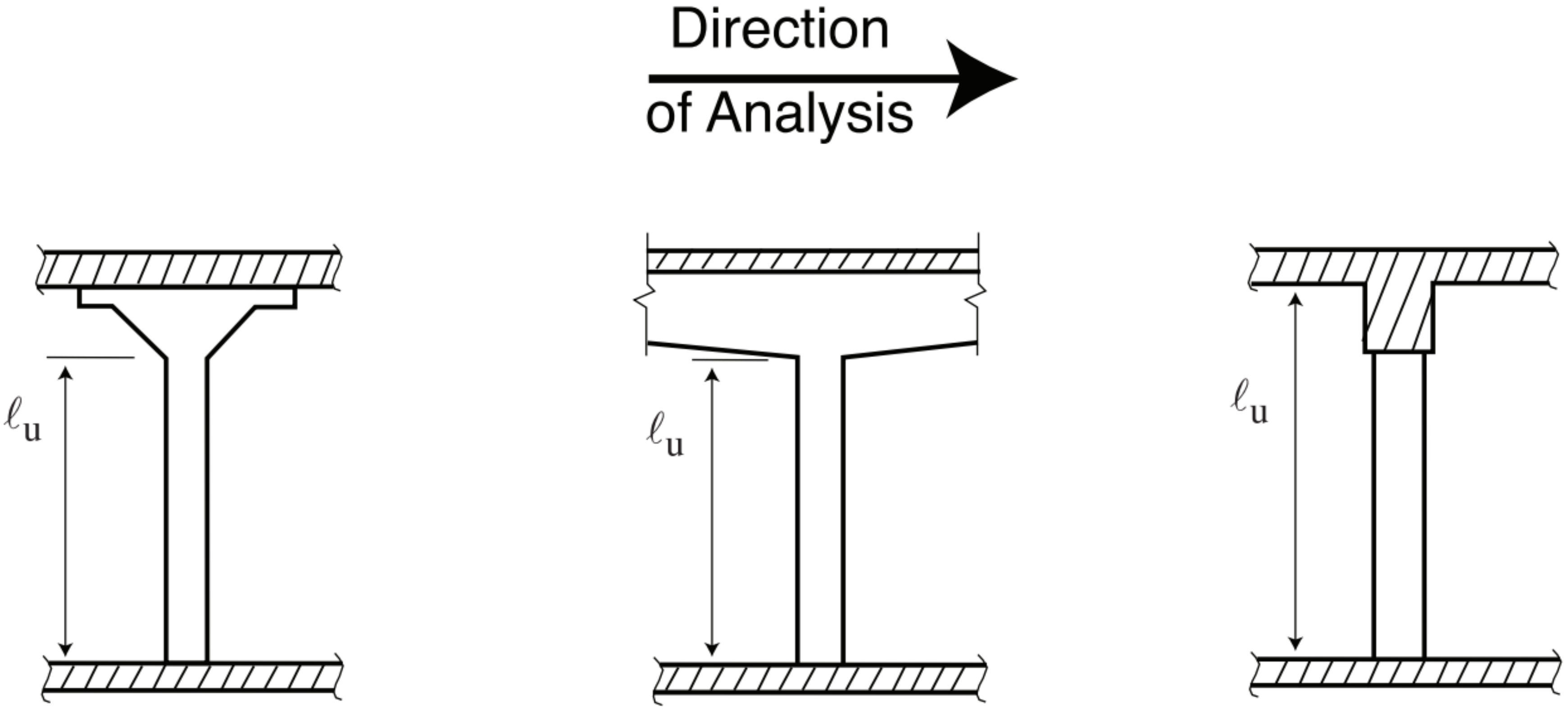
1. Short blocks and piers $L/r \leq 3$

2. "Short" columns (small secondary moments)

3. "Long" columns (significant secondary moments)

Unsupported Length (ℓ_u)

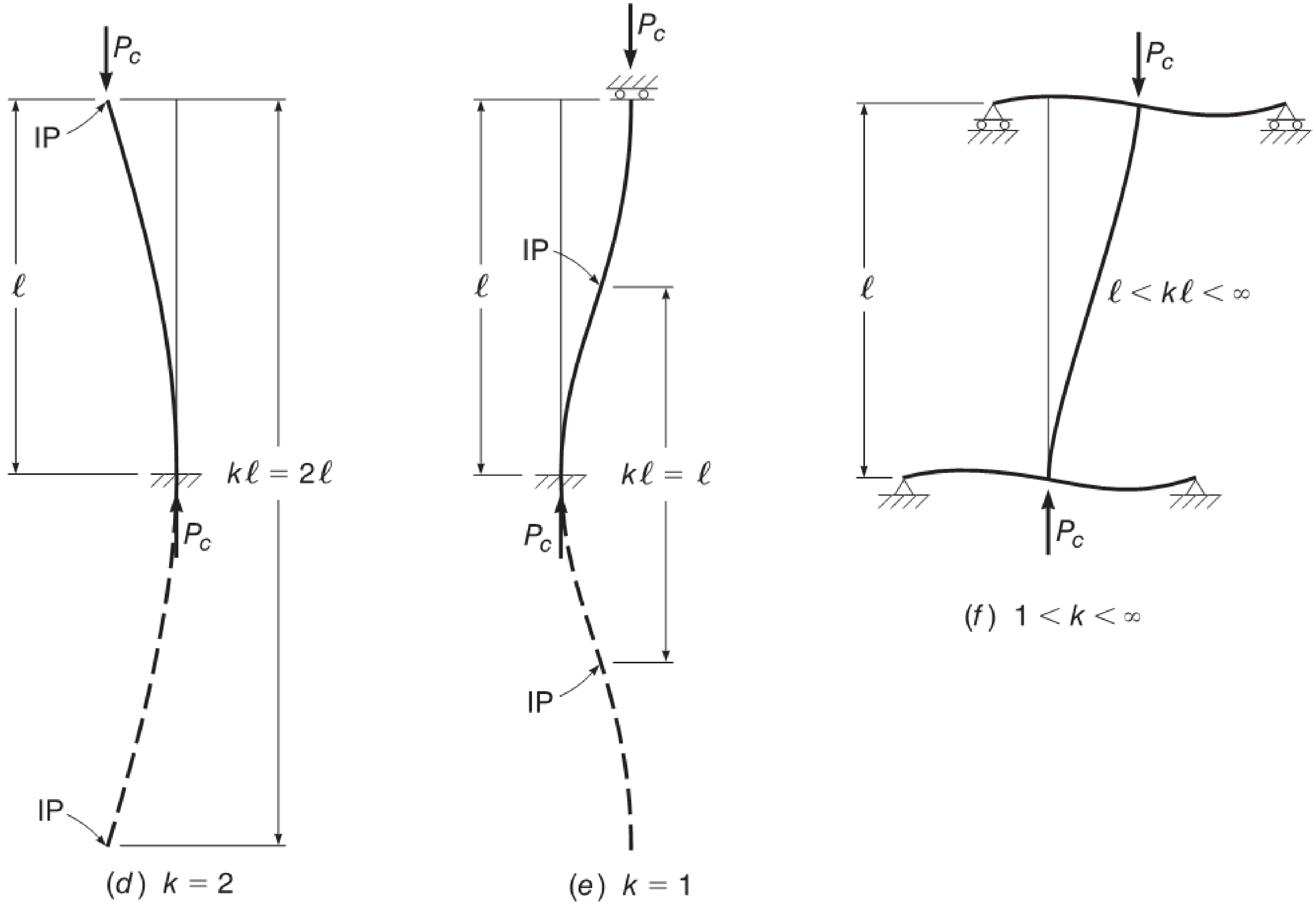
Clear distance between lateral supports



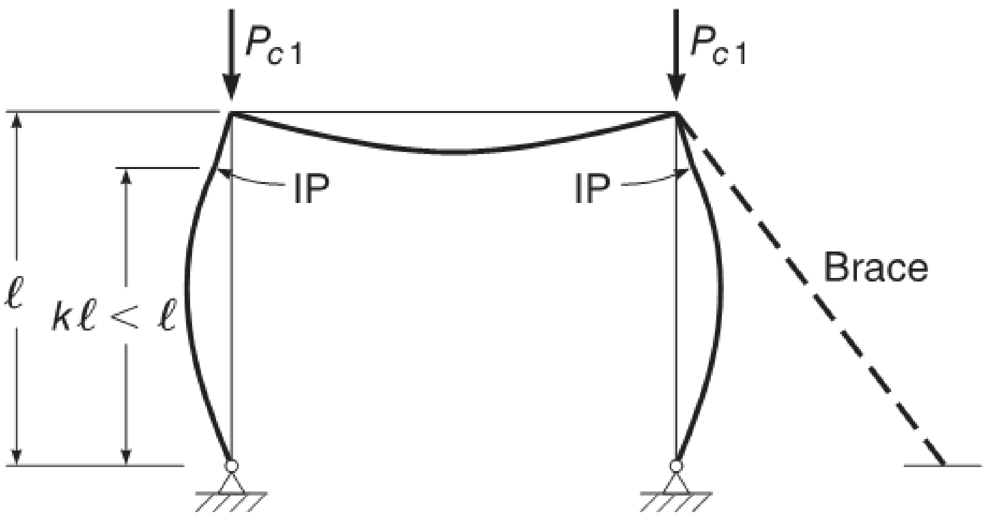
Effects of End Restrainted on Buckling

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2}$$

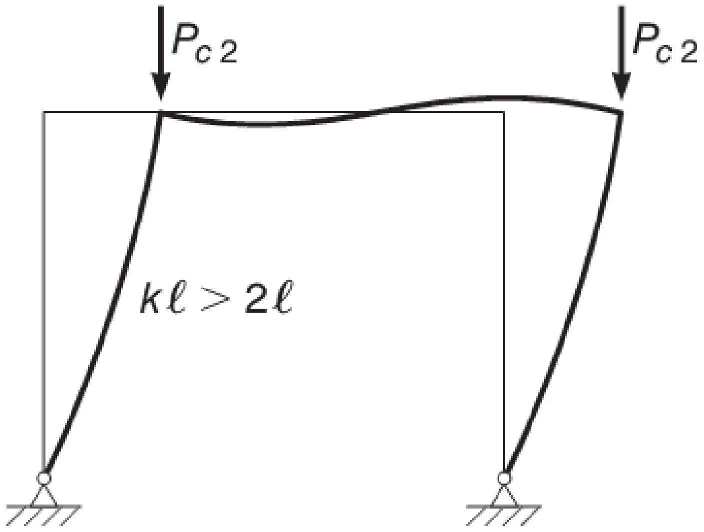
k = effective length factor



Rigid Frame Buckling



Laterally braced frame



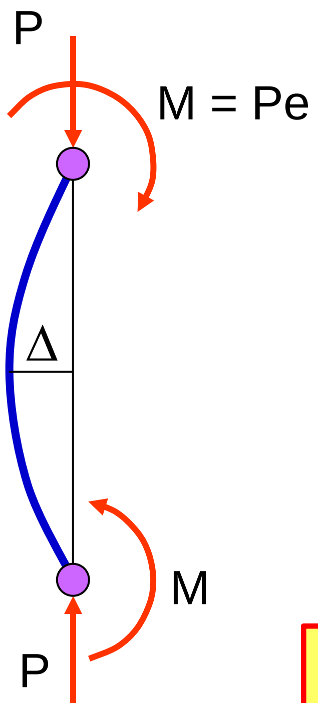
Unbraced frame

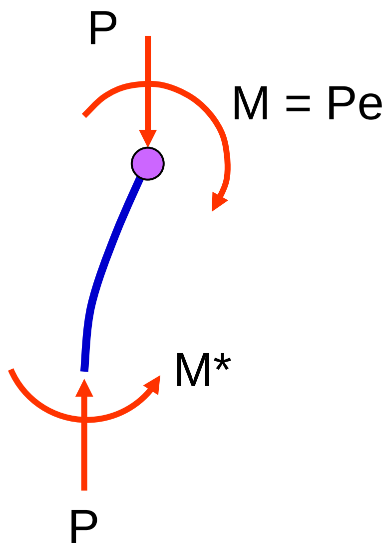
$$P_{c2} << P_{c1}$$

The unbraced frame will buckle at a radically smaller load than the braced frame

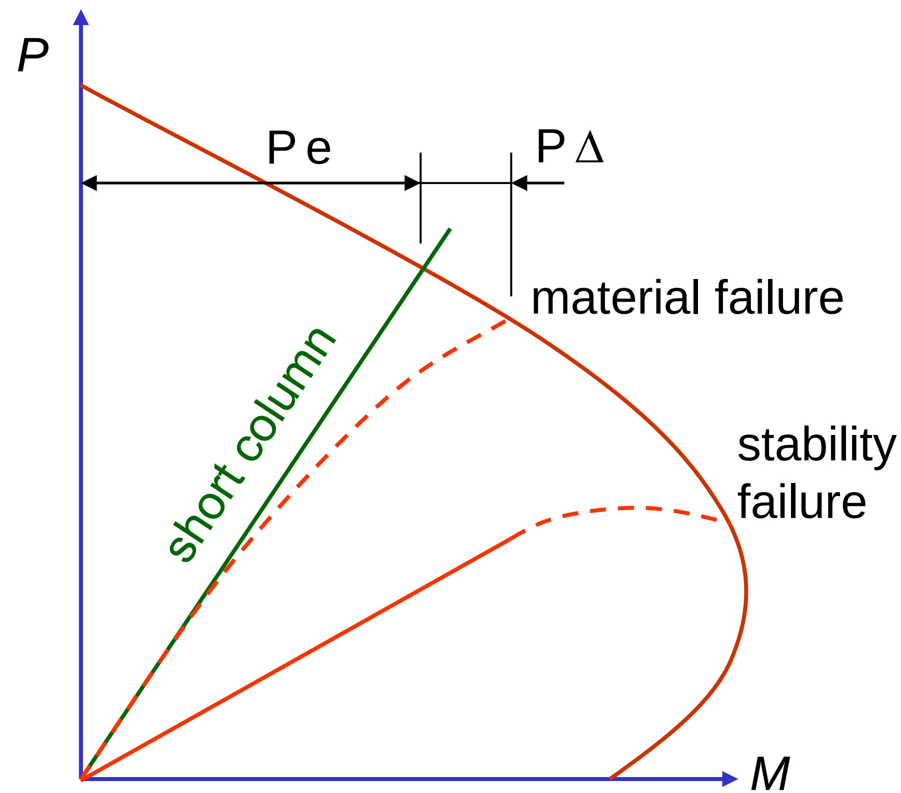
Strength Interaction for Slender Columns

เมื่อเสารับน้ำหนักบรรทุก **P** และโมเมนต์ **M = Pe** เกิดการโก่งตัว **Δ** ทำให้มีโมเมนต์ลำดับที่สองเพิ่มขึ้น **PΔ**



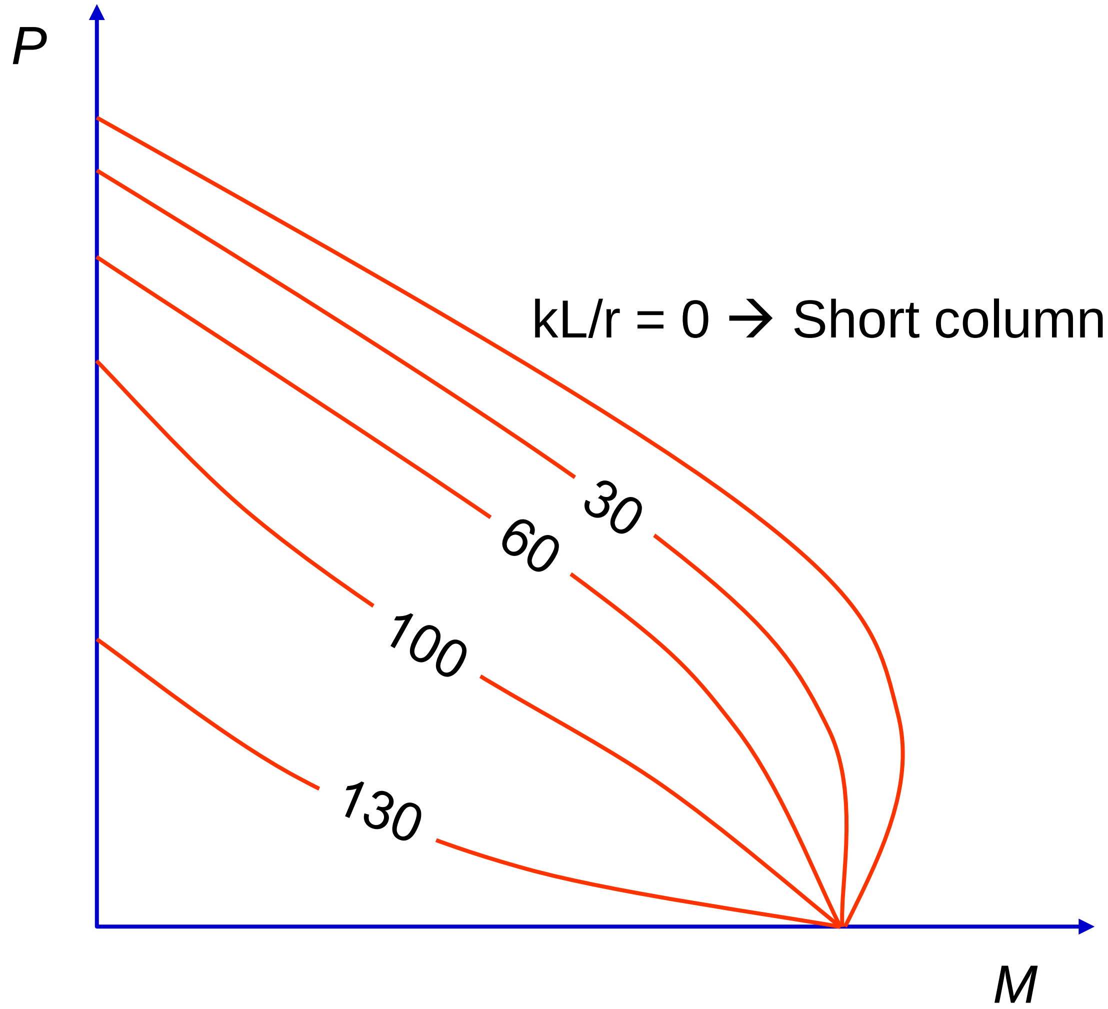


$$M^* = M + P\Delta$$

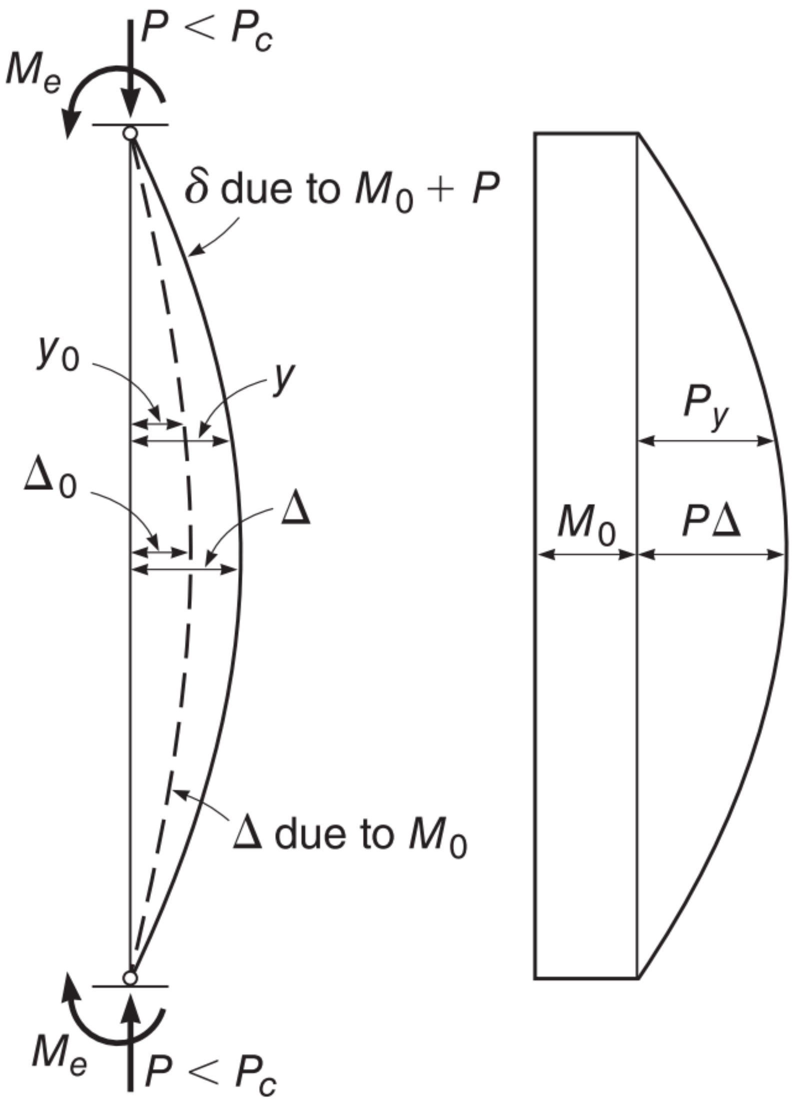


ถ้ามีความชะลูดมาก อาจโก่งตัวเพิ่มมากขึ้นจนเกิดการวิบัติในที่สุด

Strength Interaction Diagrams for Slender Columns



Compression plus Bending



Moments in slender column plus bending, bent in single curvature

At any point, total moments is

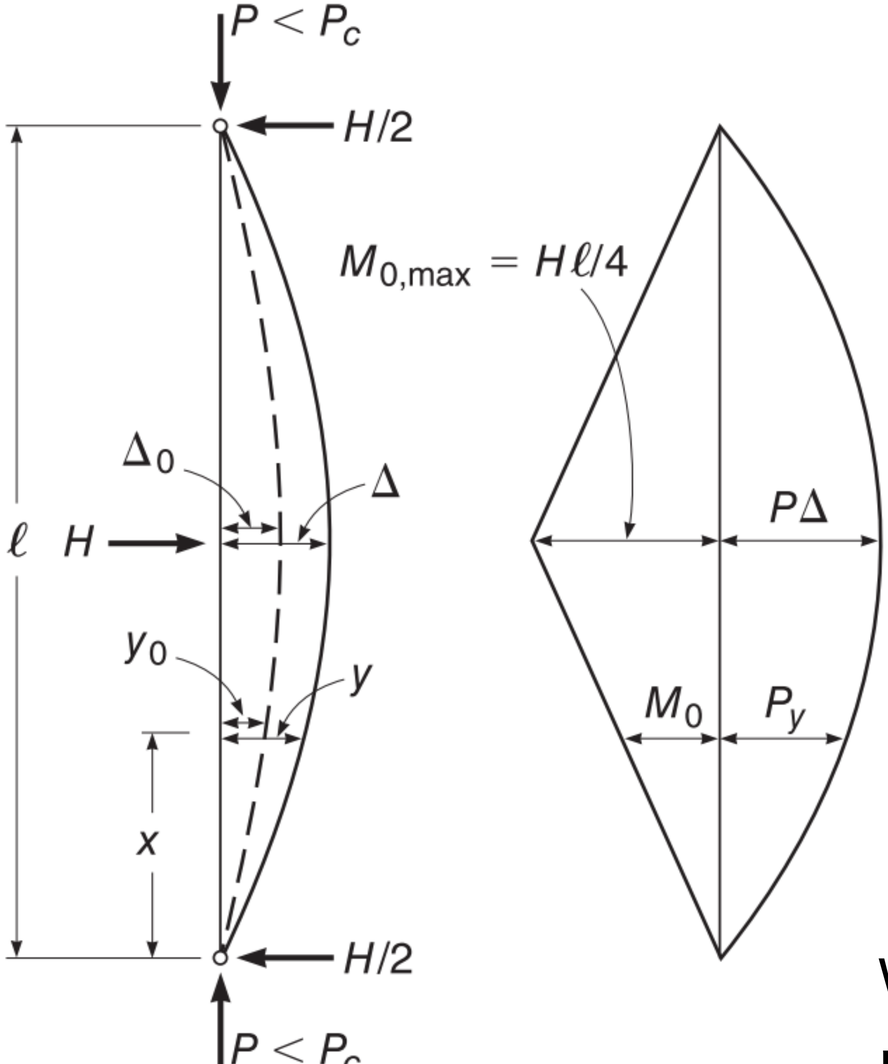
$$M = M_0 + Py$$

where y is deflection at any point

P-Δ Effect

Compression plus Bending

Similar situation of bending caused by transverse force H



Max. moment at midspan :

$$M_{0,max} = \frac{H\ell}{4}$$

Deflection $y = y_0 \dfrac{1}{1 - P/P_c}$

Δ is the deflection at M_{max}

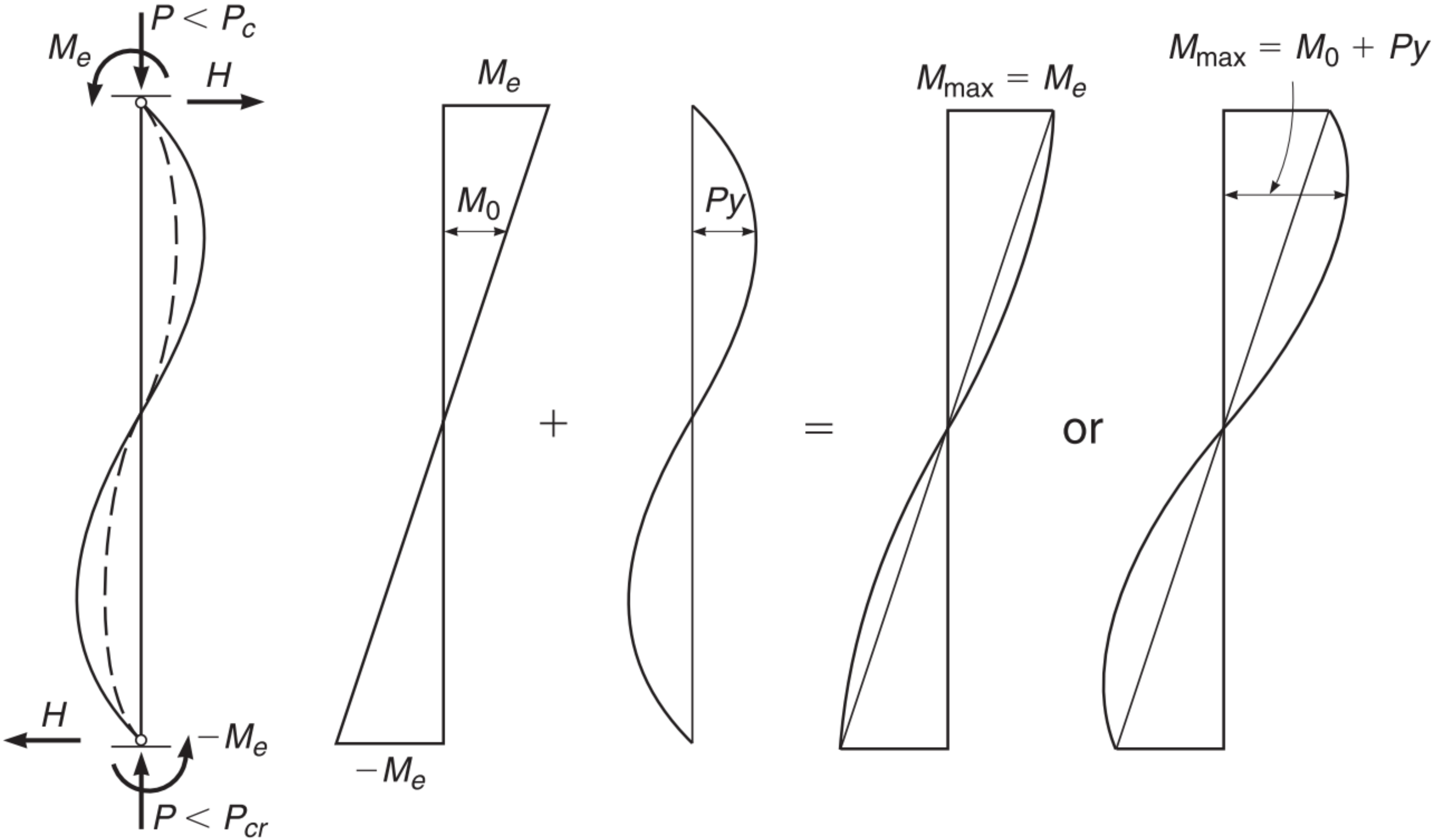
$$M_{max} = M_0 + P\Delta$$

$$\approx M_0 \frac{1}{1 - P/P_c}$$

where $1/(1 - P/P_c)$ is the moment magnification factor

Compression plus Bending

Moments in slender column plus bending, bent in double curvature.



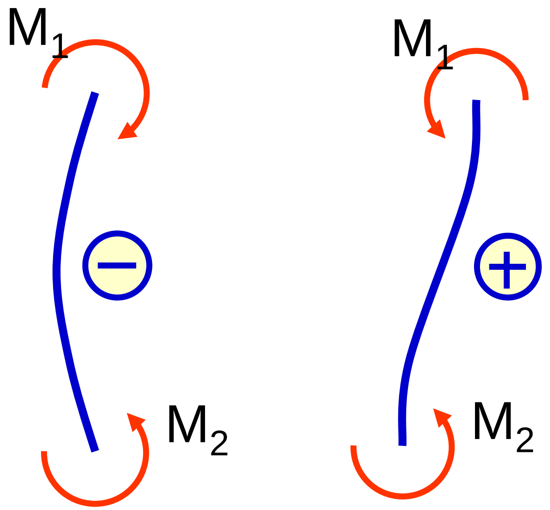
Modified Maximum Moment

Including effects of end moments

$$M_{max} = M_0 \frac{C_m}{1 - P/P_c}$$

- For columns without transverse loads applied between support

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \geq 0.4$$



M_1 = numerically smaller end moment

M_2 = numerically larger end moment

$M_{2,min} = P_u(1.5 + 0.03h)$

Negative M_1/M_2 produces single curvature

Positive M_1/M_2 produces double curvature

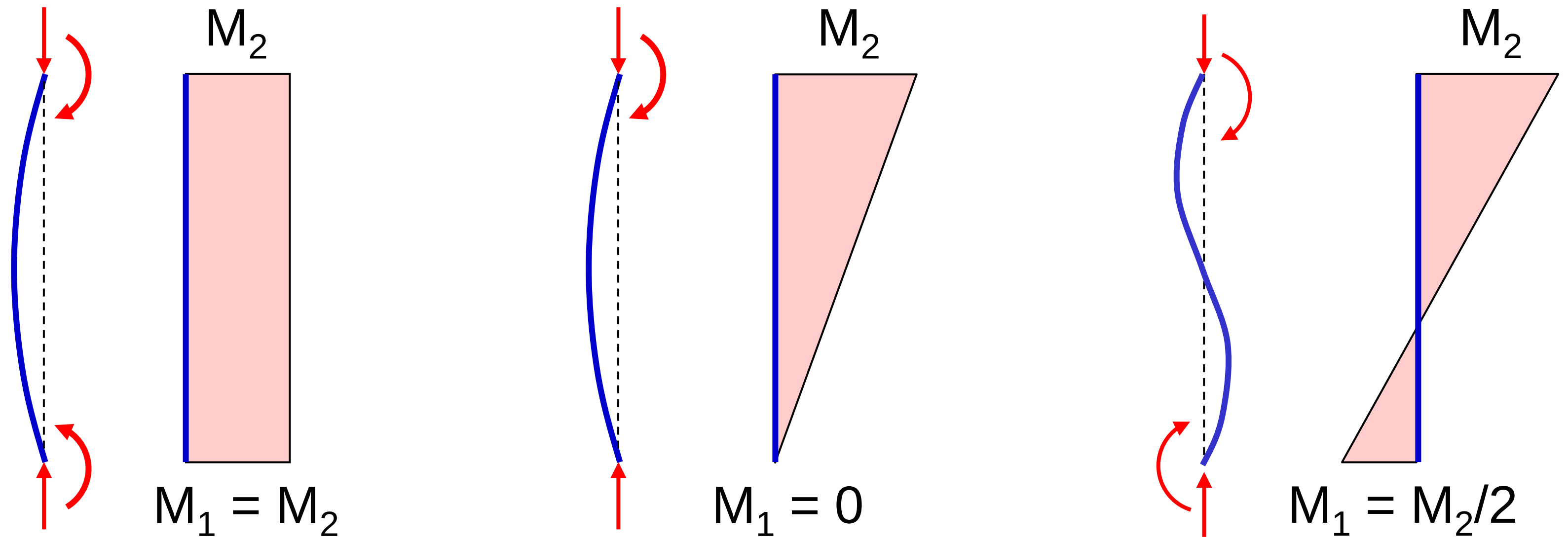
- For columns with transverse loads applied between support $C_m = 1.0$

*M_1/M_2 sign changed from ACI318-11 to ACI318-14

Values of C_m for Slender Columns

in Nonsway Frames

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \geq 0.4$$

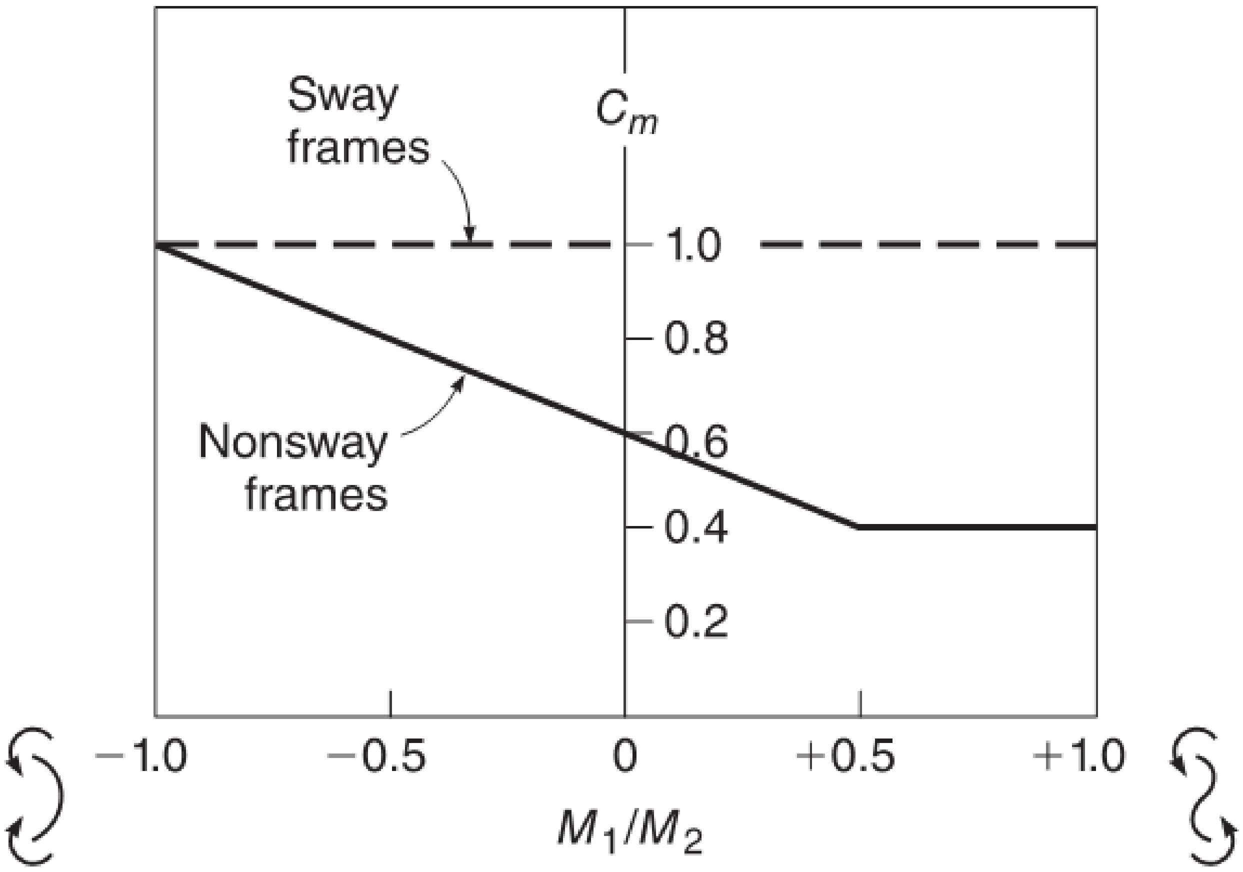


$M_1/M_2 = -1.0$	$M_1/M_2 = 0$	$M_1/M_2 = 0.5$
↓	↓	↓
$C_m = 1.0$	$C_m = 0.6$	$C_m = 0.4$
MAX		**MIN**

Values of C_m for Slender Columns

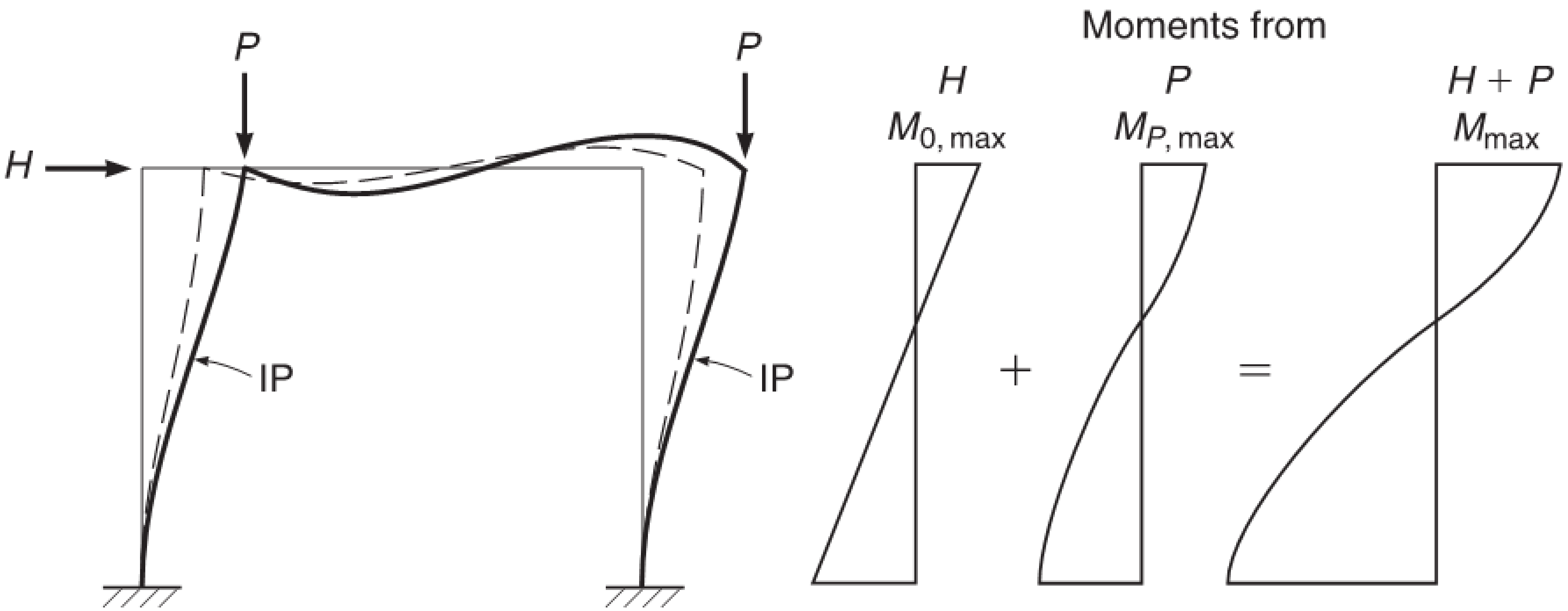
in Sway and Nonsway Frames

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \geq 0.4$$



Unbraced Frame

Sidesway can occur only for the entire frame simultaneously, not for individual columns in the frame.



Force H alone causes the frame deformation in a dash line and the moment M_0.

When force P is added, additional moment M_p increase deformation in a solid line.

Slenderness effects

ACI318-11 10.10 Slenderness effects in compression members

ACI318-14 6.2.5 Slenderness effects shall be permitted to be neglected if **(a)** or **(b)** is satisfied:

(a) For columns not braced against sidesway

$$\frac{kL_u}{r} \leq 22$$

(b) For column braced against sidesway

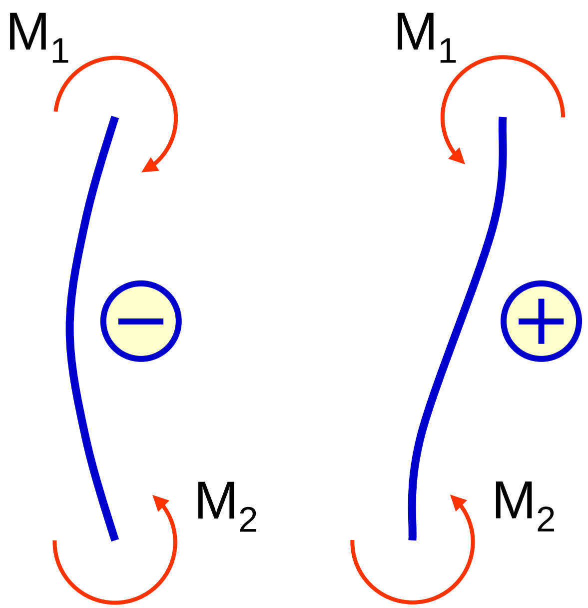
$$\frac{kL_u}{r} \leq 34 + 12\frac{M_1}{M_2} \leq 40$$

where **M_1/M_2** is negative if the column is bent in single curvature, and positive if the member is bent in double curvature.

Long - Short Columns

Short Columns:

$$\frac{kL}{r} \begin{cases} \leq 22 & \text{Unbraced members} \\ \leq 34 - 12\dfrac{M_1}{M_2} & \text{Braced members} \end{cases}$$



$$\frac{M_1}{M_2} \begin{cases} - & \text{for single curvature} \\ + & \text{for double curvature} \end{cases}$$

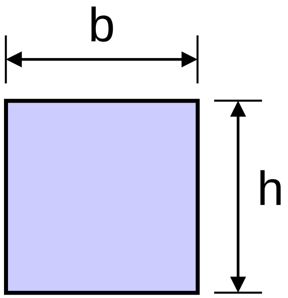
$$M_2 = \text{Largest of two moments}$$

Long Columns: Otherwise

Radius of gyration (r)

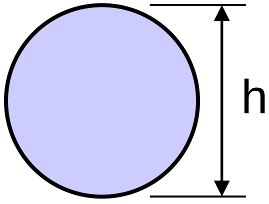
$$r = \sqrt{\frac{I_g}{A_g}}$$

Rectangular column: width = b, depth = h



$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{\frac{1}{12}bh^3}{bh}} = 0.288h \approx 0.30h$$

Circular column: diameter = h



$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{\pi h^4 (4)}{64\pi h^2}} = 0.25h$$