THE SCIENCE CUBE

Calculating Torque & Finding its Direction

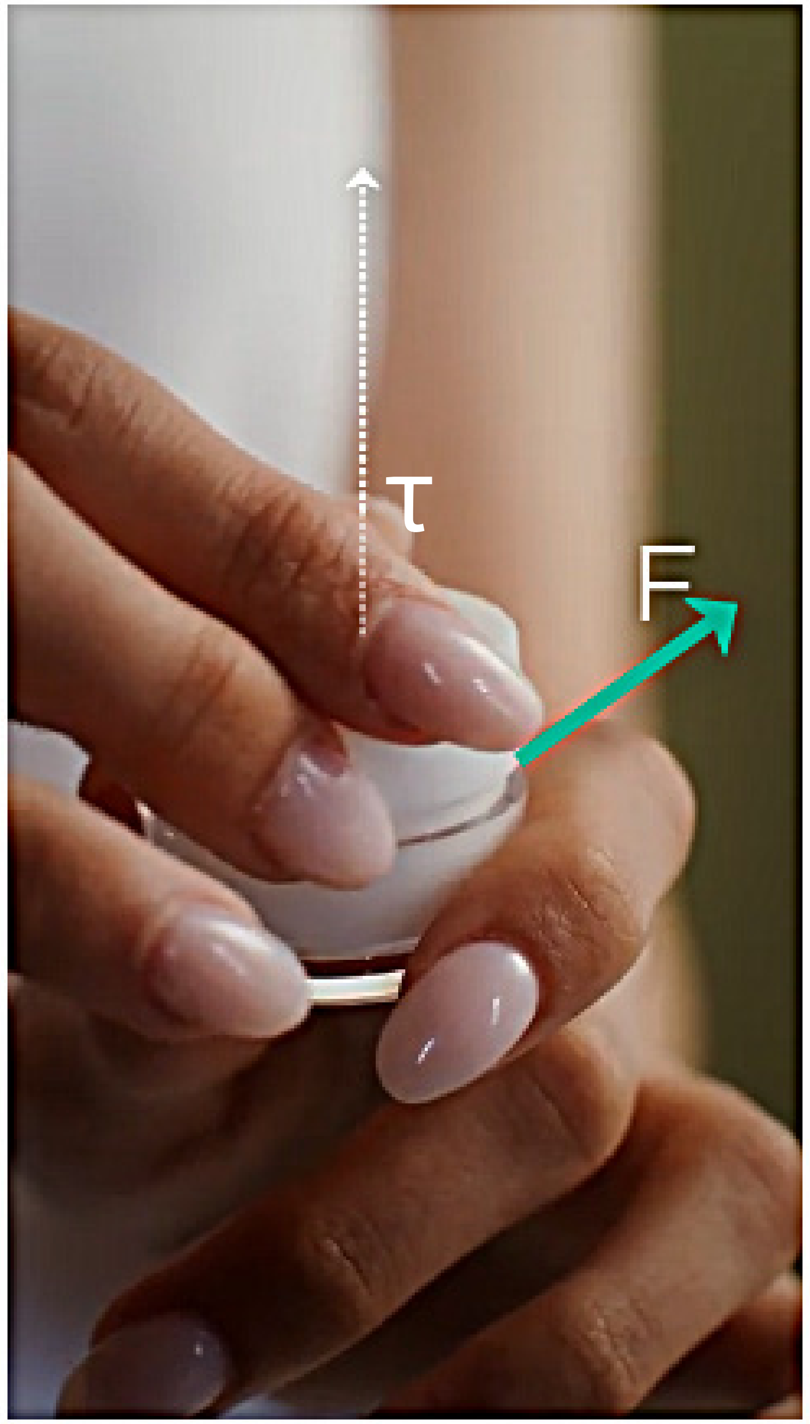
1. Introduction to Torque on a Particle
2. Defining Torque Mathematically
3. Finding the Direction & Magnitude of Torque
4. Why Do We Use sin(ϕ)?
5. Perpendicular Force Component (Interpretation 1)
6. Moment Arm (Interpretation 2)
7. Key Formulas & Equations
8. Common Mistakes and Misconceptions:

TORQUE ON A PARTICLE ABOUT A POINT

1. Torque is the rotational equivalent of force. It causes angular acceleration quite the way force cause linear acceleration

2. In a way, torque measures the rotational force's effectiveness in causing angular acceleration.

   - A larger torque leads to a greater tendency for rotation

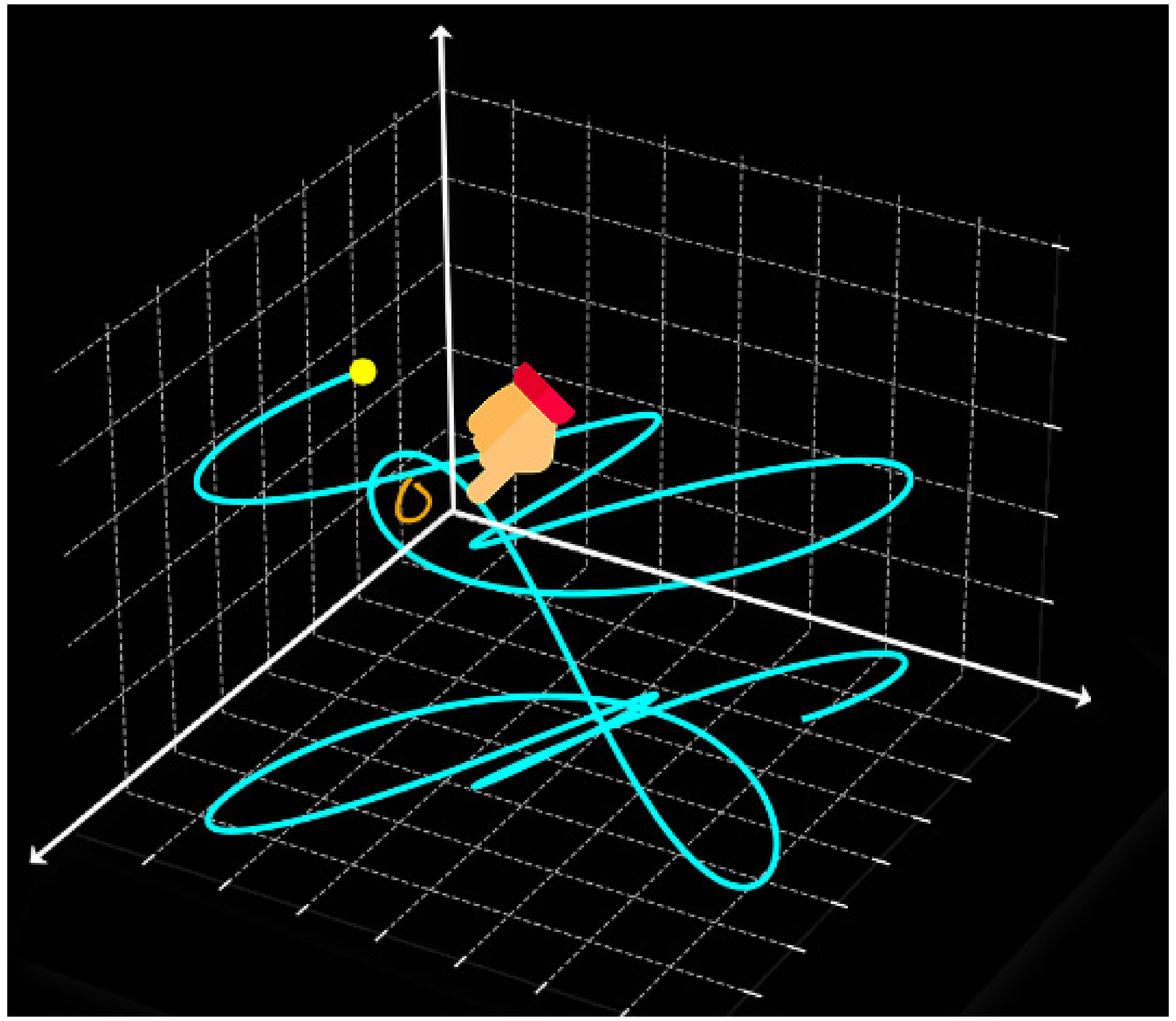


Torque vector points perpendicular to the force vector. (unexpected!)

TORQUE ON A PARTICLE ABOUT A POINT

3. It can be calculated with respect to any reference point
   - It is not necessary to have a physical connection to the force
   - Example - torque on a particle moving along any trajectory relative to a fixed point "O"
   - The path of motion does not have to be a circle.

4. Torque, being a vector, can point in any direction depending on the situation.



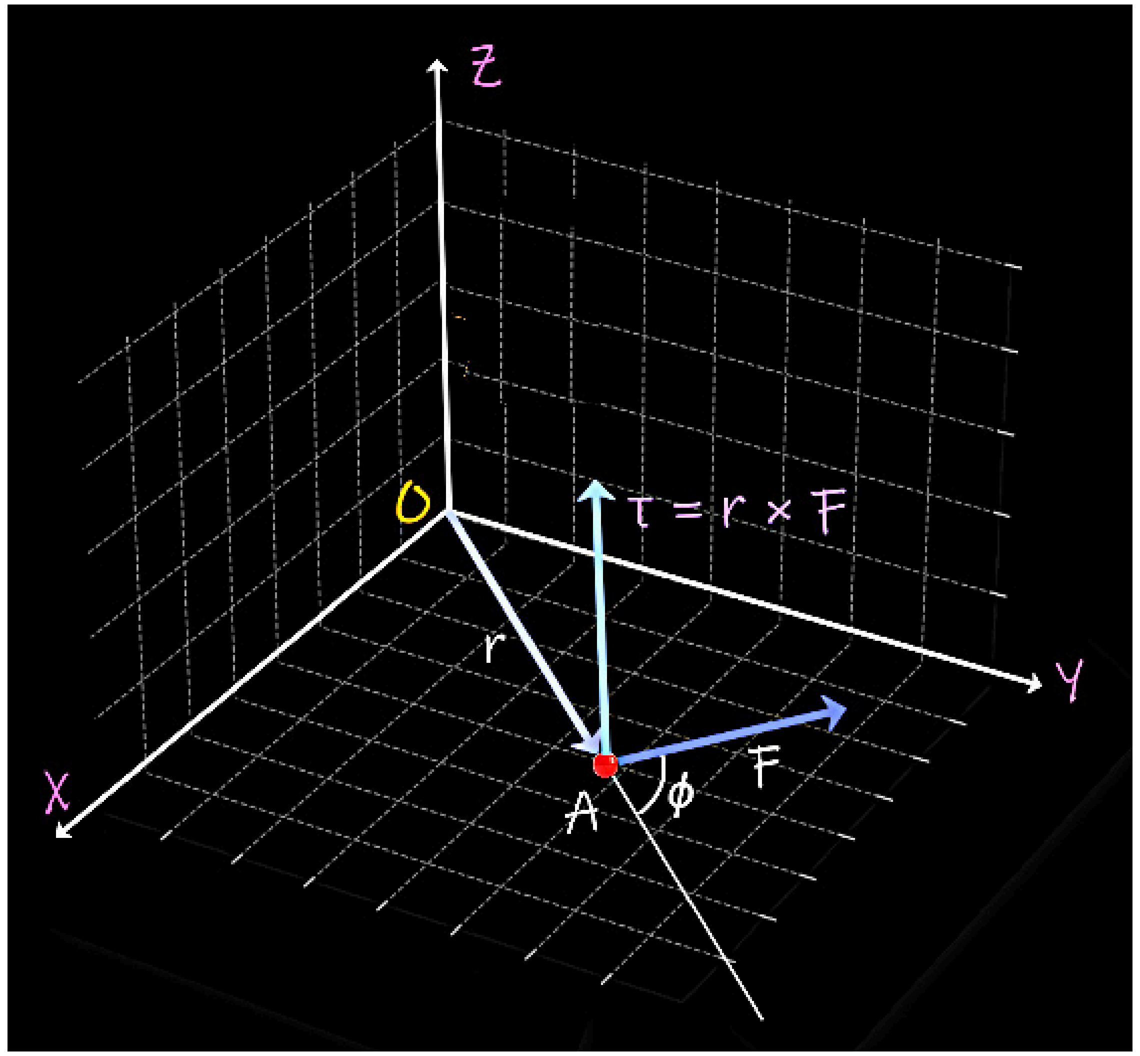
Here torque is being calculated about "O". It can be found at any point of the trajectory of the particle

DEFINING TORQUE MATHEMATICALLY

1. Consider a particle at point A in the xy plane with a force F acting on it , then -
   - If the position vector of the particle is r, the torque relative to a fixed point O is given by

$$\boxed{\tau = r \times F} \qquad \text{(cross product)}$$

$$\tau = |r|\,|F| \sin \phi \qquad \text{(Magnitude)}$$



2. Key points to remember:
   - Torque is a vector quantity,
   - It is always perpendicular to the plane formed by r and F
   - Order of cross product is r × F, <u>not</u> F × r

FINDING THE DIRECTION OF TORQUE

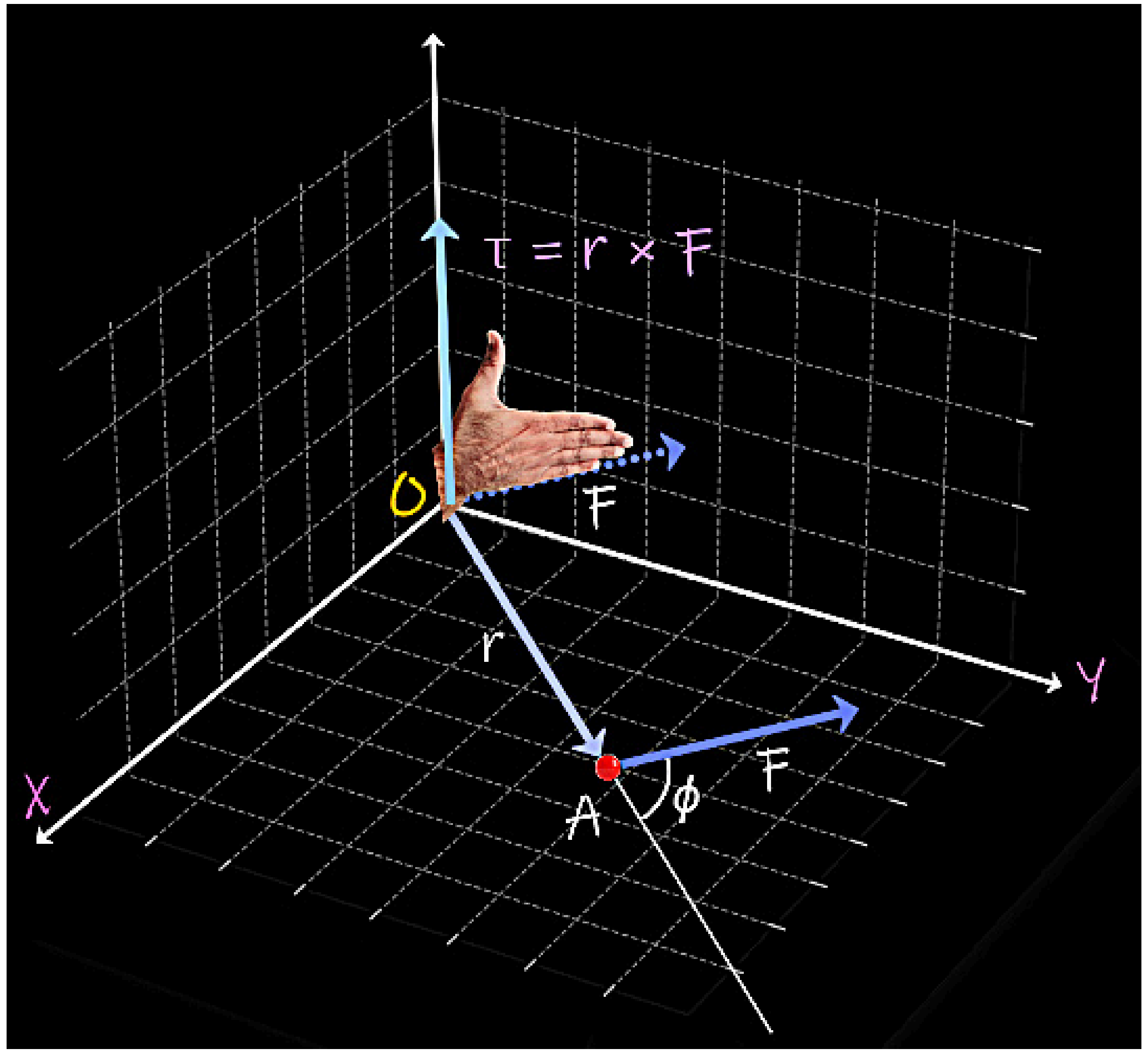
The direction of torque is determined using the right-hand rule:

a. Align the vectors tail-to-tail.

b. Point your right-hand fingers in the direction of r (the position vector)

c. Sweep them towards F

d. Your thumb points in the direction of torque

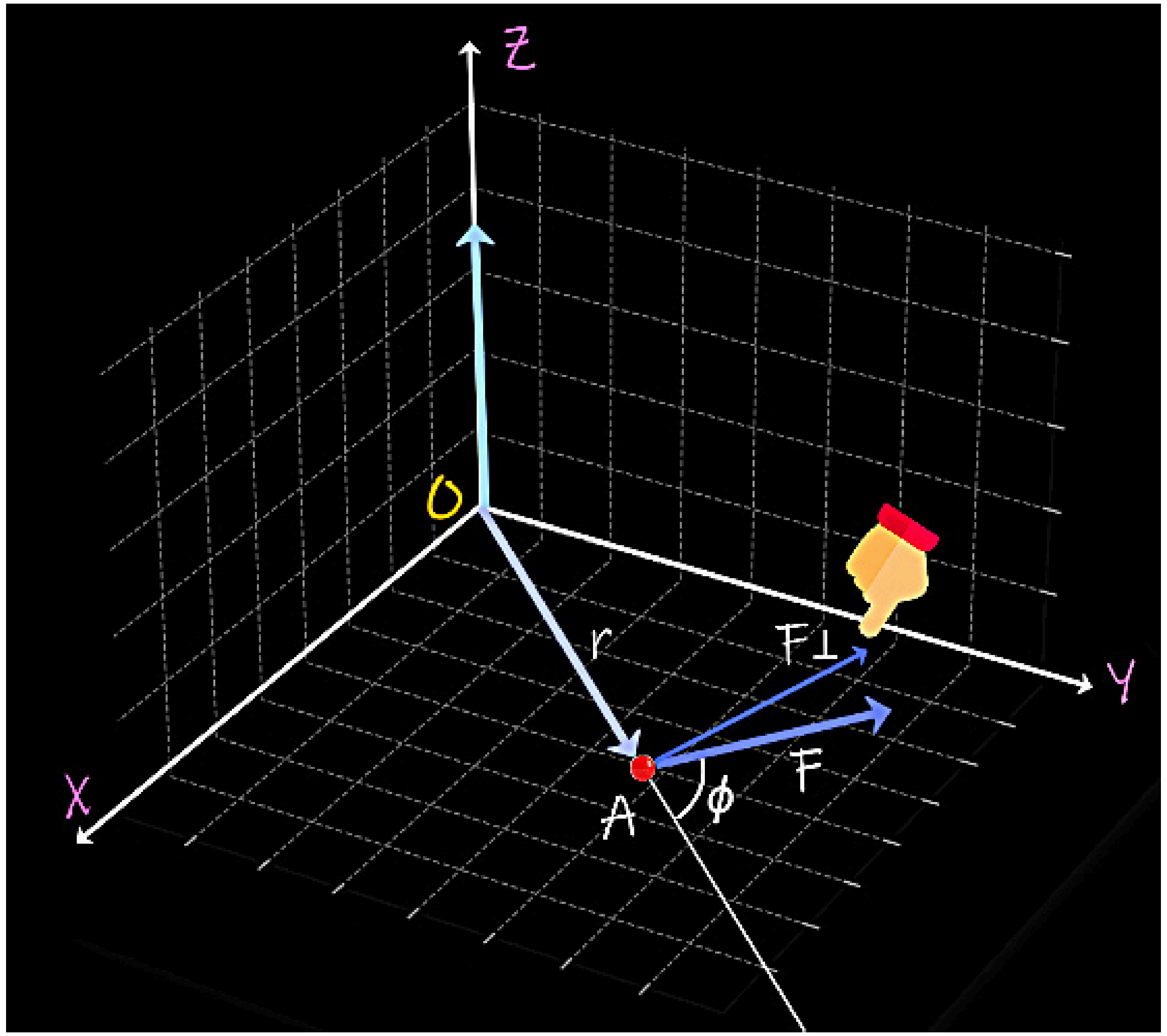


WHY DO WE USE SIN(ϕ)?

1. The cross product isolates the part of F that is perpendicular to r i.e. Fsin ϕ

2. Only the perpendicular force component contributes to rotation.

3. If F is parallel to r, there is no perpendicular component → No torque → No rotation.



PERPENDICULAR FORCE ($F_\perp$) COMPONENT

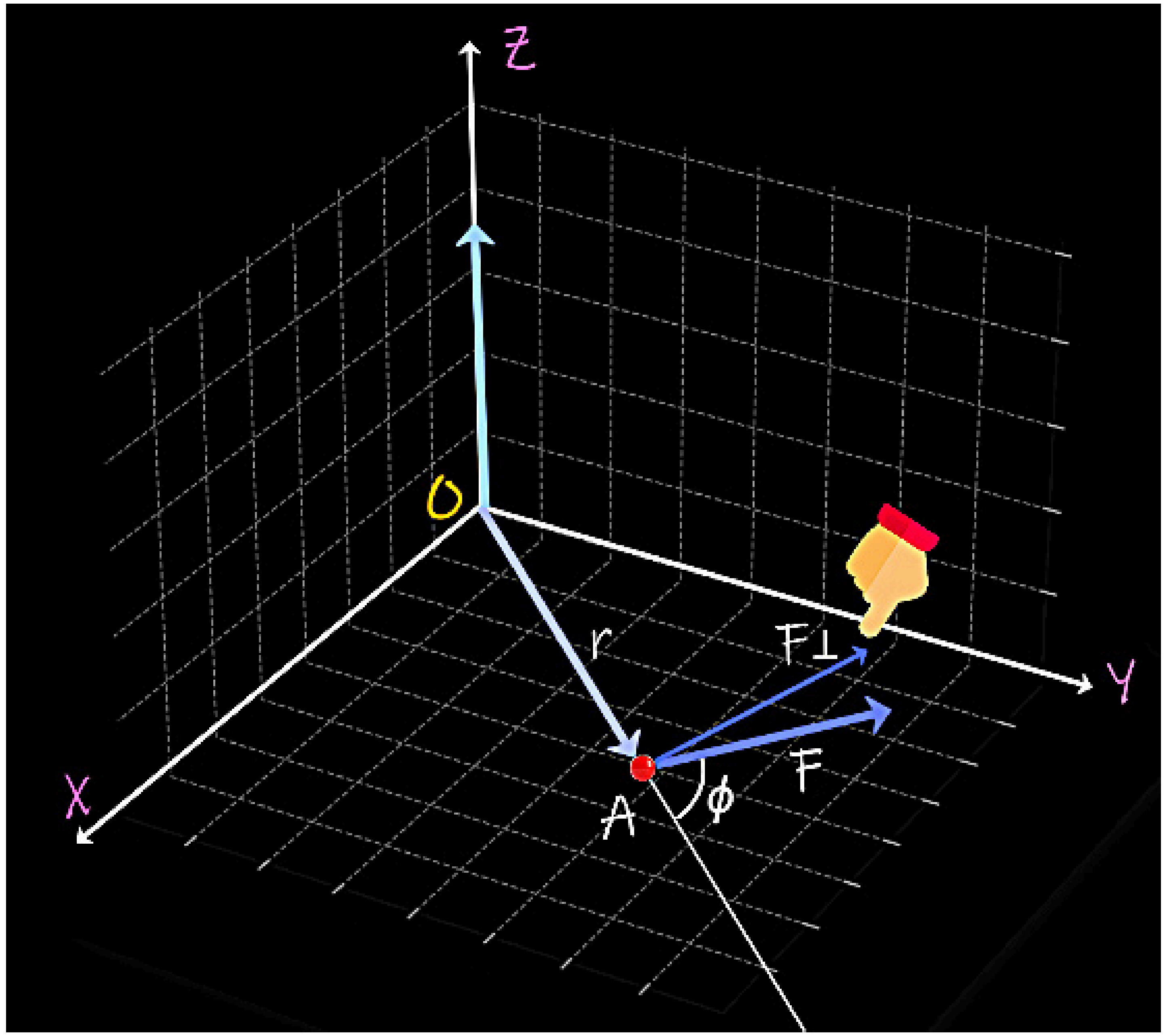
Perpendicular force component method is another way to interpret torque:

$$\tau = r F \sin \phi$$

$$F_\perp = F \sin \phi$$

$$\boxed{\tau = r F_\perp}$$

This can be interpreted as $F_\perp$ is the only effective push causing rotation



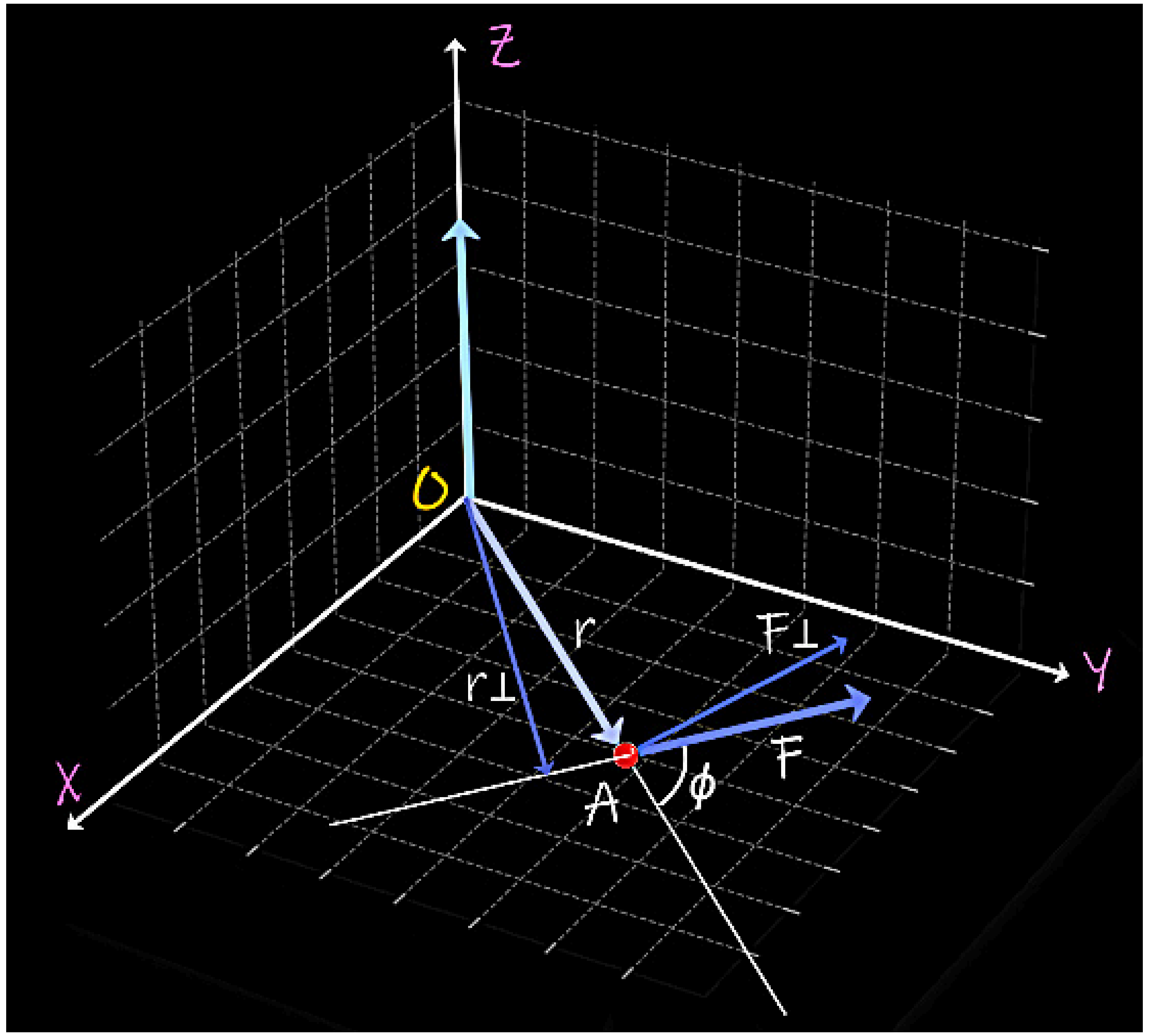
MOMENT ARM ($R\perp$) APPROACH

Moment arm component method is another way to interpret torque:

$$\tau = r F \sin \phi$$

$$r\perp = r \sin \phi$$

$$\boxed{\tau = r\perp F}$$



$r\perp$ (moment arm) is the perpendicular distance from point O to the line of action of F

This can be interpreted as - A larger moment arm ($r\perp$) makes it easier to rotate

KEY FORMULAS & EQUATIONS

Equation	When to Use	Notes
$\tau = r \times F$	General definition of torque as a vector quantity.	Torque is found as the cross product of r and F. Order matters: $r \times F$, not $F \times r$.
$\tau = r F_{\perp} \sin \phi$	To calculate the magnitude of torque using the angle between r and F.	ϕ is the smaller angle between r and F. If $\phi = 0$, torque is zero (no rotation).
$\tau = r F_{\perp}$	To determine torque using the perpendicular component of force.	$F_{\perp} = F \sin \phi$, which represents the perpendicular component of force that contributes to rotation.
$\tau = r_{\perp} F$	To determine torque using the moment arm (shortest distance between point O and force).	$r_{\perp} = r \sin \phi$, which represents the moment arm affecting the torque.

COMMON MISTAKES AND MISCONCEPTIONS

1. Misconception: Torque and Force are Identical

Clarification: While both torque and force influence an object's motion, they are distinct concepts. Force causes linear motion, whereas torque causes rotational motion. Torque is the product of force and the perpendicular distance from the pivot point (lever arm).

2. Misconception: Torque is Always in the Plane of Rotation

Clarification: Torque is a vector quantity resulting from the cross product of the position vector and the force vector. Its direction is perpendicular to the plane formed by these vectors, as determined by the right-hand rule.

COMMON MISTAKES AND MISCONCEPTIONS

3. Misconception: Torque & Energy are Same Because They Share Same Units

Clarification: Although torque and energy both have units of Newton-meters (N·m), they represent different physical quantities. Energy is a scalar quantity measuring the capacity to perform work, while torque is a vector quantity measuring the tendency of a force to cause rotational motion.

4. Misconception: Increasing Force Always Increases Torque

Clarification: Torque depends not only on the magnitude of the applied force but also on the lever arm's length and the angle between the force and the lever arm. Applying force closer to the pivot point or at a less effective angle can result in less torque, even if the force magnitude increases.

5. Misconception: Torque and Force Have the Same Direction

Clarification: Torque is a vector quantity resulting from the cross product of the position vector (r) and the force vector (F). Its direction is perpendicular to the plane formed by these vectors, as determined by the right-hand rule. Therefore, torque does not necessarily have the same direction as the applied force