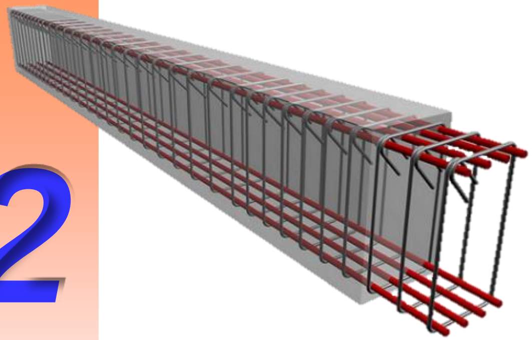
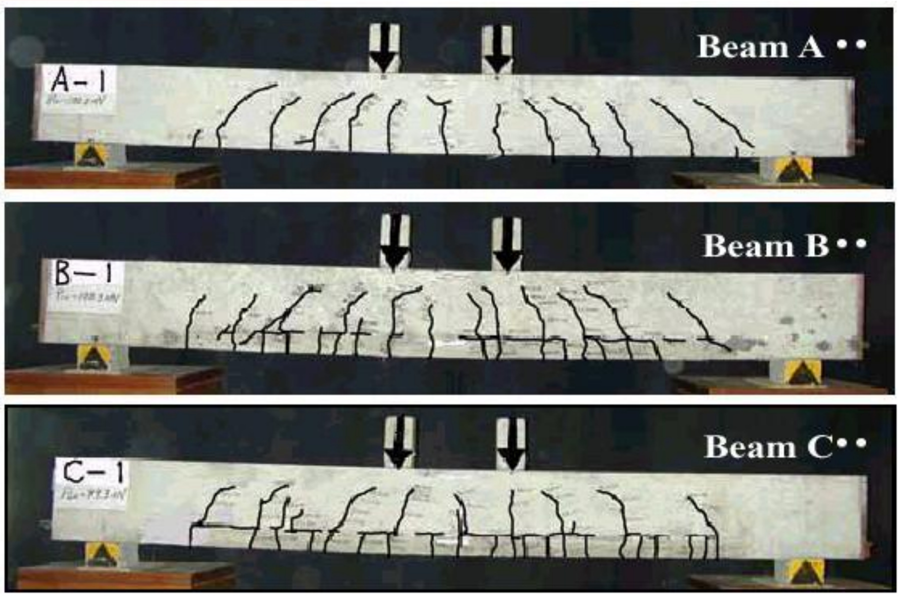
1-2



Advanced RC Structures

Flexural Behavior of RC Beam



- Yield Moment (Nonlinear)
- Nominal Moment
- Moment-Curvature Plot
- Response 2000
- M-C Plot of Double RC

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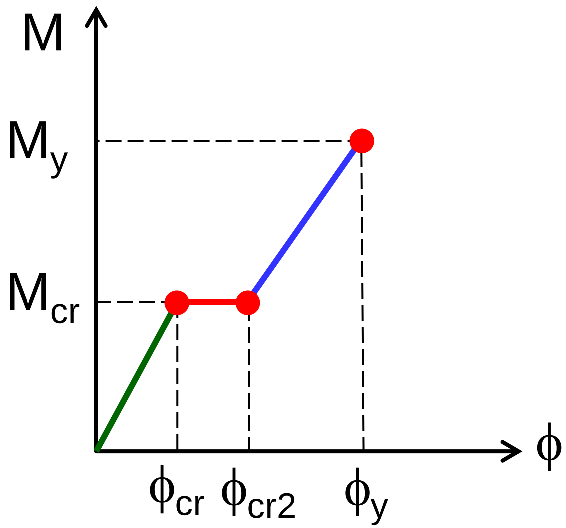
SURANAREE UNIVERSITY OF TECHNOLOGY

INSTITUTE OF ENGINEERING

SCHOOL OF CIVIL ENGINEERING

Example-4 : Yield Moment (nonlinear)



Given: $f'_c = 280$ ksc, $f_y = 4{,}000$ ksc

$$A_s = 3(4.91) = 14.73 \text{ cm}^2$$

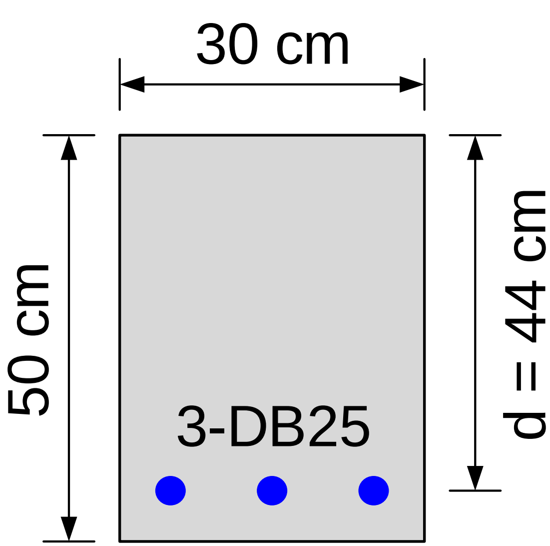
$$\varepsilon_y = f_y/E_s = 4{,}000/2.04\text{x}10^6 \approx 0.002$$

$$\varepsilon_0 \approx 0.002 \rightarrow \varepsilon_y/\varepsilon_0 \approx 1.0$$

$$[T = C] \quad A_s f_y = b f'_c c \left[\left(\frac{\varepsilon_y}{\varepsilon_0}\right)\left(\frac{c}{d-c}\right) - \frac{1}{3}\left(\frac{\varepsilon_y}{\varepsilon_0}\right)^2 \left(\frac{c}{d-c}\right)^2 \right]$$

$$14.73 \times 4.0 = 30 \times 0.28 c \left[\left(\frac{c}{44-c}\right) - \frac{1}{3}\left(\frac{c}{44-c}\right)^2 \right]$$

$$4c^3 - 111c^2 - 1{,}852c + 40{,}733 = 0$$



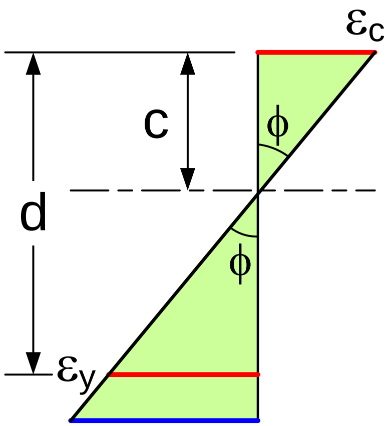
MATLAB

```
>> roots([4 -111 -1852 40733])
ans =
   -20.1842
    32.3282
    15.6061
```

$15.6061 \rightarrow c$

compare with kd = 15.12 cm from Ex.3

Curvature :



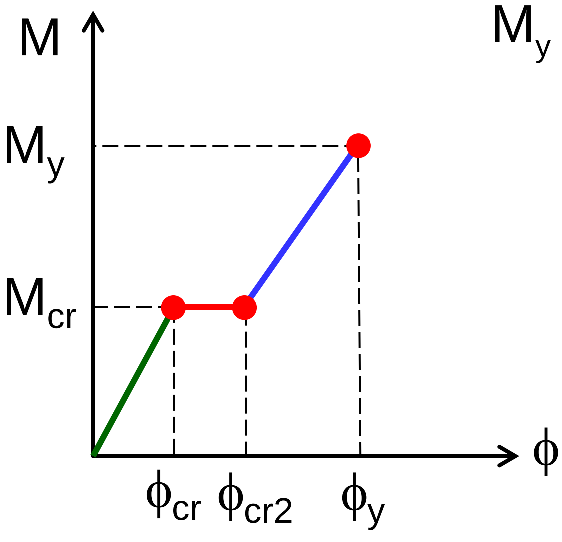
$$\phi_y = \frac{\varepsilon_y}{d-c} = \frac{4{,}000 / 2.04\times10^6}{44-15.61} = \mathbf{6.91\times10^{-5}\ /cm}$$

$$\varepsilon_c = \phi_y c = 6.91\times10^{-5}\times15.61 = \mathbf{0.0011}$$

From Ex.3
$\varepsilon_c = 0.001$

$$\bar{y} \approx \frac{2}{3}c = \frac{2}{3}(15.61) = 10.41\ \text{cm}$$

Yield Moment :



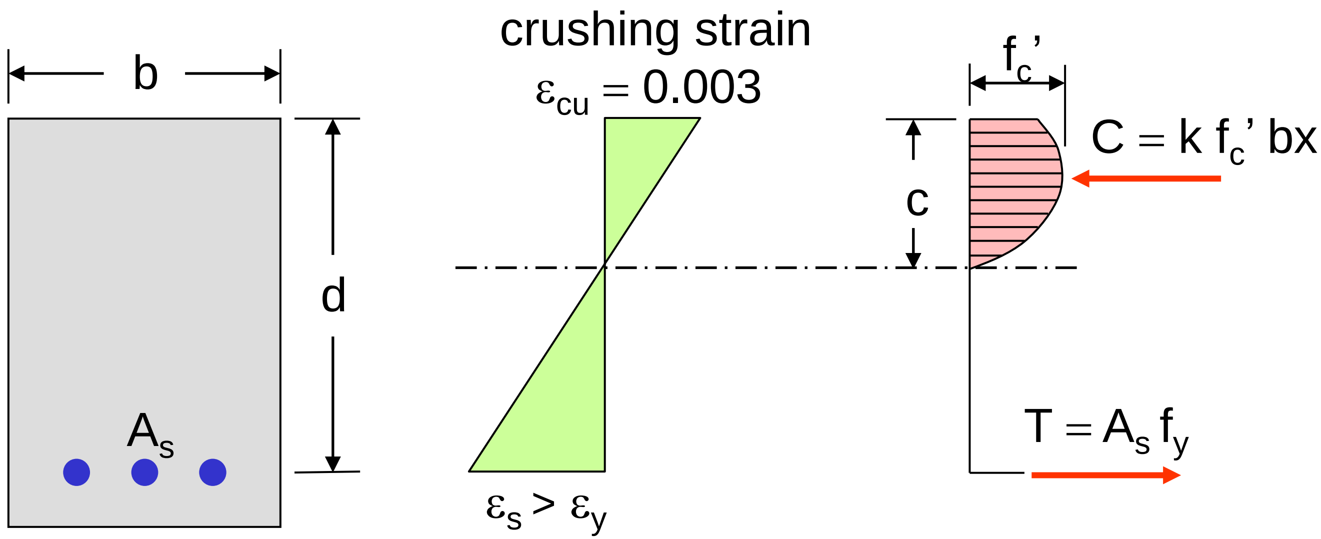
$$M_y = A_s f_y (d - c + \bar{y})$$

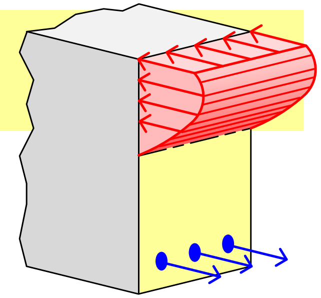
$$= 14.73(4{,}000)(44-15.61+10.41)$$

$$= 2{,}286{,}096\ \text{kg-cm} = 22{,}861\ \text{kg-m}$$

From Ex.3
$M_y = 22{,}952$

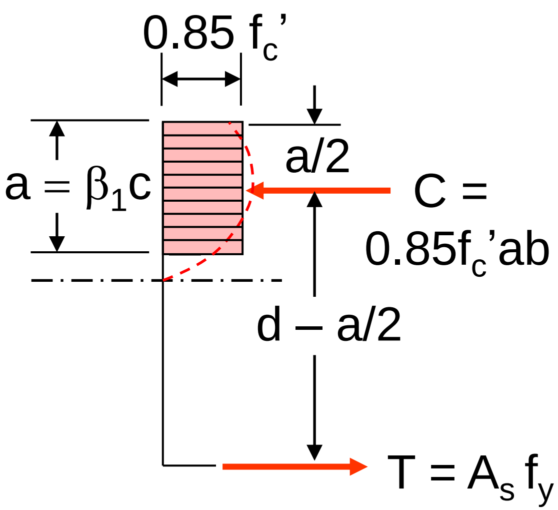
④ Nominal Moment (ACI M_n)





Equivalent stress block

$[C = T]\ \ 0.85\, f_c'\, a\, b = A_s\, f_y$

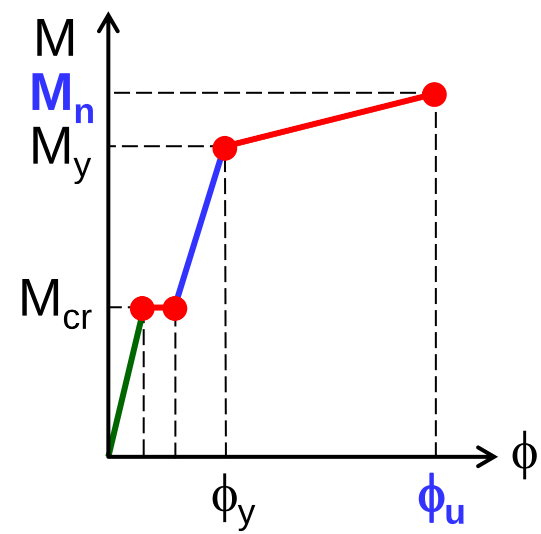


$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} = \frac{\rho\, f_y\, d}{0.85\, f_c'}$$

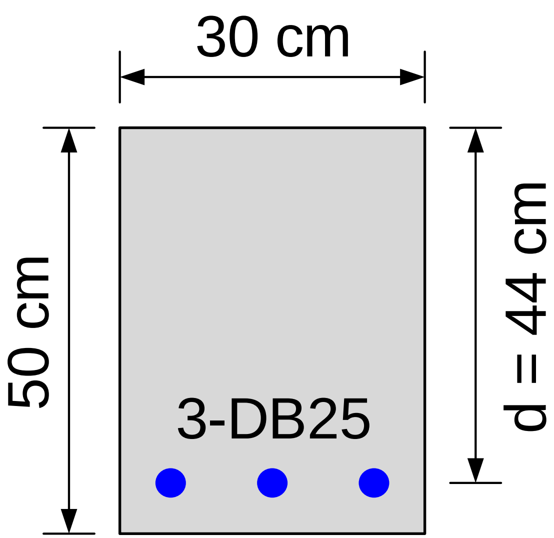
$$\phi_u = \frac{\varepsilon_{cu} = 0.003}{c = a/\beta_1}$$

Check : $\varepsilon_s = \phi_u (d - c) > \varepsilon_y$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$



Example-5 : Nominal Moment (ACI M_n)



Given: $f'_c = 280$ ksc, $f_y = 4{,}000$ ksc

$$A_s = 3(4.91) = 14.73 \text{ cm}^2$$

$$[T = C] \quad A_s f_y = 0.85 f'_c a b$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{14.73 \times 4{,}000}{0.85 \times 280 \times 30} = 8.25 \text{ cm}$$

$$c = a / \beta_1 = 8.25 / 0.85 = 9.71 \text{ cm}$$

$$\phi_u = \frac{\varepsilon_{cu}}{c} = \frac{0.003}{9.71} = \mathbf{3.09 \times 10^{-4} \ /cm}$$

$$\varepsilon_s = \phi_u (d - c) = 3.09 \times 10^{-4} (44 - 9.71)$$

$$= \mathbf{0.0106} > [\, \varepsilon_y = 0.00196 \,]$$

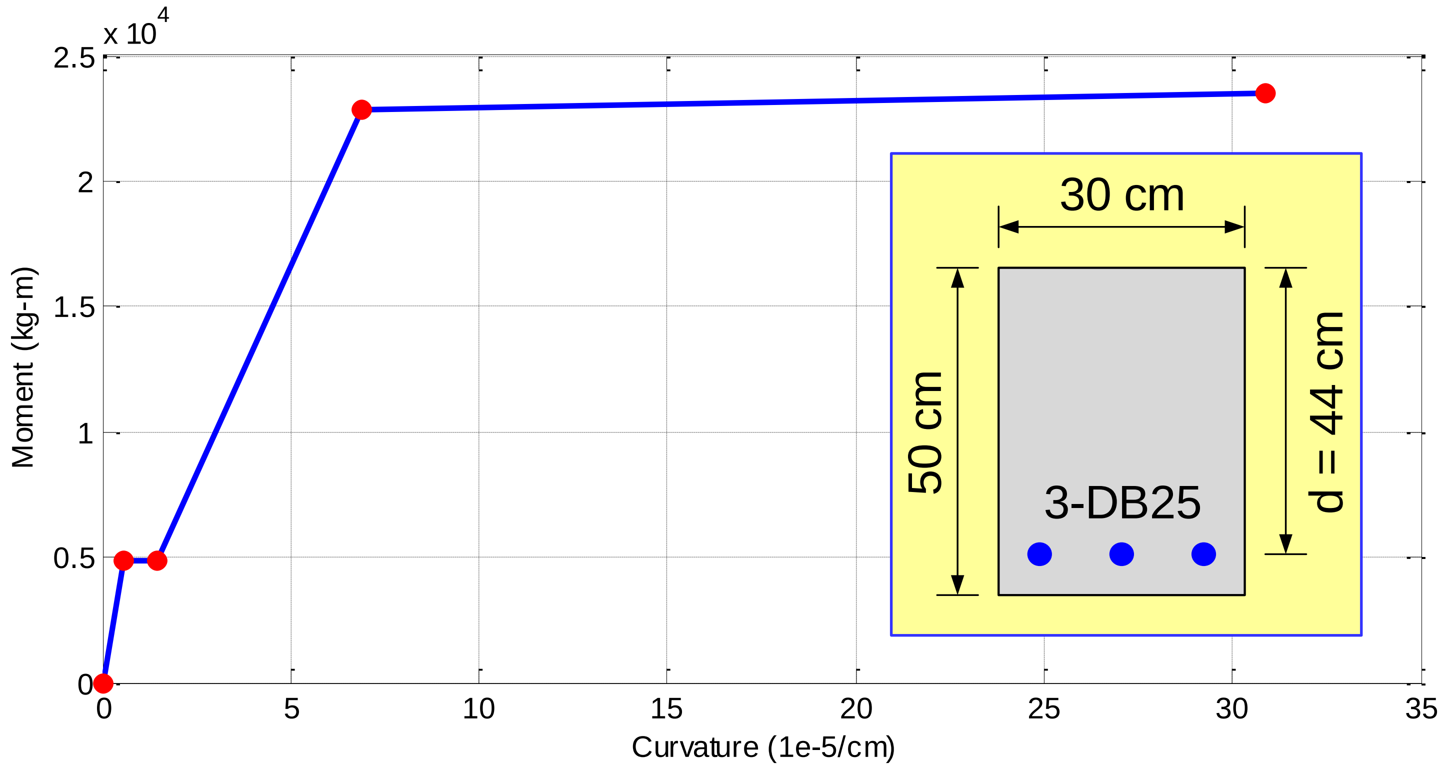
$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = 14.73 \times 4{,}000 \left(44 - \frac{8.25}{2} \right) = 2{,}349{,}435 \text{ kg-cm}$$

$$= \mathbf{23{,}494 \text{ kg-m}}$$

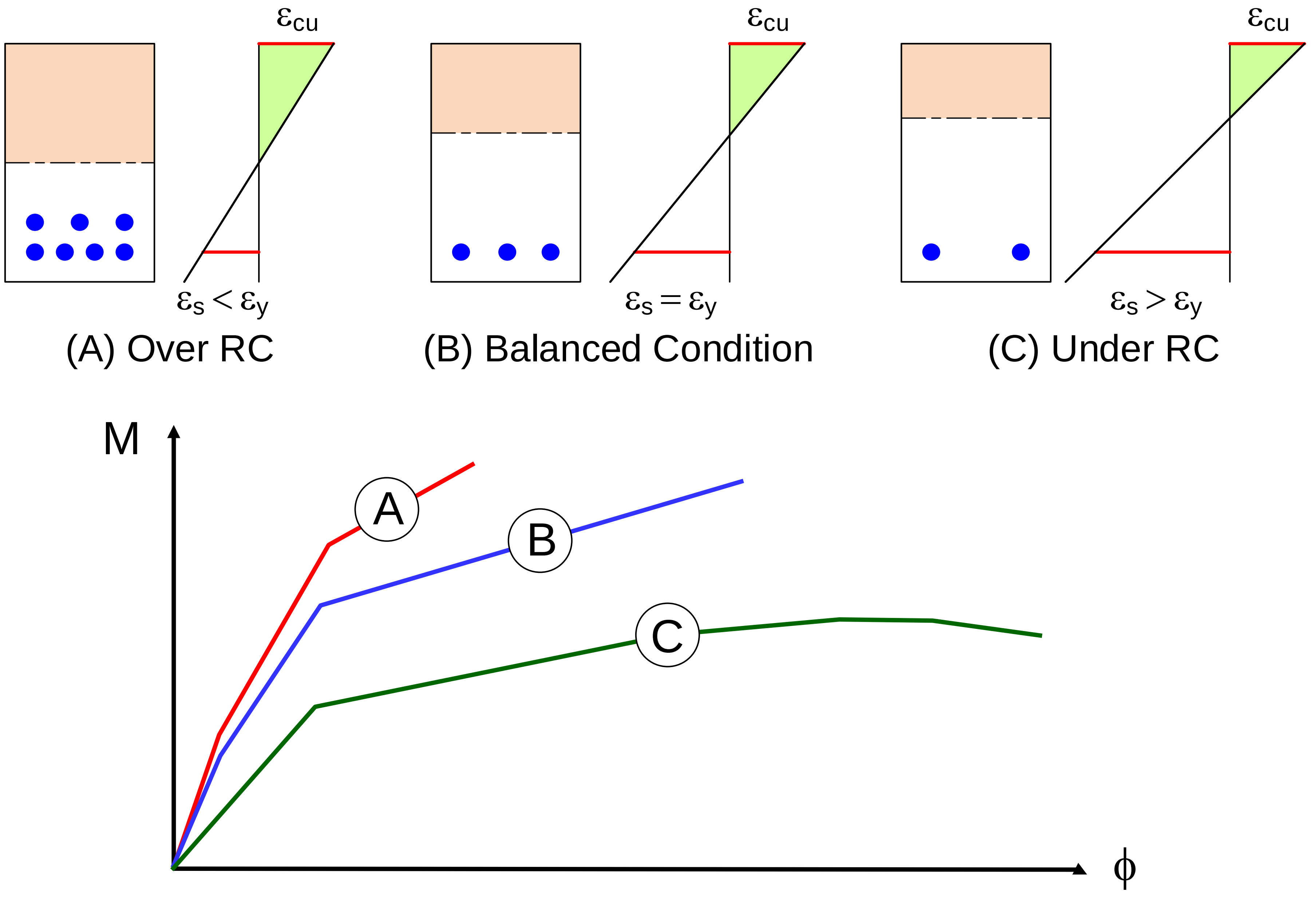
Moment – Curvature Plot

Pt.	$\phi \times 10^{-5}$ (cm^{-1})	Moment (kg-m)
1	0.56	4,895
2	1.45	4,895
3	6.91	22,861
4	30.9	23,494

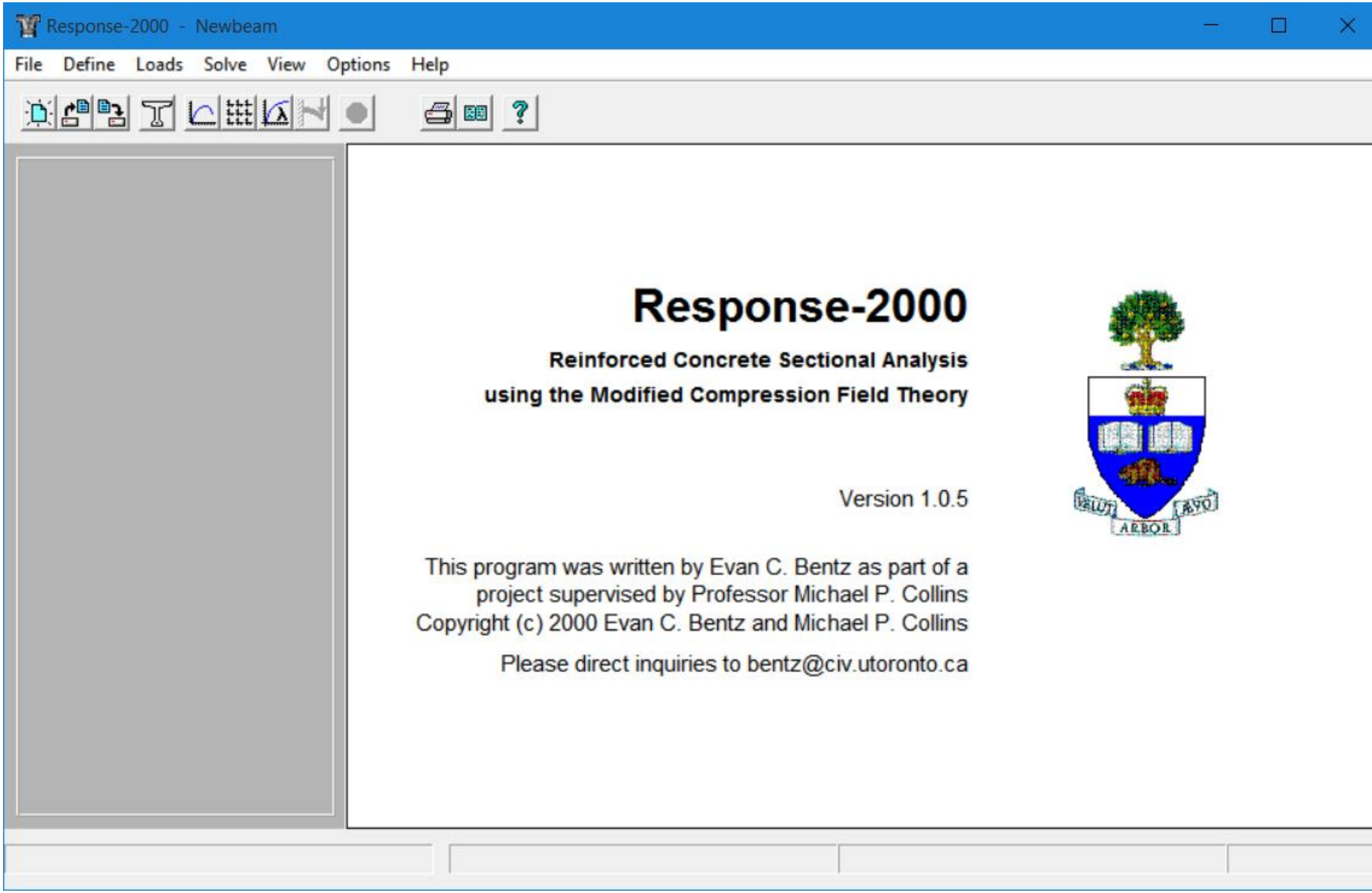
```
>> x = [0 0.56 1.45 6.91 30.9]
>> y = [0 4895 4895 22861 23494]
>> plot(x,y,'o-')
>> grid
>> xlabel('Curvature (1e-5/cm)')
>> ylabel('Moment (kg-m)')
```

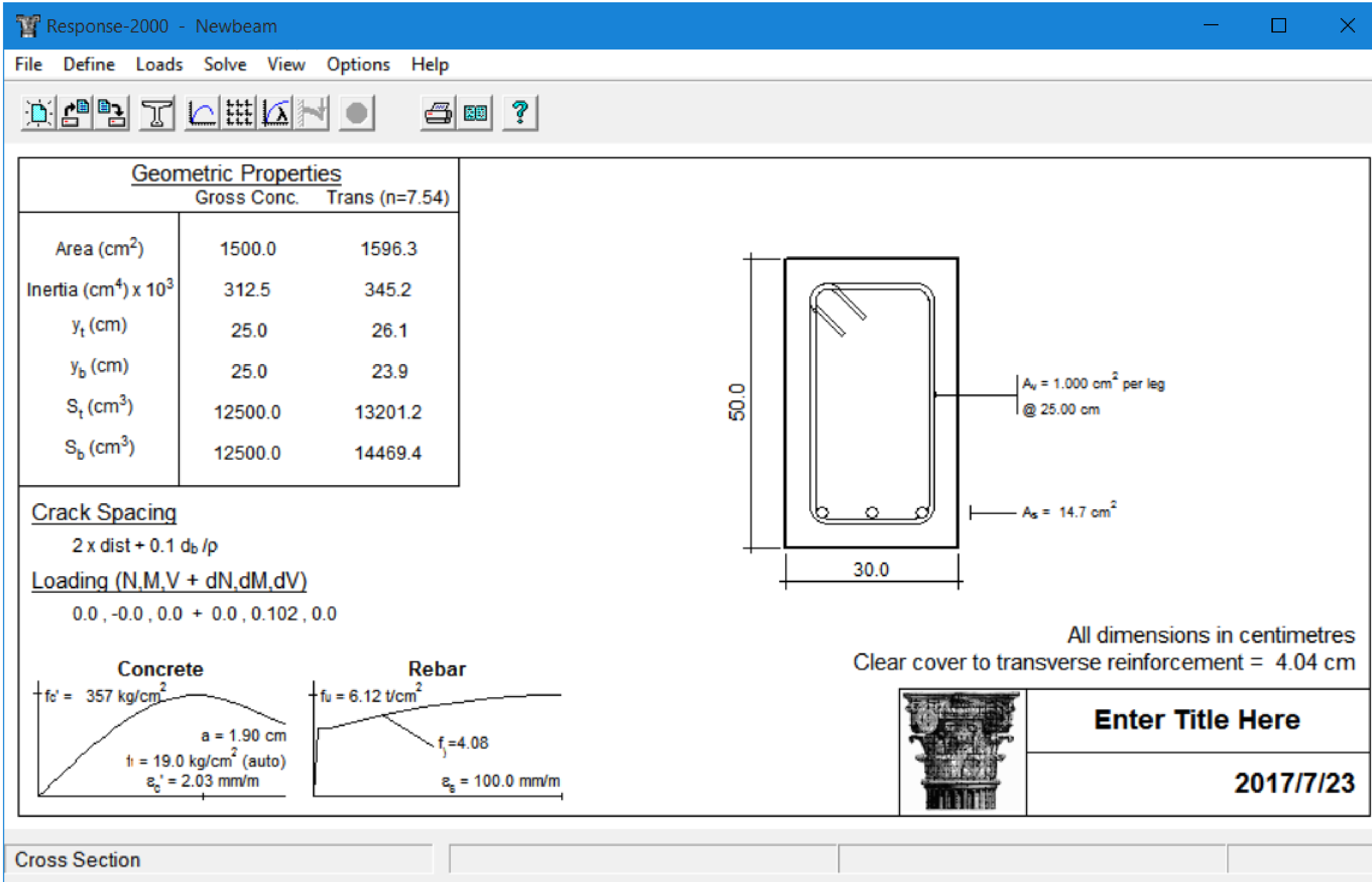


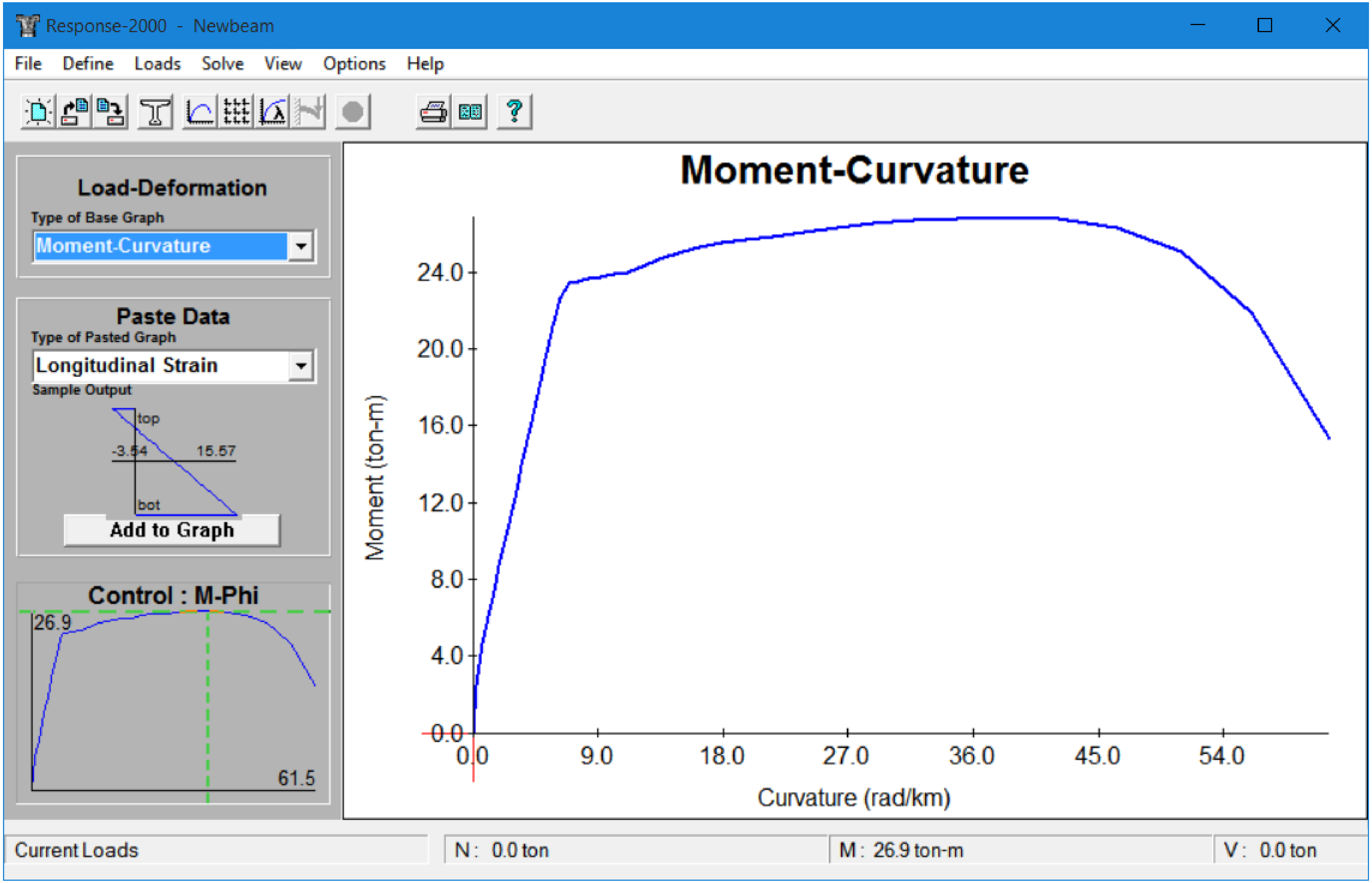
Under v.s. Over RC



http://www.ecf.utoronto.ca/~bentz/r2k.htm

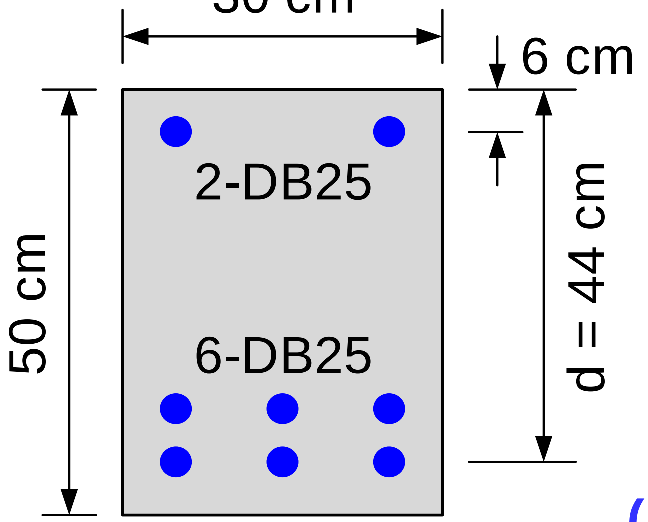

Response-2000 - Newbeam
File Define Loads Solve View Options Help
Response-2000
Reinforced Concrete Sectional Analysis
using the Modified Compression Field Theory
Version 1.0.5
This program was written by Evan C. Bentz as part of a
project supervised by Professor Michael P. Collins
Copyright (c) 2000 Evan C. Bentz and Michael P. Collins
Please direct inquiries to bentz@civ.utoronto.ca
VELUT ARBOR ÆVO


Response-2000 - Newbeam
File Define Loads Solve View Options Help
Geometric Properties
Gross Conc. Trans (n=7.54)
Area (cm2) 1500.0 1596.3
Inertia (cm4) x 103 312.5 345.2
yt (cm) 25.0 26.1
yb (cm) 25.0 23.9
St (cm3) 12500.0 13201.2
Sb (cm3) 12500.0 14469.4
Crack Spacing
2 x dist + 0.1 db /ρ
Loading (N,M,V + dN,dM,dV)
0.0 , -0.0 , 0.0 + 0.0 , 0.102 , 0.0
Concrete
fc' = 357 kg/cm2
a = 1.90 cm
ft = 19.0 kg/cm2 (auto)
εc' = 2.03 mm/m
Rebar
fu = 6.12 t/cm2
fy=4.08
εs = 100.0 mm/m
50.0
30.0
Av = 1.000 cm2 per leg
@ 25.00 cm
As = 14.7 cm2
All dimensions in centimetres
Clear cover to transverse reinforcement = 4.04 cm
Enter Title Here
2017/7/23
Cross Section



Response-2000 - Newbeam
File
Define
Loads
Solve
View
Options
Help
Load-Deformation
Type of Base Graph
Moment-Curvature
Paste Data
Type of Pasted Graph
Longitudinal Strain
Sample Output
top
-3.54
15.57
bot
Add to Graph
Control : M-Phi
26.9
61.5
Moment-Curvature
Moment (ton-m)
24.0
20.0
16.0
12.0
8.0
4.0
0.0
0.0
9.0
18.0
27.0
36.0
45.0
54.0
Curvature (rad/km)
Current Loads
N: 0.0 ton
M: 26.9 ton-m
V: 0.0 ton

Example-6 : Moment Curvature of Double RC



Given: $f'_c = 280$ ksc, $f_y = 4{,}000$ ksc

$$A'_s = 2(4.91) = 9.82 \text{ cm}^2$$

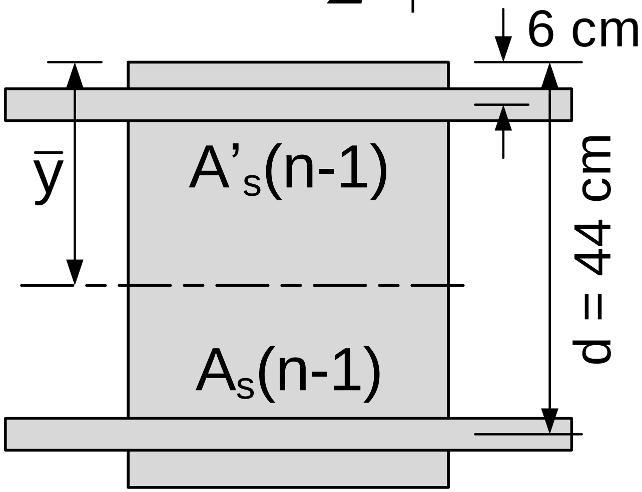
$$A_s = 6(4.91) = 29.46 \text{ cm}^2$$

$$E_c = 15{,}100\sqrt{280} = 252{,}671 \text{ ksc}$$

$$n = \frac{E_s}{E_c} = \frac{2.04 \times 10^6}{252{,}671} = 8.07$$

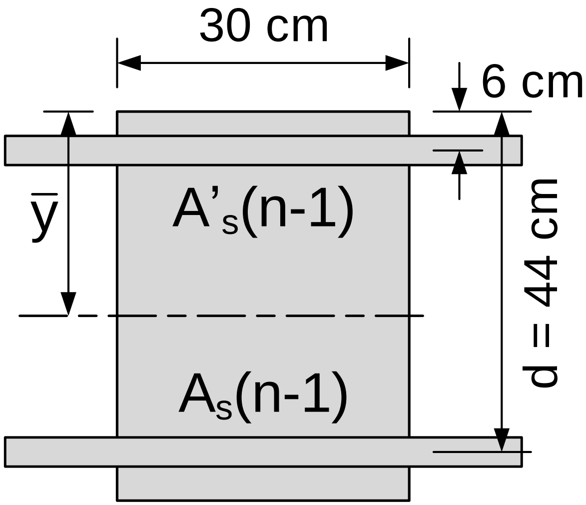
(1) Uncracked transformed section :

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{(30)(50)(25) + (8.07-1)(9.82)(6) + (8.07-1)(29.46)(44)}{(30)(50) + (8.07-1)(9.82) + (8.07-1)(29.46)}$$



$$\bar{y} = \frac{47{,}081}{1{,}778} = 26.48 \text{ cm}$$

$$I_{uctr} = \sum (I_i + A_i d_i^2)$$



$$I_{uctr} = \sum (I_i + A_i d_i^2)$$

$$= \frac{(30)(50)^3}{12} + (30)(50)(26.48 - 25)^2$$

$$+ (8.07 - 1)(9.82)(26.48 - 6)^2$$

$$+ (8.07 - 1)(29.46)(44 - 26.48)^2$$

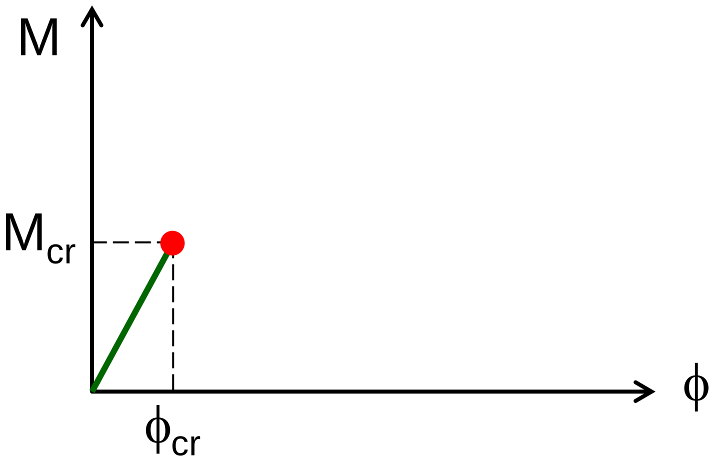
$$= \mathbf{408{,}838\ cm^4}$$

Cracking Moment:

$$f_t = 2.0\sqrt{f'_c} = 2.0\sqrt{280} = 33.47 \text{ ksc}$$

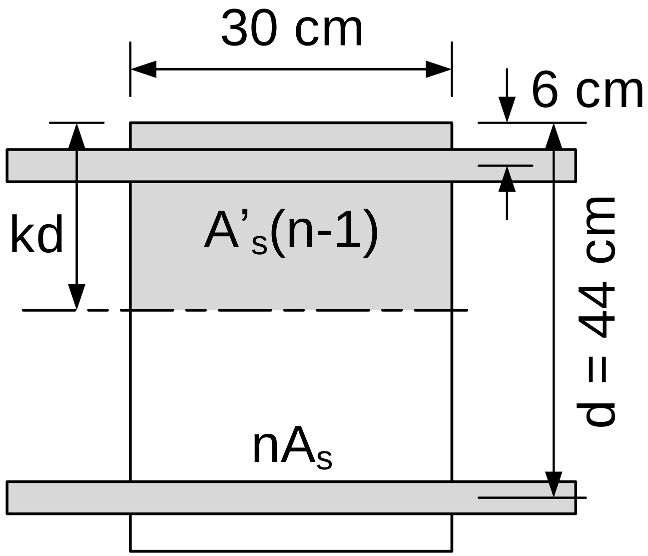
$$M_{cr} = \frac{f_r I_{uctr}}{y_t} = \frac{33.47 \times 408{,}838}{50 - 26.48}$$

$$= 581{,}795 \text{ kg-cm} = 5{,}818 \text{ kg-m}$$



$$\phi_{cr} = \frac{(f_t / E_c)}{y_t} = \frac{(33.47 / 252{,}671)}{(50 - 26.48)} = 5.63 \times 10^{-6} \text{ / cm}$$

(2) Cracked transformed section :



$$b(kd)(kd/2) + (n-1)A'_s(kd - d') = n A_s(d - kd)$$

$$(30)\frac{(kd)^2}{2} + (8.07-1)(9.82)(kd-6) = (8.07)(29.46)(44-kd)$$

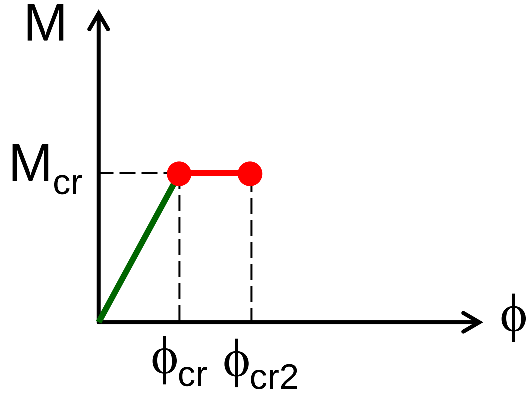
$$15(kd)^2 + 307.2(kd) - 10{,}878 = 0$$

$kd = 18.57$ cm

$$I_{crtr} = \sum(I_i + A_i d_i^2) = \frac{(30)(18.57)^3}{3} + (8.07-1)(9.82)(18.57-6)^2 + (8.07)(29.46)(44-18.57)^2$$ **$= 228{,}752\ cm^4$**

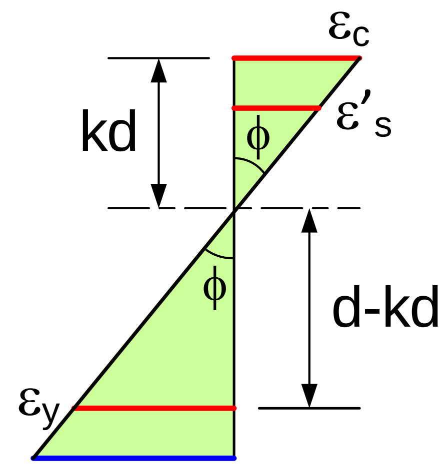
$$\phi_{cr2} = \frac{M_{cr}}{E_c I_{crtr}} = \frac{581{,}795}{252{,}671 \times 228{,}752}$$

$= 1.01\times10^{-5}$ /cm



(3) Yield of steel : Cracked transformed section valid until $f_s = f_y$ or $f_c = 0.7f'_c$

Steel yield



$$\varepsilon_y = f_y/E_s = 4{,}000/2.04\text{x}10^6 = 0.00196$$

$$\phi_y = \frac{\varepsilon_s}{d - kd} = \frac{0.00196}{44 - 18.57} = \mathbf{7.71{\times}10^{-5}\ /cm}$$

$$M_y = \phi_y E_c I_{crtr} = 7.71 \times 10^{-5} \times 252{,}671 \times 228{,}752$$

$$= 4{,}456{,}303 \text{ kg-cm} = \mathbf{44{,}563 \text{ kg-m}}$$

Concrete nonlinear

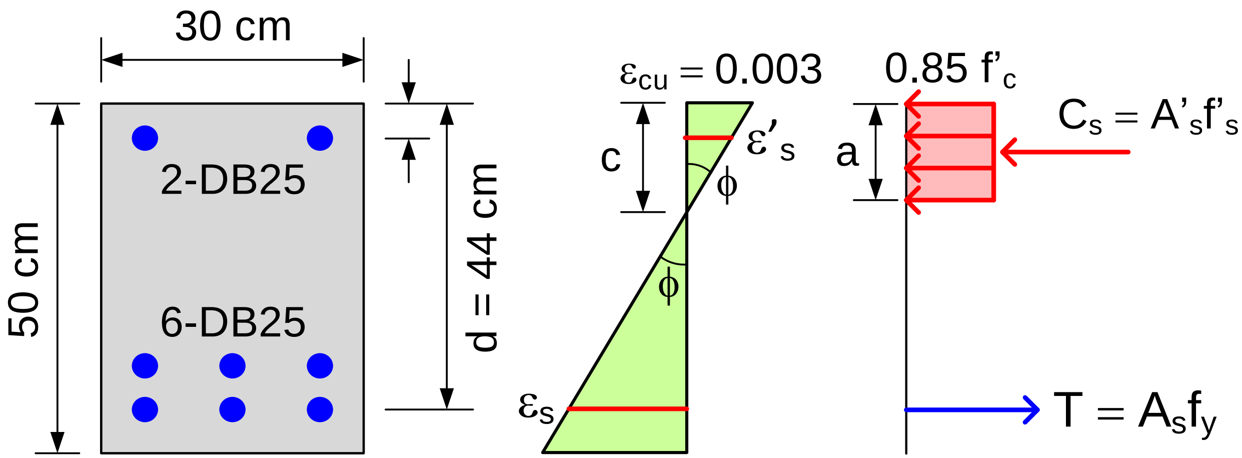
$$\phi_{0.7f'_c} = \frac{0.7f'_c / E_c}{kd} = \frac{0.7(280) / 252{,}671}{18.57} = \mathbf{7.75{\times}10^{-5}\ /cm} > \phi_y$$

$$M_{0.7f'_c} = \phi_{0.7f'} E_c I_{crtr} = 7.75 \times 10^{-5} \times 252{,}671 \times 228{,}752$$

$$= 4{,}420{,}676 \text{ kg-cm} = \mathbf{44{,}207 \text{ kg-m}} > M_y$$

$\therefore$ *Steel yield before concrete becomes nonlinear*

(3) Nominal strength:



$$\varepsilon'_s = 0.003\left(\frac{c-d'}{c}\right)$$

$$f'_s = \varepsilon'_s E_s = 0.003 E_s \left(\frac{c-d'}{c}\right)$$

$[T = C]$

$$A_s f_y = 0.85 f'_c b \beta_1 c + A'_s f'_s$$

check comp. steel yield condition:

$$\rho - \rho' \geq \frac{0.85\beta_1 f'_c d'}{f_y d}\left(\frac{6{,}120}{6{,}120 - f_y}\right)$$

$$\rho' = \frac{2(4.91)}{(30)(44)} = 0.00744$$

$$\rho = \frac{6(4.91)}{(30)(44)} = 0.0223$$

$$\frac{0.85 \times 0.85 \times 280 \times 6}{4{,}000 \times 44}\left(\frac{6{,}120}{6{,}120 - 4{,}000}\right) = 0.0199$$

Since $\rho - \rho' = 0.0149 < 0.0199$, comp. steel not yield $f'_s < f_y$

$$\varepsilon'_s = 0.003\left(\frac{c-d'}{c}\right) < \varepsilon_y$$

$$f'_s = E_s \varepsilon'_s = 6{,}120\left(\frac{c-d'}{c}\right)$$

$[T = C]$

$$A_s f_y = 0.85 f'_c \beta_1 b c + 6{,}120 A'_s \left(\frac{c-d'}{c}\right)$$

จัดรูปสมการใหม่ $c^2 + \dfrac{6{,}120A'_s - A_s f_y}{0.85 f'_c \beta_1 b} c - \dfrac{6{,}120 A'_s d'}{0.85 f'_c \beta_1 b} = 0 \longrightarrow c^2 + 2Rc - Q = 0$

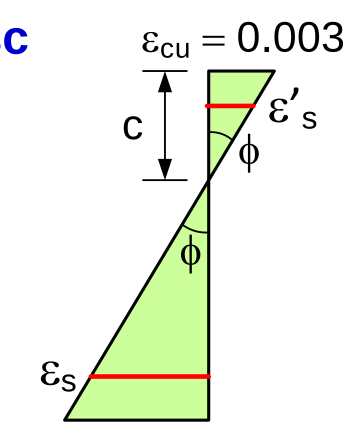
กำหนดให้ $R = \dfrac{6{,}120 A'_s - A_s f_y}{1.7 f'_c \beta_1 b}$ และ $Q = \dfrac{6{,}120 A'_s d'}{0.85 f'_c \beta_1 b}$

$= \dfrac{6{,}120(9{,}82) - (29.46)(4{,}000)}{1.7(280)(0.85)(30)}$ $\quad = \dfrac{6{,}120(9{,}82)(6)}{0.85(280)(0.85)(30)}$

$= -4.757$ $\quad = 59.42$

$$c = -R \pm \sqrt{R^2 + Q} = 4.757 \pm \sqrt{4.757^2 + 59.42} = \mathbf{13.72\ cm}$$

$$f'_s = 6{,}120\left(\frac{c - d'}{c}\right) = 6{,}120\left(\frac{13.72 - 6}{13.72}\right) = \mathbf{3{,}444\ ksc}$$



$$\phi_n = \frac{\varepsilon_{cu}}{c} = \frac{0.003}{13.72} = \mathbf{2.19 \times 10^{-4}\ /\ cm}$$

$$a = \beta_1 c = 0.85(13.72) = 11.66\ \text{cm}$$

$$M_n = 0.85 f'_c a b \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')$$

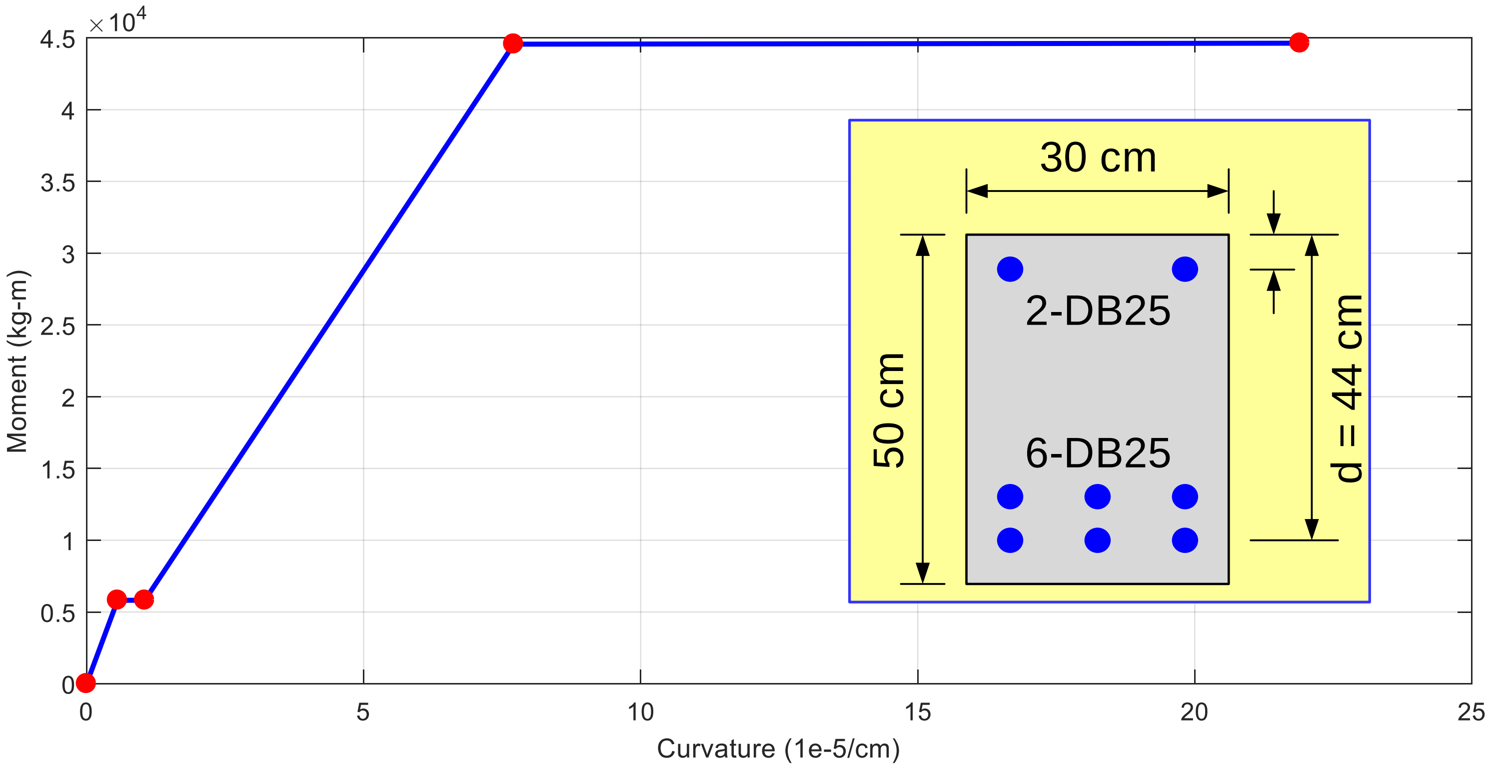
$$= 0.85(280)(11.66)(30)\left(44 - \frac{11.66}{2}\right) + 9.82(3{,}444)(44 - 6)$$

$$= 4{,}462{,}907\ \text{kg-cm} = \mathbf{44{,}629\ kg\text{-}m}$$

Moment – Curvature Plot

Pt.	$\phi \times 10^{-5}$ (cm^{-1})	Moment (kg-m)
1	0.56	5,818
2	1.05	5,818
3	7.71	44,563
4	21.9	44,629

```
>> x = [0 0.56 1.05 7.71 21.9]
>> y = [0 5818 5818 44563 44629]
>> plot(x,y,'o-')
>> grid
>> xlabel('Curvature (1e-5/cm)')
>> ylabel('Moment (kg-m)')
```



End of Lecture