PROLOGUE

This teacher-parent manual in the Spiritual Syllabus Series outlines 12 Mathematics unit lessons in Class 5 and 6 for 11 and 12-year-olds. The maths stream is essentially taught in the morning main lessons, for the first 2 hours of the day. There are 3, 3-week maths main lessons per year; about 90 hours teaching – or 'fun with number'!

But there is also a corollary stream, the Numeracy, which is presented as 3 middle lesson units per year. These occupy the 1 ½ hour lesson between break and lunch, and are again sequential, of 3-week duration. The time scheduled being some 60 hours teaching over the year.

So mathematics/numeracy has a 150 hour allotment every year, right through primary school. The miracle manifest in this broad program calls on the *astral* forces of the learner – it calls on imagination, or 'Imaginative Cognition' as Rudolf Steiner called it. This is the most important faculty to be developed in the coming age – the new Age of Abraham, beginning in the 3rd Millennium.

An intelligent and image filled maths education is essential for this sublime and imperative step on the long road of the evolution of consciousness – this small offering is directed to these high ends. The following is a suggested curriculum structure, emphasizing the many aspects of the human being awakened by the various number paths – aspects detailed in the 12 articles in this book:

*Full Primary Curriculum in my book *La Pleroma.*
* Full High School Curriculum in *A Steiner High School?.*
*Full Programming details in *A Steiner Homeschool?.*

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	CLASS 5		MATHS – ASTRAL	CLASS 6		
MATHEMATICS – HEAD	MAIN LESSONS					
	BODY	SOUL	SPIRIT	BODY	SOUL	SPIRIT
	MENSURATION	NUMERATION	CALCULATION	MENSURATION	NUMERATION	CALCULATION
	VOLUME, MASS WEIGHT Page 3	GRAPHS Page9	DECIMALS Page 17	SOLID GEOMETRY Page 48	RATIO AND PROPORTION Page 56	PERCENTAGES Page 63
NUMERACY – CHEST	MIDDLE LESSONS					
	WILL	FEELING	THINKING	WILL	FEELING	THINKING
	APPLIED NUMBER	DIAGRAMM. NUMBER	PROBLEM SOLVING	APPLIED NUMBER	DIAGRAMM. NUMBER	PROBLEM SOLVING
	MATHS IN NATURE Page 26	GEOMETRIC PATTERNS Page 32	FINANCE Page 38	MATHS IN ENGINEERING Page 71	GEOMETRIC CONSTRUCTION Page 77	SETS AND PATTERNS Page 83

See *Grammatika Grammatikos* for full Class 5 and 6 curriculum.

ATRIUM TO THE 3RD DIMENSION
Weight & Mass – Class 5 – Main Lesson

The stars are very bright indeed; and I have an imponderable yearning to return whence I came. But I'll stay; someone must have instilled a sense of *gratitude* into my soul as a child. This transformed naturally in early adult life as *love* – love of children, teaching and subject in that order. This gratitude/love in turn metamorphosed, in my early dotage, into a sense of *duty*. Thus is Rudolf Steiner's depiction of the moral 'Kokoda Track' of life.

So instead of leaving the 3rd dimension for the 4th, I'm writing this brief exposition on the Volume, Weight and Mass main lesson in Class 5. This is a subject very much connected, indeed expressing as no other, the 3rd dimension.

This is a maths lesson, and as such calls on the supersensible forces of the astral body. Moses was an astral initiate; concerning himself very much with incarnating the principles of measure, weight and number into human consciousness; hence his 'initiate's horns' in the astral – the 3rd – vertical quadrant of the heard, as depicted in Michelangelo's mighty and insightful sculpture.

In the primary maths main lesson stream, there are 3 strands, Mensuration, Numeration and Calculation. These express body, soul and spirit characteristics respectively. Volume, weight and mass is a mensuration, or measuring, lesson. As we encourage these 11-year-olds to measure their world – consciously – they are also measuring their own bodies. Every principle studied is mirrored within; and should be made clear to the class – a 'body' lesson indeed!

The 3-dimensional human being – and the whole world – is composed, according to Archimedes, essentially of 4 solids. The first, representing the Physical Body, is the *sphere* – found in the eyeball for instance. The sphere is the first creation; that from which all 3-dimensional forms are derived.

That expressing the Etheric Body in this 'solid' quartet, is the *cone.* Again the eyes have it; in the conic light receptors on the retina in fact! Light? Etheric? Of course – the cone is that mystical ether (the etheric is the 'light ether') form which, when sectioned 4 ways, creates the 4 arithmetic curves. The circle, ellipse, parabola and hyperbola are the blueprint for life itself. Another epithet of the ether is the 'life body'.

Ah, then there's the *cylinder*, the Astral solid. This is found in the body, especially where fluid travels – liquid *is* the astral element – such as the vascular system, with its miles of cylindrical tubing! The children accept and absorb these form/body relationships so easily.

The 4th solid is that of the Ego, the cube, and all those forms which derive from it. These include the rectangular, triangular and hexagonal prisms – and even the more obscure geometric solids like rhomboids. The generic term for this multifarious ego manifestation is *polyhedron*.

A marvelously instructive classroom activity is to have the children make the 4 solids out of beeswax. If colored, they should be, to avoid in inner conflict of color and form: sphere-red; cone-yellow; cylinder-blue; and cube-violet.

These are the forms/colors of physical, etheric, astral and ego respectively. How the children enjoy struggling to achieve symmetry I these mystical forms. How lovely these small pure-form sculptures look in a collective display – like a miniature city on another planet! And how more easily then are the maths concepts assimilated.

Now to the gristle of the lesson, the formulas: the first is the easiest, yet that which represents the most enigmatic form/body. This is the cube, with its simple LxBxH to find volume. This works for any regular prism. The triangular prism is a logical extension from this – LxBxH ÷ 2. It is a good idea to begin with volume; in a 3-week main lesson, one could, with value and economy, devote a week to each of the 3 principles, volume, weight and mass.

Of course the human being is an *irregular* solid, making it hard to determine his/her volume; but there is a way – the water displacement method. Better perhaps in the lesson to use a stone and a calibrated beaker! Place water in 10cm; therefore volume of stone is equivalent to 5cm of water. Then calculate volume using formula for cylinder volume $\pi\ R^2\ H$. If beaker radius is 5cm, then volume of stone is (roughly for these pre-decimal children) 3x5x5x5 = 375 cubic centimeters.

There is an even easier way; no, not to perceive the stone's supersensible reality in the astral world, but to *weigh* the water displaced. Say this was 15 grams; 1 gram of water = 1 cubic centimeter Therefore 15 grams is 15 cubic centimeters, the volume of the stone!

Turning to the 'real' world, children should be made aware of the vast spectrum of volume use; from the tiny volumes used by apothecaries (a visit to a chemist might be interesting) – to people who deal in vast volumes, like wheat storage.

A trip to the supermarket is a must in this unit; especially when armed with a simple formula to cut the Gordian Knot of volume/prices – this formula is price over weight $\frac{P}{W}$. What is the better value; 250 grams for $1.20, or 280 grams for $2? You simply find the cost per gram of item 1 - $\frac{120}{240}$ = ½ cent per gram. For item 2 it is $\frac{200}{280} = \frac{5}{7}$ of a cent per gram. Item one is the better value. How edifying to get the edge on those cunning, confusing pricing managers!

Apart from this obvious empowerment, it is good for children to see the relative volumes of various products; how big is a kilo of butter? Of potato chips? A good homework task is for the class to bring in *marked* containers – bottle, cans, bags – of various sizes for a classroom display. If not especially lovely, it is certainly instructive – for pupil and teacher alike! A good teaching aid for demonstrating cube numbers, so necessary for an understanding of volume measurement, is toddlers' cubic building blocks. 1^3 is one block; 2^3 is 8 blocks; 3^3 27; 4^3 64…! How much easier this rather complex idea is to grasp if it's visual.

Archimedes arranged to have inscribed on his tomb a sphere enclosed (to fit) in a cylinder. This is an occult symbol of the globe of the physical (the world, the body) enfolded by the astral world; that to which he was so sadly dispatched by a witless Roman soldier. A blood cell moving through a capillary is a sphere (of a kind) in a cylinder.

The great mathematician was no doubt aware of this duality of reality, and it is a memorial to him that we *introduce* the young Greeks before us to these two volume formulas embodied in the sacred symbol. Already mentioned is that for the cylinder, but the sphere is almost as easy – 2/3 of its encompassing cylinder in fact $(\pi\, r^2\, h)\,\frac{2}{3}$.

We don't really expect 11-year-olds to understand these formulae fully, but seed sowing is important for later on. Some introductory concepts on the ratio of surfaces and volumes is also valuable at this time. A mouse has a greater surface-to-volume ratio than a bear – both having about the same body form. However if both are dropped from 100 feet, the mouse lands safely – the bear is strawberry jam! The mouse's high s-t-v ratio acts like a parachute, breaking its fall.

This ratio is not always an advantage through, the high surface area means greater heat loss – the mouse has to eat half its weight per day just to cut even. The bear, even in Arctic climes, sleeps for months without a meal.

Having described the reality, we proceed to demonstrate the abstract: 1 cubic unit (mouse) has 6 square surface units – the ratio is 1:6. The bear with 8 cubic units has a mere 24 surface units - $\frac{24}{8}$ = 3. The ratio 1:3 is half that of the mouse.

Now what of *weight*? A browse through the Guinness Book of Records provides plenty of stunning facts and figures on heaviest and lightest. The heaviest human was 100 stone; have 20 children, each of around 5 stone, cluster together to approximate this mountain. The smallest adult was only 13 pounds; this homunculus was only the size of a large doll! Sorry about the empirical measurements; naturally one teaches in the metric system; but the more organic – and hence spiritual – should be at least introduced to children, even if only as an anachronism. Many will calculate in empirical throughout life in spite of being taught in metric; a confirmation of its higher reality.

A brief history of weight systems provides colorful content for this unit – ask your local librarian to do the spade work for you! How did humanity come by: pounds; ounces; quarts; hundredweights; drams; pecks; stones? Maybe the class should know something of the fabled French nobleman, Tallyrand. He it was who, immersed in rational soul consciousness, created the metric system. While in France, a description of the Paris Measurement Standards is of interest; accurate now to infinitesimal atomic weights!

No simple post office scales for the sophisticated French – but nothing could be better for Class 5. How they delight in placing those beautiful brass weights in the amazingly accurate (to a single nail in some cases) scales. Again there should be weighing apparatus of all kinds in the classroom for this unit – how about a railway station scales – or gold balances?!

As well there should be a lot of *estimation*; what do the children weigh for instance? – in stones and pounds as well! Of course the metric terminology is that with which modern young learners must be most familiar – it's a metric world out there. (Oh for a world governed by realty rather than rationality!)

Liters; milliliters; kiloliters and even megaliters must be the argot of this unit; as well as grams; kilograms; tonnes…*tonnes!* What a terrible word – aarrhhgg. I hate metric!!

Finally we move on to *mass*, the density of a subject or object – the relationship between volume and weight. Initially a scale of substances should be described – demonstrated even. Take a cube of uranium, the mostest in the massest; and compare the weight of a cube of wood the same size. Okay, for the purists replace the 'yellow cake' with lead or gold! The children love those old jokes about 'What is heavier, a kilo of lead or a kilo of feathers?' With mass we can again draw on the human body as a living reference.

Two people the same size can be different weights, because they have different mass (kg/c^3). One is soft and obese, the other stocky and muscular. The fat person is lighter than 'muscles'; whose bulk is well-served by a generous blood supply. There is decreased vascular action in cellulite, a deposited fatty substance. Blood has a similar mass to water; fat or oil is far less dense – has less mass. This allows it to float on water. Indeed our plump friend bobs around effortlessly in the pool, whilst the musclebound showoff sinks like a brick!

Every 11-year-old should learn the ultimate example of mass; the story of Archimedes and the King of Syracuse. And this is where you teach it. The king has a crown of gold made; he suspected that his metallurgists had snuck some less-valuable silver into the mix. So he passed the problem to Archimedes, who decided to have a bath instead. (These Greeks were a sanitary lot!).

As he lowered his tired old body into the steaming water, the level rose – he leapt out with a shout. Was the water too hot? Probably; but also he was ejected by a visionary breakthrough in mathematical law. Racing back to the palace, naked as a robin's egg and yelling 'Heureka!!' (I have discovered), Archimedes shocked the court by plonking the crown in a vessel of water – his birthday suit didn't seem to bother the august assembly. (The Greeks were also a liberal lot!)

He then placed a gold block, the same weight as the crown, in the water. Alas the water level was lower. The mass (density) of metal in the crown was less than the gold. Silver, having less mass, has greater volume for the same weight, therefore displaced more water.

This exercise, expressed as kg per c^3, should be done in class using various substances and objects; but make sure they sink! The inspiration streaming into the soul of Archimedes (meaning 'first Mede' or Persian. The Persian Stream brought science to Western culture; as opposed to the artistic 'Buddhist'.) from the Number Ether will, with the aid of imagination, humor, anecdote and crystal-bright intelligence, infuse this wonderful unit.

This oblique methodology in maths teaching will illumine the golden hearts of your young Greeks in Class 5. May they too rejoice – fully clothed! – at their own small discoveries; enriching a love of subject born of the *gratitude* of infancy; later to mature into *love* – leading finally to a sense of *duty*. Now my duty's done on this particular main lesson, and I'm off to the 4^{th} dimension, in the oh-so-starry sky. Goodnight.