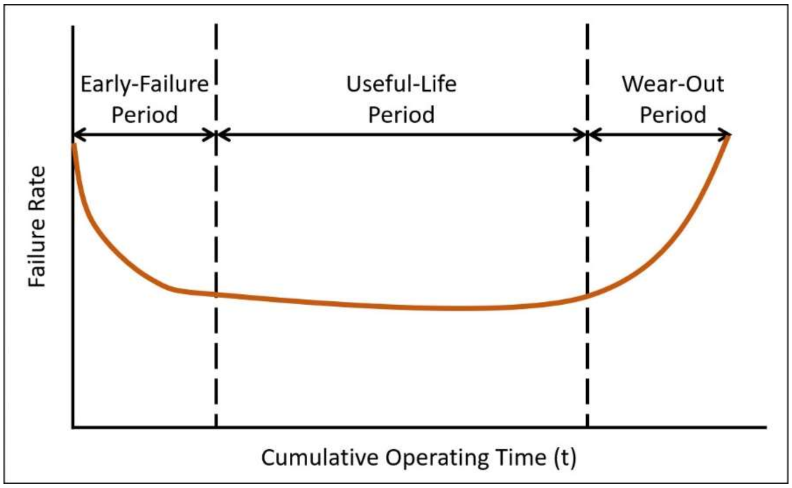
End of Life Region

We're going to skip over the Useful Life Region of the bathtub curve for now and head to the last region, the Wear Out Period. The end of life region is also known as the wear out period because of the increase in the failure rate of your product due to wear and tear.

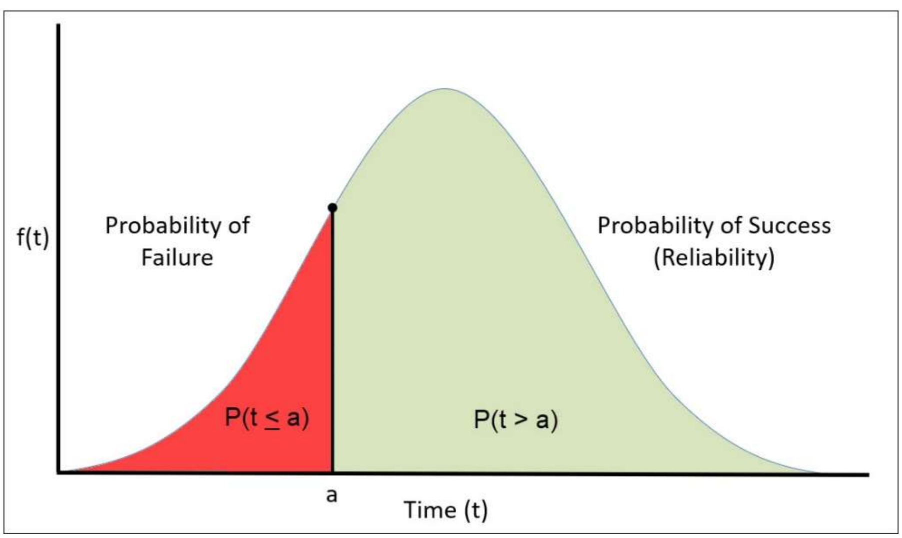


In this time period, with an increasing failure rate, the appropriate PDF (Probability Density Function) to use in making Reliability estimations is the **Normal Distribution** or the **Weibull Distribution**.

Utilizing the Normal Distribution Curve, you can make reliability predictions by calculating the area under the curve at any given moment in time.

To do this requires that you know or be able to estimate the mean (μ) & standard deviation (σ) associated with this distribution.

Once you know these two parameters, you can use the Z-Tables associated with the normal distribution to calculate the reliability at any given time (t_1) as the area to the right of the time value ($t_1 = a$).



This is identical to the example discussed above where the area to the left of the time value ($t_1 = a$) is the probability of failure and the area to the right of ($t_1 = a$) is the probability of success or Reliability.

$$Z = \frac{(t_1 - \mu)}{\sigma}$$

For example, let's say you know you've got a motor that's in the wear out period and your test data indicates that the mean and standard deviation associated with the wear out period is 6,500 hours and 500 hours respectively.

What is the reliability of the motor at 7,200 hours? This is the probability that the motor has not failed yet.

$$\boldsymbol{Z = \frac{(t_1 - \mu)}{\sigma} = \frac{(7,200 - 6500)}{500} = 1.4}$$

Using the Z-Tables, *the area under the curve at Z = 1.4 is .9192*.

Remember though that the Z-Score and the resulting probability represent the area to the left of the time value (7,200 hours). The reliability is the area to the right of the curve, which is 1 - .9191 = 0.0809.

Therefore there is an 8.1% probability that the motor has not yet failed after 7,200 hours. Or, said differently, 8.1% of the original population of motors are likely still operational after this amount of time.

We can do another example real quick.

Let's say you've got the same motor and you know it's in the wear out period where the mean and standard deviation associated with the wear out period is 6,500 hours and 500 hours respectively.

If you did not want the motor to exceed a reliability of 75%, at what time should you stop running the motor and perform some maintenance?

Using the standard Z-Tables, we can look up the Z-Score associated with .25, which is Z = 0.67

$$\boldsymbol{Z = \frac{(t_1 - \mu)}{\sigma} = 0.67 = \frac{(t_1 - 6,500)}{500}}$$

This equation can be re-arranged to solve for t_1.

$$\boldsymbol{t_1 = 0.67 * (500) + 6,500 = 6,835 \; hours}$$

This means that after 6.835 hours, the reliability equals 75% and we should perform maintenance on the motor.

The Useful Life Region of the Bathtub Curve

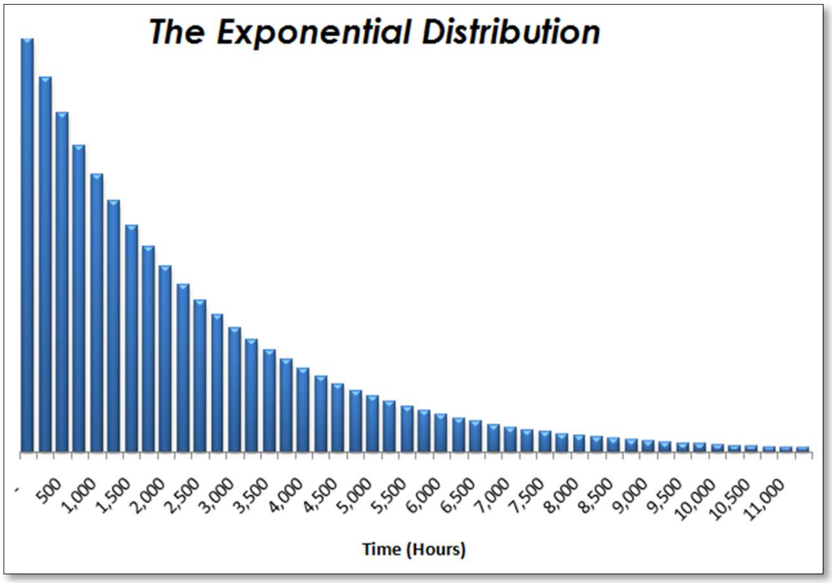
Alright, on to the final region of the bathtub curve, the Useful Life region. the Useful Life region is characterized by a period of time where the failure rate of your product is constant, due to random failures of your product.

This region is typically the longest region in terms of time spent in that region, and it's also the time period where you will want to make the most reliability estimations. *The appropriate PDF for this region is the Exponential Curve, discussed below.*

The Exponential Distribution

The Exponential Distribution is considered the most basic and widely used distribution model within reliability.

This distribution is used to model a system that is within its Useful Life Region or period where the system is experiencing a constant failure rate.



The beauty of this region of the bathtub curve and the exponential distribution is the ease at which you can make reliability calculations.

Below are the PDF, CDF & reliability equations associated with the Exponential Distribution:

$$\boldsymbol{PDF}\text{: } \mathrm{f(t)} = \lambda \mathrm{e}^{-\lambda \mathrm{t}}$$

$$\boldsymbol{CDF}\text{: } F(t) = 1 - \mathrm{e}^{-\lambda \mathrm{t}}$$

$$\boldsymbol{Reliability}\text{: } R(t) = e^{-\lambda t} = e^{\frac{-t}{\theta}}$$

$$\boldsymbol{Failure\ Rate} = \boldsymbol{\lambda} = \frac{\mathbf{1}}{\boldsymbol{\theta}}$$

$$\boldsymbol{\theta} = \boldsymbol{MTBF}\ (or\ \boldsymbol{MTTF})$$

Example Reliability Calculation Using the Exponential Distribution

So let's expand the example we used above, where we've tested 20x units and found that our MTBF is 2,996 Hours. What is the reliability of our product at 1,200 hours of operation, let's use the equation:

$$\boldsymbol{R(t)} = \boldsymbol{e^{-\lambda t}} = \boldsymbol{e^{\frac{-t}{\theta}}} = \boldsymbol{e^{\frac{-1,200}{2,966}}} = \boldsymbol{.6673}$$

So, the probability that our product will perform successfully past the 1,200 hour mark is approximately 66%.

This reliability calculation can be applicable to the entire population of product; i.e. 66% of the population can be expected to surpass the 1,200 hour mark.

2nd Reliability Calculation Using the Exponential Distribution

We can do another quick example; let's say we wanted to perform preventative maintenance on our product or equipment when the likelihood for failure is 25% (Reliability = 75%); let's take a look at how we would calculate that.

$$\boldsymbol{Failure\ Rate\ or\ F(t) = .25}\ \&\ we\ know\ that\ Reliability = R(\boldsymbol{t}) = \boldsymbol{F(t) - 1 = .75}$$

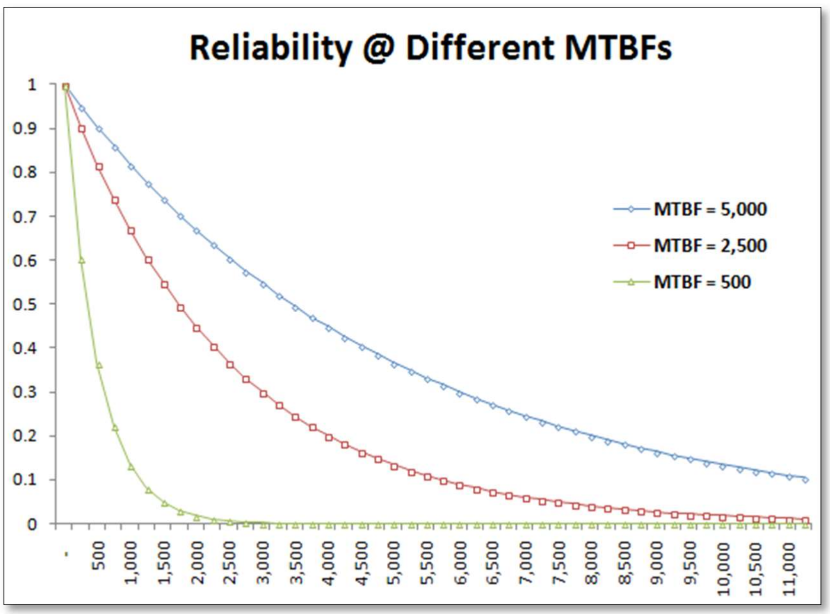
$$\boldsymbol{R(t) = e^{-\lambda t} = .75 = e^{\frac{-t}{2{,}966}}}$$

We can take the Natural Log of both sides of the equation, and we get:

$$\boldsymbol{LN(.75)} = \frac{\boldsymbol{-t}}{\boldsymbol{2,966}}, therefore\ \boldsymbol{t} = \boldsymbol{-2.966 * LN(.75) = 853\ Hours}$$

This allows us to solve the equation where we find that the reliability of our product hits 75% at 853 hours. This also means that at 853 hours, the likelihood for product failure is 25%.

Below is a graph of the **Reliability Curve** for products with different MTBF values. You'll see that the larger the MTBF the better the reliability over time.



This can be seen at the far right of the graph where at 11,250 hours the reliability is still greater than 10%, where as the reliability of the other two curves (**MTBF = 500** & **MTBF = 1,500**) are approaching 0%.

One question you might ask when looking at this graph is, why doesn't the MTBF for a given curve appear to equal 50% reliability?

Well, first let's calculate the Time value associated with 50% reliability and see how that relates to the MTBF. Let's say we've got a system with the MTBF of 5,000, at what point in time is the reliability equal to 50% (0.50):

$$\boldsymbol{R(t) = e^{-\lambda t} = .50 = e^{\frac{-t}{5{,}000}}}$$

$$\boldsymbol{LN(.50)} = \frac{\boldsymbol{-t}}{\boldsymbol{5,000}}, therefore\ \boldsymbol{t} = \boldsymbol{-5,000 * LN(.50) = 3,465\ Hours}$$

So, we were able to confirm that the time value associated with 50% reliability did not equal the MTBF - what gives?

The MTBF value for the exponential distribution is a measure of Central Tendency, however it is not like the Normal Distribution where the measure of central tendency is also the mid-point of the distribution (50% reliability).

Well then what is the reliability when time has reached the MTBF value:

$$R(t) = e^{-\lambda t} = e^{\frac{-5,000}{5,000}} = e^{-1} = 36.79\%$$

This tells us that the MTBF (5,000 Hours in the example directly above) for the Exponential distribution always occurs at the point in time where 36.79% of the population is still operations (Reliable) & 63.21% of the population has failed.

This result is the same for all products or processes that follow the exponential distributions. The measure of central tendency (MTBF) is always equal to the point in time where 63.21% of the population has failed, and where 36.79% of the product is still functional (reliable).

How Can A Constant Failure Rate Result in an Exponential Curve?

If the Exponential distribution is supposed to represent the point in time where a machine has a constant failure rate, why haven't I seen any graphs with any straight (non-linear) lines on them?

Fair question.

The exponential distribution graph above assume that once a given unit fails, it is removed from the population without replacement.

So the overall population is decreasing over time, meaning that, with a constant failure rate, less units are likely to fail as time moves on.

You can see this below in the table showing the failures of 100x motor over time.

As you can see in year 1, once 10% of the populations has failed, there are only 90 motors left.

At the constant failure rate of 10% there will only be 9 motors that fail in year 2.

Same for year 2.

Only 81 motors are present to start the year, and thus only 8.1 motors will fail that year (10% failure rate).

So, while the failure rate is constant (10%) the distribution has an exponential curve to it.

Time (Years)	Units Remaining	Failures Per Year	Cumulative Failures F(t)	Reliability Calculation R(t)
0	100	0.0	0.0	1.000
1	90.0	10.0	10.0	0.905
2	81.0	9.0	19.0	0.819
3	72.9	8.1	27.1	0.741
4	65.6	7.3	34.4	0.670
5	59.0	6.6	41.0	0.607
6	53.1	5.9	46.9	0.549
7	47.8	5.3	52.2	0.497
8	43.0	4.8	57.0	0.449
9	38.7	4.3	61.3	0.407
10	34.9	3.9	65.1	0.368
11	31.4	3.5	68.6	0.333
12	28.2	3.1	71.8	0.301
13	25.4	2.8	74.6	0.273
14	22.9	2.5	77.1	0.247
15	20.6	2.3	79.4	0.223
16	18.5	2.1	81.5	0.202
17	16.7	1.9	83.3	0.183
18	15.0	1.7	85.0	0.165
19	13.5	1.5	86.5	0.150
20	12.2	1.4	87.8	0.135