G.C.E. 'A' Level

PHYSICS

Worked Solutions

Classified Topic by Topic

(Latest Edition – H2 Syllabus)

NOVEMBER
2002 – 2010

Phang Yu Hon
B.Eng (1st Class Hons.)

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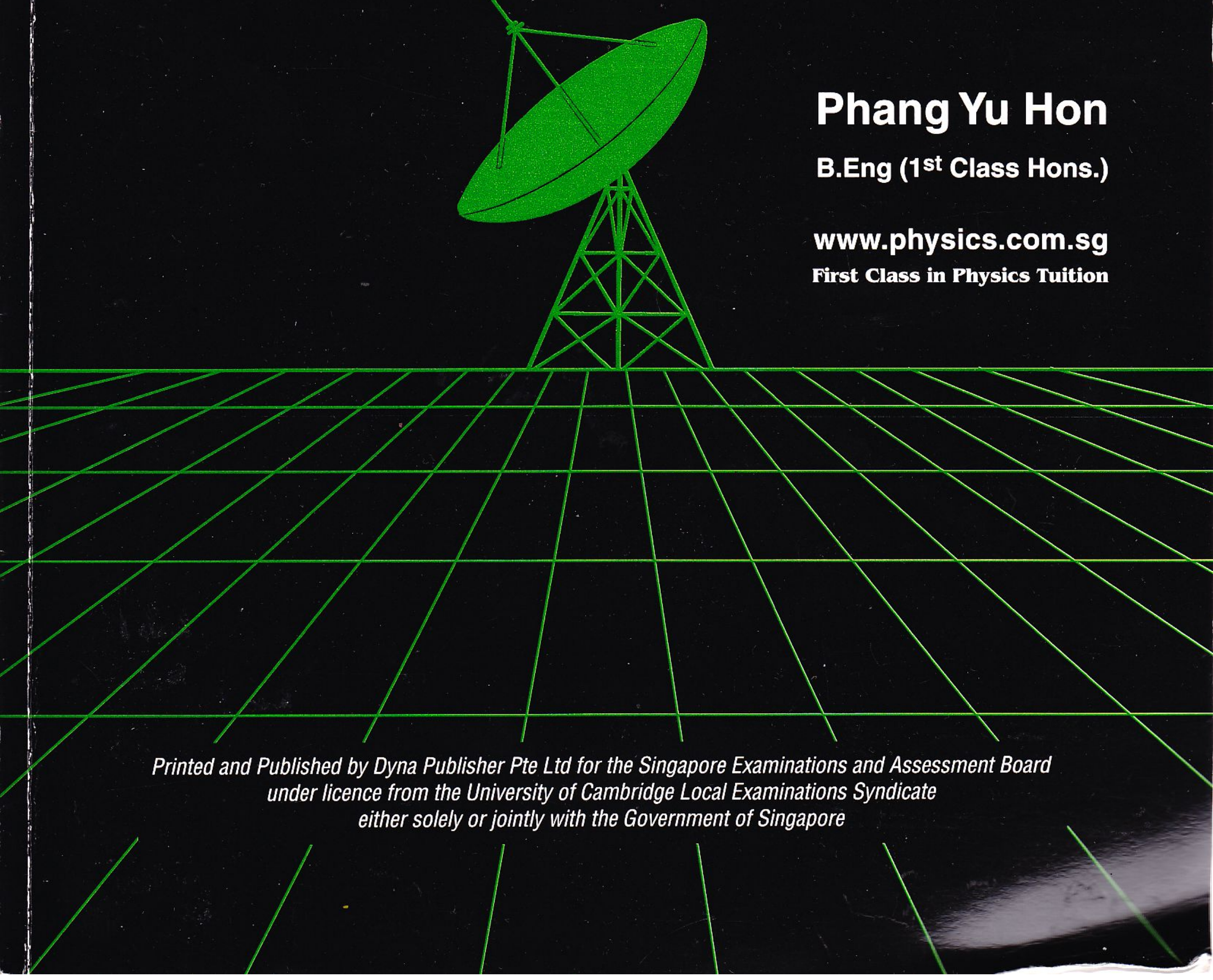

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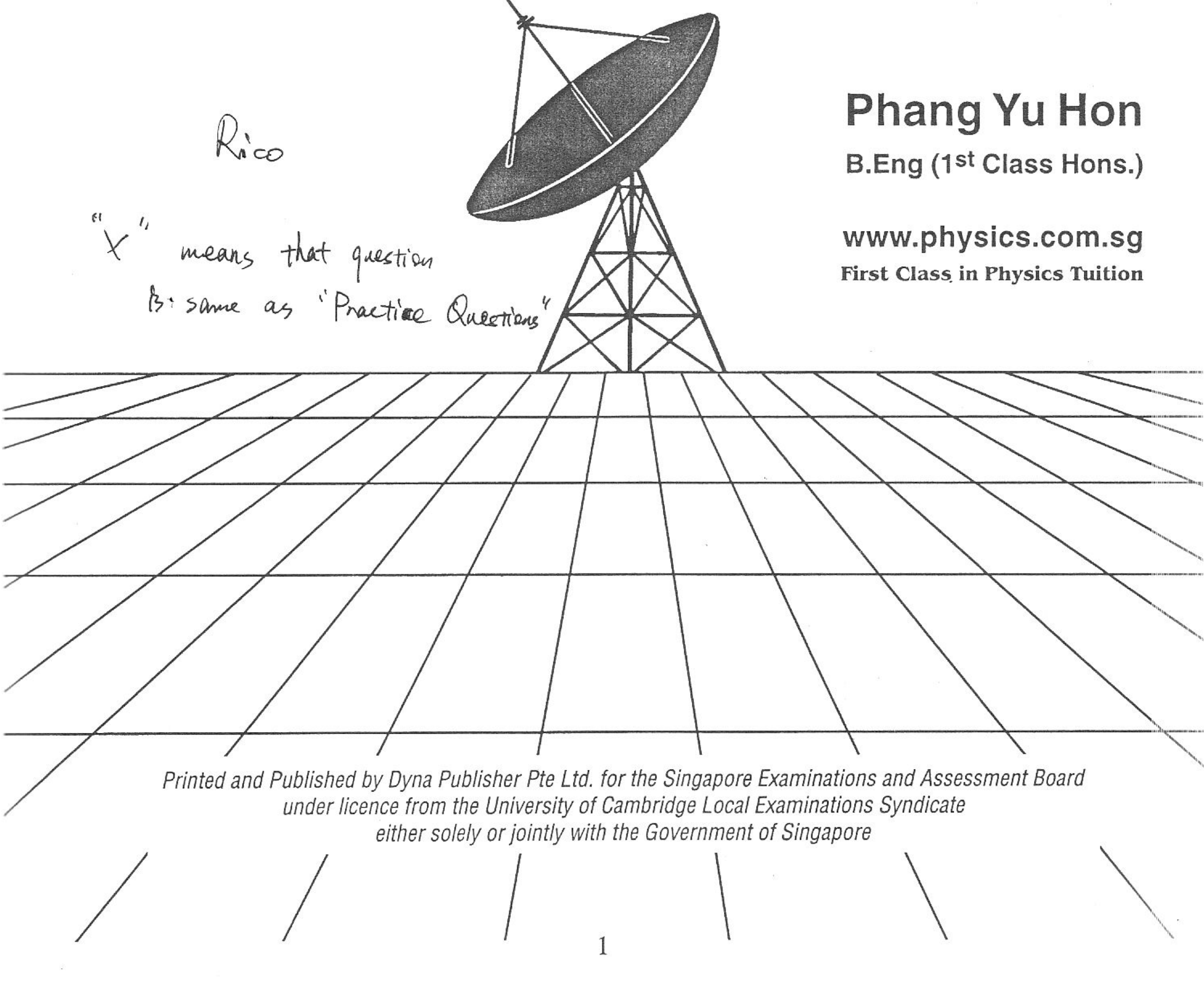
"X" means that question is same as "Practice Questions"



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CONTENTS

N.B.: Questions marked with an asterisk (*) require knowledge of later topics. They should be attempted later.

DATA AND FORMULAE

Data

speed of light in free space	c	=	3.00×10^{8} m s^{-1}
permeability of free space	μ_o	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	ε_o	=	8.85×10^{-12} F m^{-1}
			$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	e	=	1.60×10^{-19} C
the Planck constant	h	=	6.63×10^{-34} Js
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron	m_e	=	9.11×10^{-31} kg
rest mass of proton	m_p	=	1.67×10^{-27} kg
molar gas constant	R	=	8.31 J K^{-1} mo^{-1}
the Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant	k	=	1.38×10^{-23} J K^{-1}
gravitational constant	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
work done on/by a gas	W	=	$p\Delta V$
hydrostatic pressure	p	=	ρgh
gravitational potential	ϕ	=	$-Gm/r$
displacement of particle in s.h.m.	x	=	$x_o \sin\omega t$
velocity of particle in s.h.m.	v	=	$v_o \cos\omega t$
		=	$\pm\omega\sqrt{(x_o^2 - x^2)}$
resistors in series	R	=	$R_1 + R_2 + ...$
resistors in parallel	$1/R$	=	$1/R_1 + 1/R_2 + ...$
electric potential	V	=	$Q/4\pi\varepsilon_o r$
alternating current/voltage	x	=	$x_o \sin\omega t$
transmission coefficient	T	=	$\exp(-2kd)$
	where k	=	$\sqrt{\dfrac{8\pi^2 m(U-E)}{h^2}}$
radioactive decay	x	=	$x_o \exp(-\lambda t)$
decay constant	λ	=	$\dfrac{0.693}{t_{\frac{1}{2}}}$

TOPIC 1 Measurement

SI Units

1 The drag force F experienced by a steel sphere of radius r dropping at speed v through a liquid is given by

$$F = arv,$$

where a is a constant.

What would be a suitable SI unit for a?

A N

B $N s^{-1}$

C $N m^2 s^{-1}$

D $N m^{-2} s$

N02/I/1

Solution Answer: **D**

Acceleration $a = F/rv$.
The unit for force F is the newton (N).
The unit for r is the metre (m) while that of v is ($m s^{-1}$).
Therefore, the base units for a are (N)/($m^2 s^{-1}$) = $N m^{-2} s$.

2 An alternative form of the unit of resistance, the ohm (Ω), is VA^{-1}.

Which of the following examples shows a similar correct alternative form of unit?

	unit	alternative form
A	coulomb (C)	$A s^{-1}$
B	pascal (Pa)	$N m^{-2}$
C	volt (V)	J C
D	watt (W)	J s

N04/I/1

Solution Answer: **B**

Answer B is the correct answer as the pascal (Pa) is the unit for pressure, where pressure is defined as force (N) acting per unit area (m^2).
Answer A is wrong because the coulomb (C) = A s.
Answer C is wrong because the volt (V) = $J C^{-1}$.
Answer D is wrong because the watt (W) = $J s^{-1}$.

3 What is a reasonable estimate for the volume of a wooden metre rule found in a school laboratory?

A 1.5 cm^3 B 15 cm^3 C 150 cm^3 D 1500 cm^3

N07/I/1

Solution Answer: **C**

The length of a metre rule is 100 cm. A reasonable estimate of its width would be 3 cm, while that of its thickness 0.5 cm.
The estimated volume is 100 cm x 3 cm x 0.5 cm = 150 cm^3.

4 A radio aerial of length L, when the current is I, emits a signal of wavelength λ and power P. These quantities are related by

$$P = kI^2\left(\frac{L}{\lambda}\right)^2$$

where k is a constant.
What unit, if any, should be used for the constant k?

A volt

B ohm

C watt

D no unit

N07/I/2

Solution Answer: **B**

The SI unit of the left-hand-side of the equation is [P] = W.
The SI unit of the right-hand-side of the equation is
[k](A^2)(m / m)^2 = [k](A^2).
Therefore, the SI unit of the quantity k is
[k] = W/A^2 = (W/A)/A = V/A = Ω.

5 What is the order of magnitude of the energy of an electron when it hits the screen of a cathode-ray tube?

A 10^{-22} J B 10^{-19} J C 10^{-16} J D 10^{-13} J

N09/I/2

Solution Answer: **C**

In a cathode-ray tube, the accelerating potential difference is in the kV range, hence the electrons are given a kinetic energy of a few keV (10^{-16} J).

In secondary 4, under the G.C.E 'O' level syllabus, students learned about step-up and step-down transformers. The cathode-ray tube television (the old boxy type) was one of the few household devices that use a step-up transformer (another being the mosquito zapper). The accelerating potential difference in the cathode-ray tube's electron gun is stepped up from the a.c. power supply's 240 V to the kV range.

Short Questions

6 When a solid is heated, the thermal energy required is given by the expression

gain in thermal energy = mass × c × temperature rise,

where c is a constant.

(a) Name the quantities in the expression that are SI base quantities. [2]

(b) Express, in terms of SI base units, the units of

(i) thermal energy,

(ii) the constant c. [4]

N03/II/1

Solution

(a) mass (in kilogram) and temperature rise (in kelvin).

(b) (i) Thermal energy has the same units as work (the joule), which is equal to force x distance (N m).

Force (in newtons) is mass x acceleration (kg m s^{-2}).

The base units for thermal energy are (kg m s^{-2})(m) = kg m^2 s^{-2}.

(ii) c = (thermal energy)/(mass x temperature rise).

The base units of c

= (kg m^2 s^{-2})/(kg K)

= m^2 s^{-2} K^{-1}.

Long Questions

Put the following terms into pairs, so that in each pair a quantity is given followed by its corresponding unit.

coulomb
power
volt
magnetic flux density
work
watt
potential difference
charge
tesla
joule [4] N04/III/1 (part)

Solution

power, watt
magnetic flux density, tesla
work, joule
potential difference, volt
charge, coulomb

Errors and Uncertainties

8 An object of mass 1.000 kg is placed on four different balances. For each balance the reading is taken five times.

The table shows the values obtained together with the means.

Which balance has the smallest systematic error but is not very precise?

balance	reading/kg 1	2	3	4	5	mean/kg
A	1.000	1.000	1.002	1.001	1.002	1.001
B	1.011	0.999	1.001	0.989	0.995	0.999
C	1.012	1.013	1.012	1.014	1.014	1.013
D	0.993	0.987	1.002	1.000	0.983	0.993

N02/I/2

Solution Answer: **B**

Precision refers to how close data points are from one another.
Systematic errors increase the amount by which measured values deviate from their true value.
It can be seen that the readings in B are scattered over a wide range thus having low precision.
However, the mean value in B (0.999) is very close to the true value (1.000), thus B is said to have a low systematic error.

9 Errors in measurement may be either systematic or random.

Which of the following involves random error?

A not allowing for zero error on a moving-coil voltmeter

B not subtracting background count rate when determining the count rate from a radioactive source

C stopping a stopwatch at the end of a race

D using the value of g as 10 N kg^{-1} when calculating weight from mass

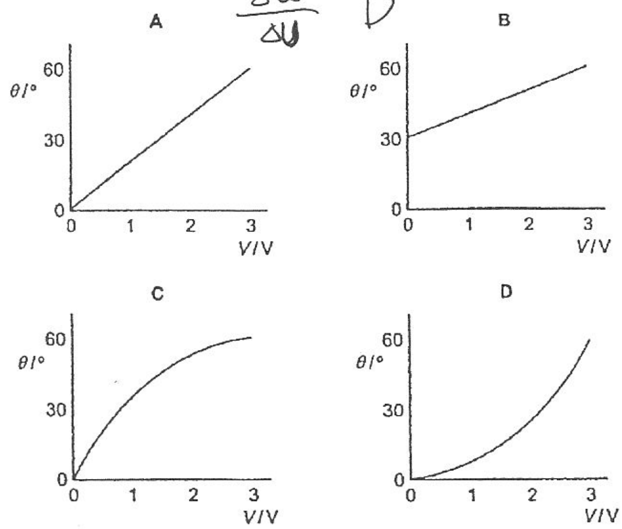
N03/I/2

Solution Answer: **C**

Random errors result in the readings being scattered.
Due to errors in human judgment, the stopwatch could be stopped before or after the actual end of the race.
This results in a random error in the time measurement.
The other answers involve an offset that is fixed in direction and thus result in systematic errors.

10 Calibration curves showing the variation with potential difference V of the deflection θ of the pointer of four different voltmeters are given below.

Which meter gives the largest angular change per unit potential difference at 2.5 V?

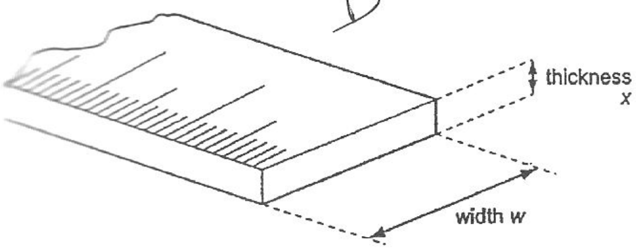


N05/I/2

Solution Answer: **D**

Answer D's gradient ($d\theta/dV$) is the steepest at 2.5 V.
Hence the rate of change of deflection θ with potential difference V is also the greatest.

11 In an experiment, the width w and the thickness x of a metre rule are to be measured as precisely as possible using normal laboratory apparatus.



Which combination of instruments is most appropriate for these measurements?

	measurement of w	measurement of x
A	half-metre rule	half-metre rule
B	half-metre rule	vernier calipers
C	vernier calipers	half-metre rule
D	vernier calipers	micrometer screw gauge

N06/I/2

Solution Answer: **D**

The precision of a half-metre rule, vernier calipers and micrometer screw gauge are respectively 1 mm, 0.1 mm and 0.01 mm.

12 Which estimate is realistic?

A The kinetic energy of a bus travelling on an expressway is 30 000 J.
B The power of a domestic light is 300 W.
C The temperature of a hot oven is 300 K.
D The volume of air in a car tyre is 0.03 m^3.

N08/I/2

Solution Answer: **D**

Using average car tyre radius R = 0.20 m and cross-sectional radius r = 0.10 m, volume = $(2\pi R)(\pi r^2)$ = 0.039 m^3.
Answer A is wrong. Using expressway speed of 70 km/h ≈ 19 m s^{-1} and ½mv^2 = 30 000 J, the mass of the bus is 166 kg, less than some human adults.
Answer B is wrong as the power rating of a domestic filament lamp is typically around 40 W (energy-saving lamps will have even lower power ratings).
Answer C is wrong because 300 K ≡ 27 °C which is room temperature.

13 The manufacturers of a digital voltmeter give, as its specification,
accuracy ± 1% with an additional uncertainty of ± 10 mV.
The meter reads 4.072 V.
How should this reading be recorded, together with its uncertainty?

A (4.07 ± 0.01) V
B (4.07 ± 0.04) V
C (4.072 ± 0.052) V
D (4.07 ± 0.05) V

N08/I/3

Solution Answer: **D**

1% of 4.072 V is 0.0407 V. Add to this the additional uncertainty of 0.010 V, the total uncertainty is 0.05 V (uncertainties recorded to 1 s.f.)

4.072 V is then rounded to 4.07 V (2 d.p.), same number of d.p.s as the uncertainty.

14 An instrument gives a numerical reading of 0.00160 ± 0.00005.
Which statement is correct?

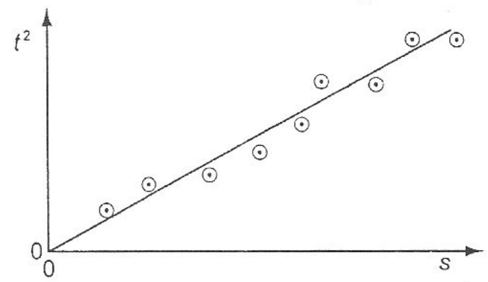
A The actual uncertainty is 5.
B The fractional uncertainty is 5 x 10^{-5}.
C The fractional uncertainty is 5/16.
D The percentage uncertainty is 3 %.

N09/I/1

Solution Answer: **D**

Percentage uncertainty is (0.00005 / 0.00160) x 100 % = 3 %.

15 An object falls freely from rest and travels a distance s in time t. A graph of t^2 against s is plotted and used to determine the acceleration of free fall g.



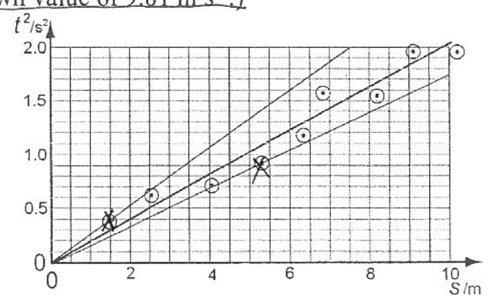
The gradient of the graph is found to be 0.204 s^2 m^{-1}.
Which statement about the value obtained for g is correct?

A It is accurate but not precise.
B It is both precise and accurate.
C It is neither precise nor accurate.
D It is precise but not accurate.

N10/I/1

Solution Answer: **A**

Use the kinematics equation $s = ut + \frac{1}{2}at^2$ where initial velocity u = 0 (from rest) and acceleration $a = g$. The equation can be rearranged into $t^2 = (2/g)s$ where the gradient $2/g$ = 0.204 s^2 m^{-1}. Using this value, g = 9.8039 m s^{-2}, which is close to the known value of g (9.81 m s^{-2}) and thus accurate. However, there is significant scattering of data points around the straight line hence the value is not precise.
(The judgment for precision can be subjective but by visually laying a grid over the graph, it can be seen that the gradient varies significantly. The exact values are from 0.267 s^2 m^{-1} to 0.173 s^2 m^{-1}, implying an acceleration of free fall g between 7.49 m s^{-2} and 11.56 m s^{-2}. This is significantly different from the known value of 9.81 m s^{-2}.)



16 A wire of uniform circular cross-section has diameter d and length L. A potential difference V between the ends of the wire gives rise to a current I in the wire.
The resistivity ρ of the material of the wire is given by the expression

$$\rho = \frac{\pi d^2 V}{4LI}.$$

In one particular experiment, the following measurements are made.

$d = 1.20 \pm 0.01$ cm

$I = 1.50 \pm 0.05$ A

$L = 100 \pm 1$ cm

$V = 5.0 \pm 0.1$ V

Which measurement gives rise to the **least** uncertainty in the value for the resistivity?

A d **B** I **C** L **D** V

N10/I/3

Solution Answer: **C**

The fractional uncertainty of the resistivity:

$$\Delta\rho/\rho = 2(\Delta d/d) + \Delta V/V + \Delta L/L + \Delta I/I$$
$$= 2(0.01/1.20) + (0.1/5.0) + (1/100) + (0.05/1.50)$$
$$= 0.0167 + 0.02 + 0.01 + 0.033$$

The **least** uncertainty is contributed by the measurement of L.

Short Question

1 A student times the fall of a small metal ball. Data for the time t taken for the ball to fall a vertical distance h from rest are given below.

$$h = 266 \pm 1 \text{ cm}$$
$$t = 0.740 \pm 0.005 \text{ s}$$

(a) Use these data to determine

(i) a value, to three significant figures, of the acceleration of free fall g,
acceleration = ... m s^{-2} [2]

(ii) the percentage uncertainty, to two significant figures, of

1. the distance h,
uncertainty = ... % [1]

2. the time t,
uncertainty = ... % [1]

(b) Use your answers in (a) to determine the actual uncertainty in the value of g.
Hence give a statement of g, with its uncertainty, to an appropriate number of significant figures.
g = ... ± .... m s^{-2} [3]

(c) Suggest two reasons why, in this experiment, although the value of t is precise, it may not be accurate.

1. ....

....

2. ....

.... [2]

N07/II/1

Solution

(a) (i) Use the kinematics equation $s = ut + \frac{1}{2}at^2$.

where s is the displacement (h), u the initial velocity (0 m s^{-1}), a the acceleration (g) and t the time taken.

$$h = \tfrac{1}{2}gt^2$$
$$g = 2h/t^2 = 2(2.66 \text{ m})/(0.740 \text{ s})^2 = 9.72 \text{ m s}^{-2}$$

(ii) 1. $\Delta h/h \times 100\% = (1/266) \times 100\% = 0.38\%$

2. $\Delta t/t \times 100\% = (0.005/0.740) \times 100\% = 0.68\%$

(b) Since $g = 2h/t^2$,

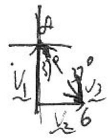
$$\Delta g/g = \Delta h/h + 2(\Delta t/t)$$
$$\Delta g/g = 0.38/100 + 2(0.68/100) = 0.0174$$
$$\Delta g = 0.0174 \times g$$
$$= 0.0174 \times 9.72 \text{ m s}^{-2} = 0.169 = 0.2 \text{ m s}^{-2} \text{ (1 s.f.)}$$

Absolute uncertainties can only have one significant figure, and the value of g has to be rounded off to the same number of decimal places as the uncertainty.

$$g = (9.7 \pm 0.2) \text{ m s}^{-2}$$

(c) 1. Human reaction time (typically around 0.3 s) causes a delay in the pressing of the stop button on the watch, resulting in an overestimation of the time of free fall.

2. Air resistance results in a smaller accelerating force than is assumed hence the time of free fall measured is an overestimation of the true value.



Scalars & Vectors

18 Which one of the following groups contains three vector quantities?

- **A** displacement, velocity, energy
- **B** displacement, velocity, momentum
- **C** velocity, acceleration, power
- **D** force, work, energy

N03/I/1

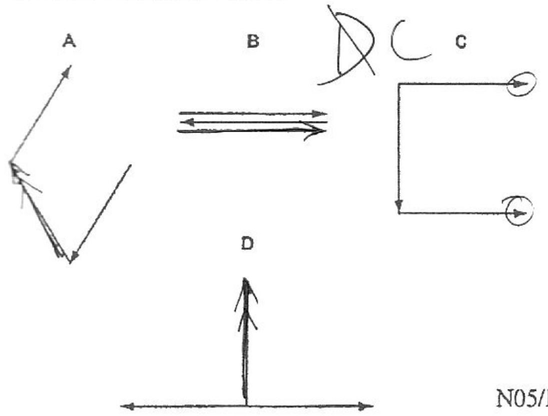
Solution Answer: **B**

The following are vector quantities because they have both magnitude and direction: displacement, velocity, momentum, acceleration. The following are scalar quantities because they have only magnitude but no direction: energy, power, work.

Note that the momentum of a body is the product of its mass and velocity, and acts in the same direction as its velocity.

19 Each diagram shows three vectors of equal magnitude.

In which diagram is the magnitude of the resultant vector different from the other three?



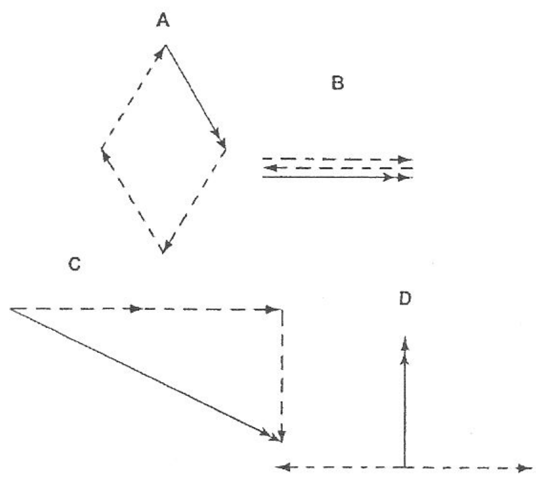
N05/I/1

Solution Answer: **C**

Answer C's resultant vector has a magnitude of √5 times that of one vector, whereas the others' resultants all have a magnitude exactly equal to that of one vector.

The resultant vector in each answer is shown below as a double-headed arrow.

The original vectors in the question are shown as dotted arrows.



20 A boat changes its velocity from 8 m s^{-1} due north to 6 m s^{-1} due east.

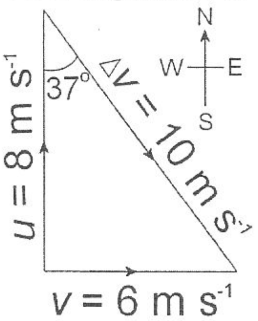
What is its change in velocity?

- **A** 2 m s^{-1} at a direction of 37° east of north
- **B** 2 m s^{-1} at a direction of 53° east of north
- **C** 10 m s^{-1} at a direction of 37° east of south
- **D** 10 m s^{-1} at a direction of 53° west of south

N10/I/2

Solution Answer: **C**

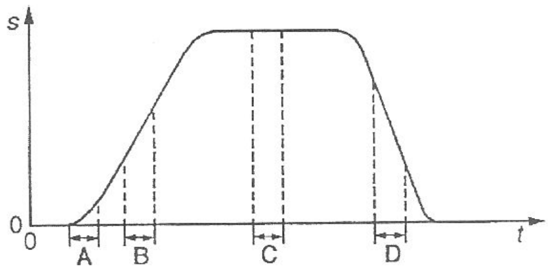
The initial velocity u, final velocity v and change in velocity Δv are shown in the vector diagram below.



TOPIC 2 Kinematics

Rectilinear Motion

1 The graph shows the variation with time t of the displacement s of a vehicle moving along a straight line.



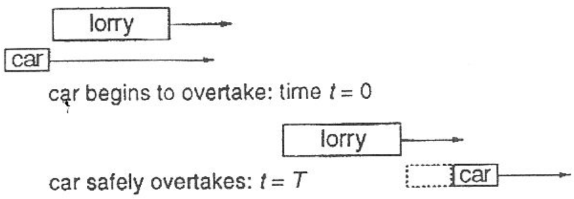
During which time interval does the acceleration of the vehicle have its greatest numerical value?

N02/I/3

Solution Answer: A

Acceleration is the second derivative of the displacement-time graph ($a = d^2s/dt^2$).
Acceleration has the greatest value when the gradient of the displacement-time graph is increasing, because that indicates the velocity is increasing (as in time-interval A).
The other intervals like B and D have constant velocity and hence zero acceleration.
For interval C, both the velocity and acceleration are zero.

2 The minimum time T required for a car safely to overtake a lorry on the motorway is measured from the time the front of the car is level with the rear of the lorry, until the rear of the passing car is a full car-length ahead of the lorry.



The car is 3.5 m long and the lorry is 17.0 m long. The graph shows the variation with time t of the speeds v of the car and the lorry.

What is the value of T?

A 0.86 s

B 1.2 s

C 2.6 s

D 3.0 s

N02/I/4

Solution Answer: D

During time T, distance travelled by car = ½(10 + 26)T = 18T while distance travelled by lorry = 10T.
The difference in distance is equal to one lorry length plus two car lengths.
Hence 18T – 10T = 17.0 m + 2(3.5m) and T = 3.0 s.

3 A particle is accelerating along a straight line.

Which statement defines the velocity of the particle at a particular instant?

A Velocity is displacement divided by time.

B Velocity is distance divided by time.

C Velocity is the rate of change of displacement.

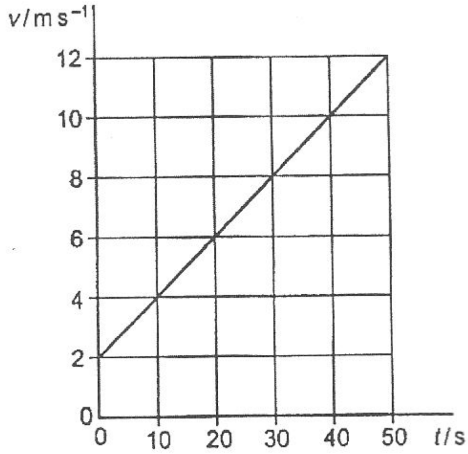
D Velocity is the rate of change of distance.

N03/I/3

Solution Answer: C

Velocity and displacements are both vector quantities whereas distance is a scalar quantity.
Thus velocity (a vector) is defined with respect to displacement (answer C), whereas speed (a scalar) is defined as the rate of change of distance (answer D).
Displacement divided by time (answer A) is the average velocity over that time interval, and not the velocity at any particular instant.
Distance divided by time (answer B) is the average speed over that time interval, and not the speed at any particular instant.

4 A train travelling at 2.0 m s^{-1} passes through a station. The variation with time t of the speed v of the train after leaving the station is shown below.



What is the speed of the train when it is 150 m from the station?

A 6.0 ms^{-1}

B 8.0 ms^{-1}

C 10 ms^{-1}

D 12 ms^{-1}

N04/I/3

Solution Answer: B

Distance travelled can be found from the area under the speed-time graph.
It can be observed that at $t = 30$ s and $v = 8.0$ m s^{-1}, the area of the trapezium under the graph is $\frac{1}{2}(2 + 8)(30) = 150$ m.
Hence the area covered is 150 m when $v = 8.0$ m s^{-1}.

Alternative method

$a = (v - u)/t = (12 \text{ m s}^{-1} - 2 \text{ m s}^{-1})/(50 \text{ s}) = 0.20 \text{ ms}^{-2}$

Use the kinematics equation $v^2 = u^2 + 2as$,

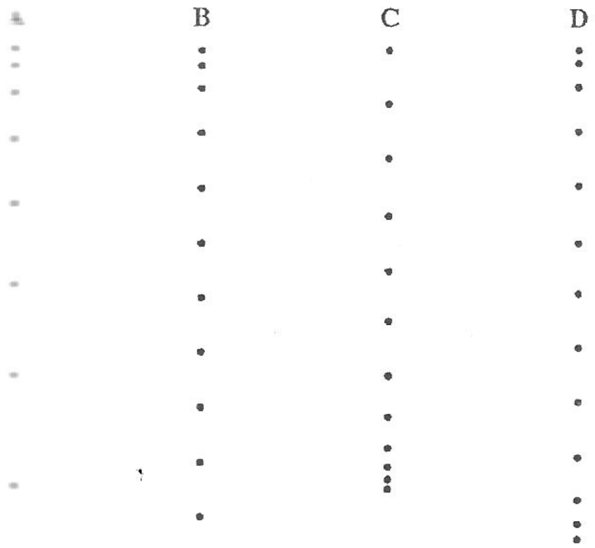
$$v^2 = (2 \text{ m s}^{-1})^2 + 2(0.20 \text{ m s}^{-2})(150 \text{ m})$$

$$v = 8.0 \text{ m s}^{-1}$$

5 A steel sphere is released from rest at the surface of a deep tank of viscous oil. A multiple exposure photograph is taken of the sphere as it falls.

The time interval between exposures is always the same.

Which of the following could represent this photograph?



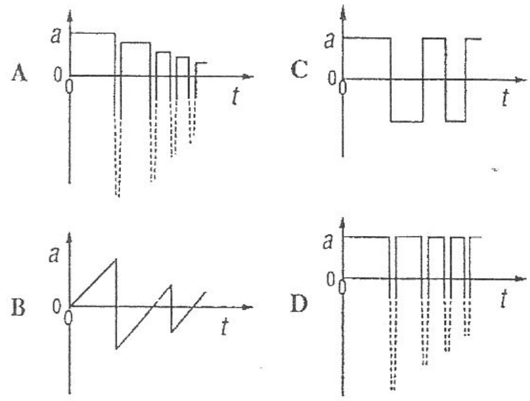
N04/I/4

Solution Answer: B

Initially the sphere's speed is low and viscous drag is negligible.
The sphere accelerates from rest hence the spacing between successive exposures increases.
However, as the sphere's speed increases, the drag force also increases and the resultant force acting on it decreases.
The sphere soon reaches terminal velocity in the viscous oil, and the spacing remains constant for the rest of the journey.

6 A steel ball is released from rest a distance above a rigid horizontal surface and is allowed to bounce.

Which graph best represents the variation with time t of acceleration a?

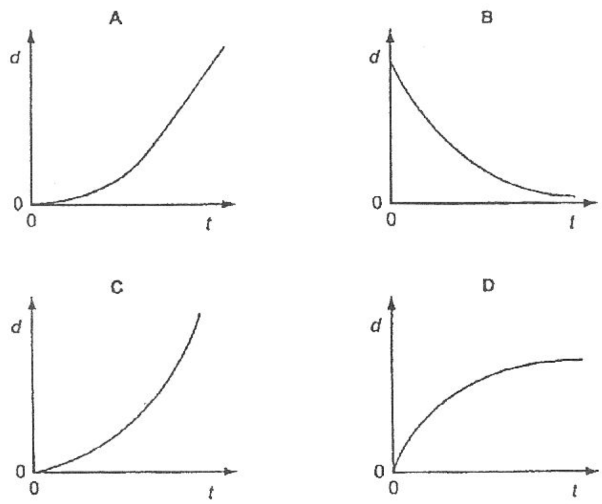


N05/I/3

Solution Answer: D

As long as the ball is in mid-air (whether on its way up or down), its acceleration a is constant (acceleration of free fall 9.81 m s^{-2}).
In that split second when the ball hits the rigid surface, it experiences an extremely large retarding force and hence a tremendous deceleration (since $a = F/m$).
Note the dotted lines indicating that these tremendous decelerations are too large to be drawn to scale.

7 Which graph shows how the distance d fallen varies with time t for a body falling from rest through air?

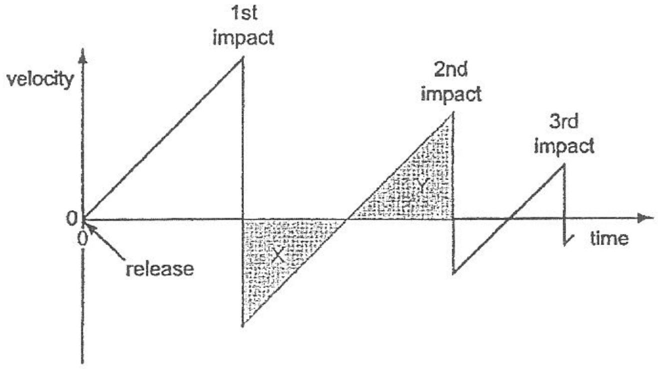


N05/I/4

Solution Answer: A

Initially the body's speed is low and air resistance is negligible hence it undergoes free fall with increasing speed. Since the gradient of the distance time graph represents the body's speed, the graph has an increasing gradient.
As the body's speed increases, the air resistance opposing its motion also increases hence its resultant force decreases and so does its acceleration.
Eventually, the sum of air resistance and upthrust is equal and opposite to the body's weight, and the resultant force as well as acceleration fall to zero.
The body then falls at a constant terminal speed hence the distance-time graph has a constant gradient (straight line).

8 A ball is released from rest above a horizontal surface. The graph shows the variation with time of its velocity.



Why are areas X and Y equal?

A For one impact, the speed at which the ball hits the surface equals the speed at which it leaves the surface.

B The ball rises and falls through the same distance between impacts.

C The ball's acceleration is the same during its upward and downward motion.

D The speed at which the ball leaves the surface after an impact is equal to the speed at which it returns to the surface for the next impact.

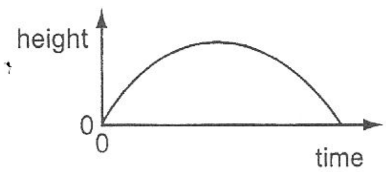
N06/I/4

Solution Answer: **B**

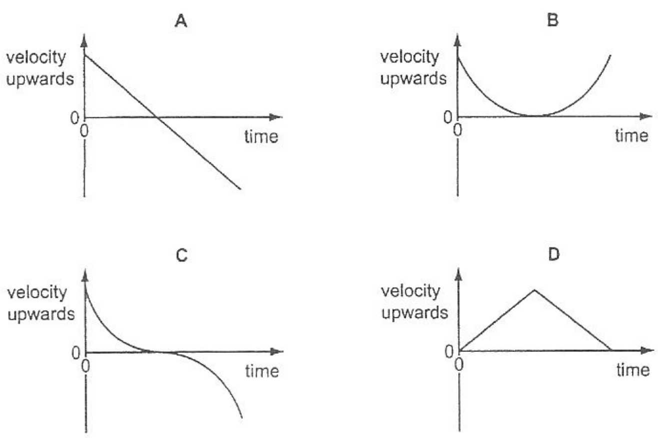
The area under a velocity-time graph represents the displacement of the body.

The reason for the areas being the same is that the ball rises and falls through the same vertical distance between impacts.

9 A ball is thrown vertically upwards and returns along the same path. The graph shows how its height varies with time.



Which velocity-time graph describes this motion?



N07/I/3

Solution Answer: **A**

The velocity-time graph must have a constant negative gradient since its gradient represents the vertical acceleration $a = -g$, which is a constant.

10 What is the definition of velocity?

A displacement per unit time
B distance travelled in unit time
C distance travelled per unit time
D speed in a particular direction

N08/I/1

Solution Answer: **A**

Velocity measures the rate of change of displacement.
Answers B and C are scalar quantities and relate to speed.
Answer D refers to velocity, but is not its definition.

11 A metal ball is dropped from rest over a bed of sand. It hits the sand bed one second later and makes an impression of maximum depth 8.0 mm in the sand.
Air resistance is negligible.
On hitting the sand, what is the average deceleration of the ball?

A 6.0×10^2 m s^{-2}
B 1.2×10^3 m s^{-2}
C 6.0×10^3 m s^{-2}
D 1.2×10^4 m s^{-2}

N08/I/4

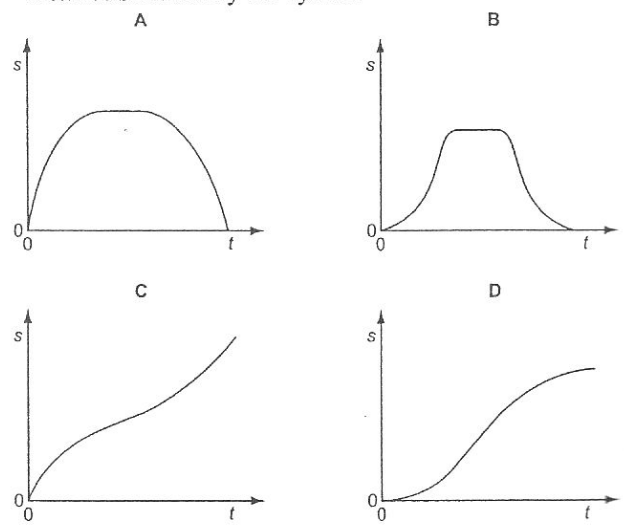
Solution Answer: **C**

The ball's speed just before hitting the sand bed can be found using $v = u + at = 0 + (9.81 \text{ m s}^{-2})(1 \text{ s}) = 9.81 \text{ m s}^{-1}$.

The average deceleration can be found using $v^2 = u^2 + 2as$.

$$0 = (9.81 \text{ m s}^{-1})^2 + 2a(8.0 \times 10^{-3} \text{ m})$$
$$a = -6.0 \times 10^3 \text{ m s}^{-2}$$

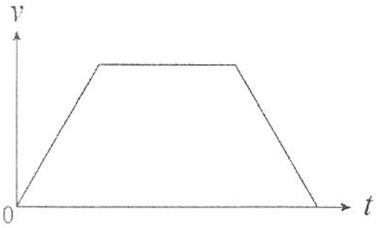
12 A cyclist accelerates down a hill and then travels at constant speed before decelerating as he climbs back up another hill.
Which graph shows the variation with time t of the distance s moved by the cyclist?



N08/I/5

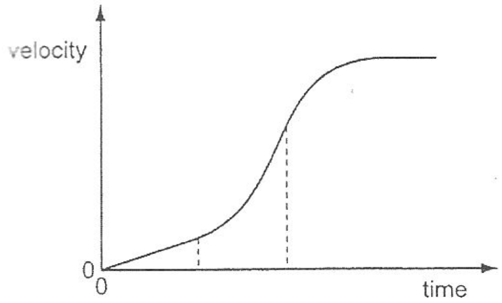
Solution Answer: **D**

The speed-time graph of the cyclist will probably look something like this (assuming uniform acceleration/ deceleration).



Since speed is the gradient of the distance-time graph, this means the gradient of the distance-time graph first increases, stays constant, and then decreases.

13 The graph shows how the velocity of a racing car changes with time.



Which statement describes the acceleration?

A A constant positive acceleration is followed by an acceleration increase and then a negative acceleration.
B The acceleration increases positively in the first two sections and then decreases to zero.
C The acceleration is positive at the start, increases, then decreases to zero.
D The acceleration starts from zero, increases, then decreases to zero.

N09/I/3

Solution Answer: C

The gradient of its velocity-time graph represents the acceleration of the racing car.

At the start, the gradient of the velocity-time graph is positive (and not zero). The graph then gets steeper with time, and finally plateaus (acceleration = 0).

14 A man stands on the edge of a cliff. He throws a stone upwards with a velocity of 19.6 m s^{-1} at time t = 0. The stone reaches the top of the trajectory after 2.00 s and then falls towards the bottom of the cliff. Air resistance is negligible.
Which row shows the correct velocity v and acceleration a of the stone at different times?

	t/s	v/m s^{-1}	a/m s^{-2}
A	1.00	9.81	9.81
B	2.00	0	0
C	3.00	9.81	−9.81
D	5.00	−29.4	−9.81

N09/I/4

Solution Answer: **D**

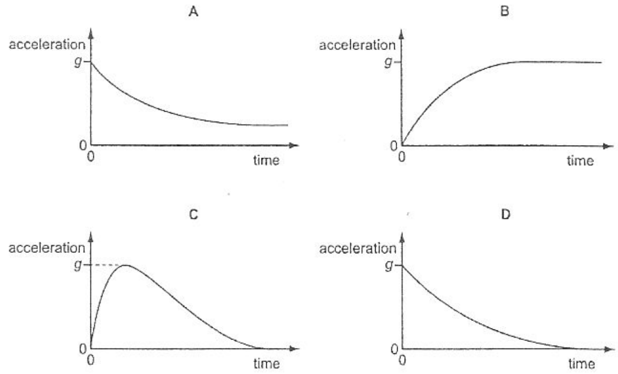
Taking upwards to be the positive direction (which this question appears to be using), the acceleration of free fall is negative since it is downwards.

At t = 5.00 s, the ball has already passed the top of its trajectory hence its velocity should be downwards as well.

Alternatively, using the kinematics equation

$$v = u + at = 19.6 + (-9.81)(5.00) = -29.4 \text{ m s}^{-1}.$$

15 An object is dropped from a great height so that air resistance becomes significant.
Which graphs shows how its acceleration varies with time?

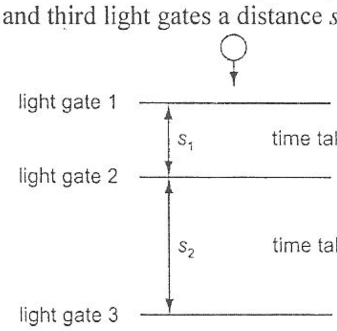


N10/I/4

Solution Answer: D

Initially, the object is at rest hence it experiences no air resistance and the resultant force acting on it is its weight alone. As a result, its acceleration is equal to g. As the object's speed increases, the air resistance opposing its motion increases thus causing its resultant force to fall. According to Newton's second law of motion, its acceleration will decrease. Eventually, when the total resistive force (air resistance and upthrust) is equal and opposite to the object's weight, its resultant force is zero and acceleration also zero (body reaches terminal velocity).

16 An object falls freely with constant acceleration a from above three light gates. It is found that it takes a time t to fall between the first two light gates a distance of s_1 apart. It then takes an additional time, also t, to fall between the second and third light gates a distance s_2 apart.



What is the acceleration in terms of s_1, s_2 and t?

A $\frac{(s_2 - s_1)}{t^2}$ B $\frac{(s_2 - s_1)}{2t^2}$ C $\frac{2(s_2 - s_1)}{3t^2}$ D $\frac{2(s_2 - s_1)}{t^2}$

N10/I/5

Solution Answer: A

Use the kinematics equation $s = ut + ½at^2$ where acceleration $a = g$. Between the first two light gates, the initial velocity $u = 0$ hence $s_1 = ½gt^2$. Between the second and third light gates, $u = gt$ hence $s_2 = (gt)t + ½gt^2 = (3/2)gt^2$. Subtracting the first equation from the second, $s_2 - s_1 = gt^2$. Hence $g = (s_2 - s_1)/ t^2$.

Short Questions

17 A student wishes to measure the length of a metal plate. The only equipment available is an electronic timer controlled by a light beam and a rod 1.00 m long.

Using the rod, the student positions the plate so that its lower edge is 1.00 m above the light beam, as shown in Fig. 1.1.

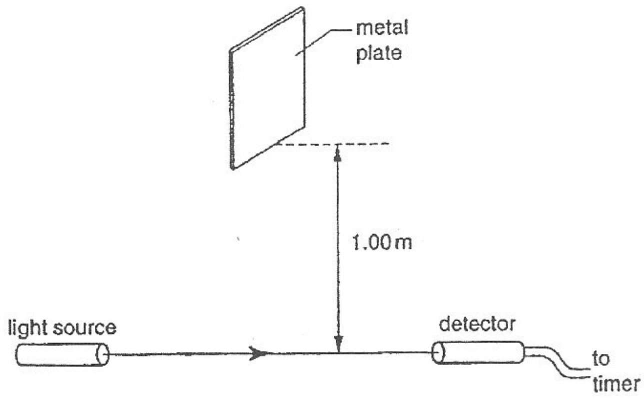


Fig. 1.1

The metal plate is released and the timer starts to record as the light beam is cut. The total time for the plate to pass through the beam is 0.052 s. The student is told that the local value for the acceleration of free fall is 9.79 m s^{-2}.

(a) (i) Show that the time for the bottom edge of the plate to reach the light beam is 0.452 s. [1]

(ii) Calculate the length of the metal plate, giving your answer to an appropriate number of significant figures.

length = ... m [4]

(b) Suggest two reasons why the time for the bottom edge of the plate to reach the light beam may differ from that quoted in *(a)*(i). [2]

N05/II/1

Solution

(a) (i) Use the kinematics equation $s = ut + ½at^2$ where s is the displacement travelled by the metal plate, u its initial velocity (0.00 m s^{-1} in this case since the plate is released from rest), a the acceleration of free fall (9.79 m s^{-2} at this location) and t the time for the bottom edge of the place to reach the beam.

$$s = ut + ½at^2$$
$$1.00 = ½(9.79 \text{ m s}^{-2})t^2$$
$$t = 0.45198 = 0.452 \text{ s}$$

(ii) Use the same kinematics equation $s = ut + ½at^2$ but with s being the length of the metal plate and u the velocity of the plate when it reaches the beam.
The initial velocity (u) of the plate when the plate starts entering the beam is equal to the final velocity (v) of the first part of the fall (through 1.00 m).

$$v = u + at$$
$$= 0 + (9.79 \text{ m s}^{-2})(0.45198 \text{ s}) = 4.4249 \text{ m s}^{-1}$$
$$s = ut + ½at^2$$
$$= (4.4249 \text{ m s}^{-1})(0.052 \text{ s}) + ½(9.79 \text{ m s}^{-2})(0.052 \text{ s})^2$$
$$= 0.2433 = 0.24 \text{ m (2 s.f.)}$$

(b) 1. In practice, the presence of air resistance decreases the acceleration of the plate and causes the time to be longer than that predicted.
2. The plate may start to tilt as it falls, causing the time for the bottom edge to reach the beam to be different from that predicted.

18 A student takes measurements to determine the acceleration of free fall. He determines the time t taken for a ball to fall a distance s from rest. Fig. 1.1 shows the variation with distance s of t^2.

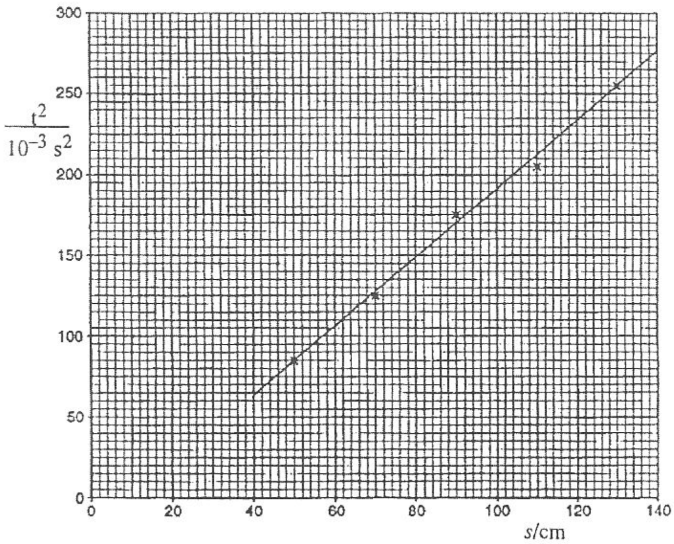


Fig. 1.1

(a) State what feature of Fig. 1.1 suggests that, in this experiment, there are

(i) random errors, [1]

(ii) systematic errors. [1]

(b) Suggest a possible source for a systematic error in this experiment. [1]

(c) Use Fig. 1.1 to determine

(i) the gradient of the line, [2]

(ii) a value for the acceleration of free fall. [2]

(d) The data collected by the student consisted of values of s and t. Plotting the variation with s of t^2 was expected to give a straight line.

(i) State what other variation could be plotted to give a straight line graph from these data. [1]

(ii) Explain why the graph in (i) gives a straight line. [1]

N06/II/1

The fact that some data points lie above while some below the expected straight line graph suggests that there are random errors, which cause the readings to be scattered.

A graph of t^2 vs s is expected to pass through the origin. The fact that it does not suggests that there are systematic errors which cause a unidirectional offset.

One possible systematic error is that the metre rule used for measuring distance has been offset, with the 10 cm mark placed at the starting point of the ball.

The gradient
$\Delta t^2 / \Delta s = (275 - 65) \times 10^{-3}\ s^2 / (140 - 40)$ cm
$= 0.210\ s^2 / 100$ cm $= 0.00210\ s^2\ cm^{-1} = 0.210\ s^2\ m^{-1}$

The equation of motion $s = ut + \frac{1}{2}at^2$.
Since the ball is released from rest, the initial velocity $u = 0$, while vertical downward acceleration $a = g$.
The equation becomes $s = \frac{1}{2}gt^2$ or $t^2 = 2s/g$.
Plotting t^2 against s should yield a straight line graph with a gradient of $2/g$.
$2/g = 0.210\ s^2\ m^{-1}$
$g = 2 / 0.210\ s^2\ m^{-1} = 9.52\ m\ s^{-2}$.

(i) Plotting log s against log t will also give a straight line graph.

(ii) Taking the log of the equation $s = \frac{1}{2}gt^2$ yields:
$\log s = \log(\frac{1}{2}g) + 2 \log t$.

A graph of log s vs log t will be a straight line with a gradient of 2 and y-intercept of log ($\frac{1}{2}$g), from which the acceleration of free fall g can also be found.

This a straight line because the gradient '2' is the power of variable t, which must be a constant since displacement s is proportional to the square of time t.

Long Questions

19 *(a)* Define *acceleration*. [1]

(b) (i) Use your definition in *(a)* to show that $v = u + at$, where v is the final velocity, u is the initial velocity and a and t are the acceleration and the time interval respectively. [2]

(ii) State the conditions that must be satisfied for the equation to be valid. [2]

(c) The graph of Fig. 1.1 shows the variation with time t of the velocity v of a ball from the moment it is thrown with a velocity of 26 m s^{-1} vertically upwards.

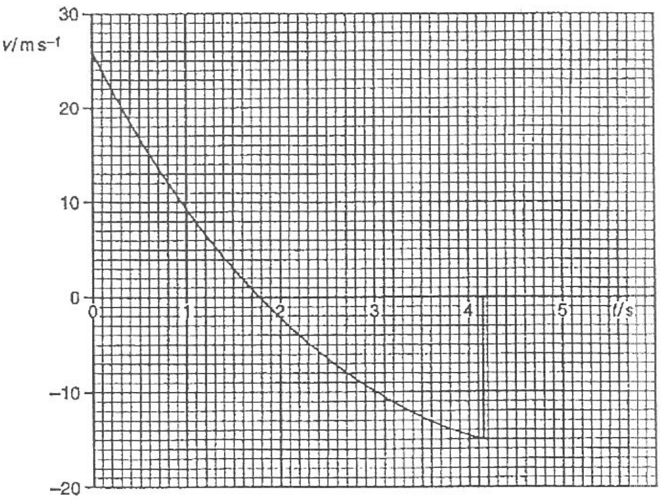


Fig. 1.1

(i) State the time at which the ball reaches its maximum height. [1]

(ii) State the feature of a velocity-time graph that enables the acceleration to be determined. [1]

(iii) Just after the ball leaves the thrower's hand, it has a downward acceleration of approximately 20 m s^{-2}. Explain how this is possible. [2]

(iv) State the time at which the acceleration is g. Explain why the acceleration has this value only at this particular time. [2]

(v) Sketch an acceleration-time graph for the motion. Show the value of g on the acceleration axis. [3]

(d) Explain why, for all real vertical throws, the time taken to reach maximum height must be shorter than the time taken to return to the starting point. [2]

(e) The ball in *(c)* starts with kinetic energy of 54 J.

(i) Calculate the mass of the ball.

(ii) Describe qualitatively how the amount of kinetic energy changes during the motion.

N03/III/1

Solution

(a) Acceleration is the rate of change of velocity.
It takes place in the direction of the change of velocity.

(b) (i) From the definition of acceleration,

$$a = dv/dt = (v - u) / t$$
$$v - u = at$$
$$v = u + at$$

(ii) The conditions are that the acceleration is constant and the motion is linear.

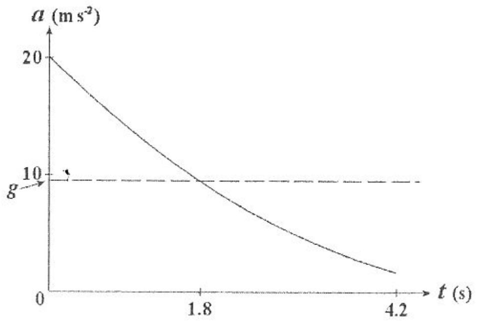
(c) (i) $t = 1.8$ s (when the ball is instantaneously at rest).

(ii) The gradient of the tangent to the velocity-time graph at any instant is the acceleration.

(iii) There is air resistance in this situation, and air resistance increases with speed.
When the ball just leaves the thrower's hand, it has a very high speed.
The total downward force acting on the ball is the gravitational force plus air resistive force.
The ball's total downward acceleration is the sum of the acceleration of free fall g (9.81 m s^{-2}) and acceleration due to air resistance (approximately 10.19 m s^{-2}).

(iv) $t = 1.8$ s.
At this instant, the ball reaches its maximum height and is instantaneously at rest.
Since air resistance is proportional to v^2 (under turbulent conditions like this), when speed is zero, the air resistive force is also zero.
The only force acting on the ball is the gravitational force hence its acceleration is the acceleration of free fall g.

(v) The acceleration-time graph for the motion is shown in the figure below (taking the downward direction as the positive direction).



(d) Air resistance is present in real vertical throws.
On its way up, air resistive force (which opposes motion) acts in the same direction as gravitational force, both acting against the direction of motion. The resultant force and hence deceleration are on average very large.
Since $t = (v - u)/a$, the time is taken is very short.
On its way down, air resistive force acts opposite to gravitational force. The resultant force and hence acceleration are on average much smaller.
Using the same concept, the time taken is much longer.

(e) (i) Kinetic energy = ½ $m v^2$ = 54 J

$$\tfrac{1}{2} m (26 \text{ m s}^{-1})^2 = 54 \text{ J}$$

$$m = 0.1598 \text{ kg} = 0.16 \text{ kg}$$

(ii) As the ball moves upwards, it decelerates and its speed decreases.
Since kinetic energy is proportional to the square of speed, its kinetic energy decreases.
When the ball reaches its maximum height such that it is instantaneously at rest, its kinetic energy is zero.
Thereafter, the ball starts accelerating downwards. As its speed increases, its kinetic energy also increases.
The rate of change in kinetic energy is non-uniform, and is greater during the ascent compared to during the descent.
This is because the rate of change of speed during the ascent is greater than that during the descent, for reasons explained in (d).

20 Explain why it is technically incorrect to define speed as distance travelled per second. Include in your answer the correct statement defining speed. [2]

N04/III/1 (part)

Solution

The second is not the only unit of time measurement, nor is it the smallest division of time interval.
"Distance travelled per second" is the average speed within a time interval of one second.
Speed should be defined as the rate of change of distance travelled.

21 A ball is thrown vertically upwards from ground level. Air resistance is **not** negligible.
The variation with time t of the vertical velocity v of the ball is shown in Fig. 6.1.

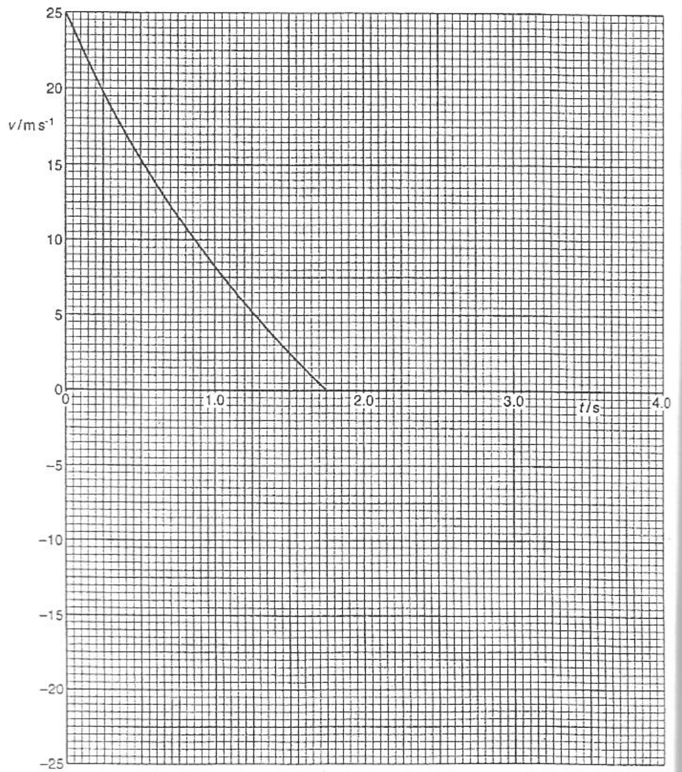


Fig. 6.1

(a) Use Fig. 6.1 to explain how it may be deduced that air resistance varies with speed.

....

....

.... [2]

(b) State and explain, without any calculation, the feature of Fig. 6.1 that enables the magnitude of the acceleration of free fall to be determined.

....

....

.... [2]

(c) (i) Use Fig. 6.1 to determine the maximum height reached by the ball.

height = ... m [4]

(ii) Use your answer in (i) to calculate the ratio

$$\frac{\text{energy lost from the ball due to air resistance during the ball's upward motion}}{\text{initial kinetic energy of the ball}}.$$

ratio = ... [4]

(d) The ball has mass 350 g.

For the instant when this ball is traveling at 10 m s^{-1},

(i) use Fig. 6.1 to show that the acceleration of the ball is approximately -13 m s^{-2},

(ii) calculate the magnitude of the force due to air resistance on the ball.

force = ... N [3]

(e) On Fig. 6.1, sketch the graph to show the variation with time t of the velocity v of the ball as it falls back to ground level. [3]

N09/III/6

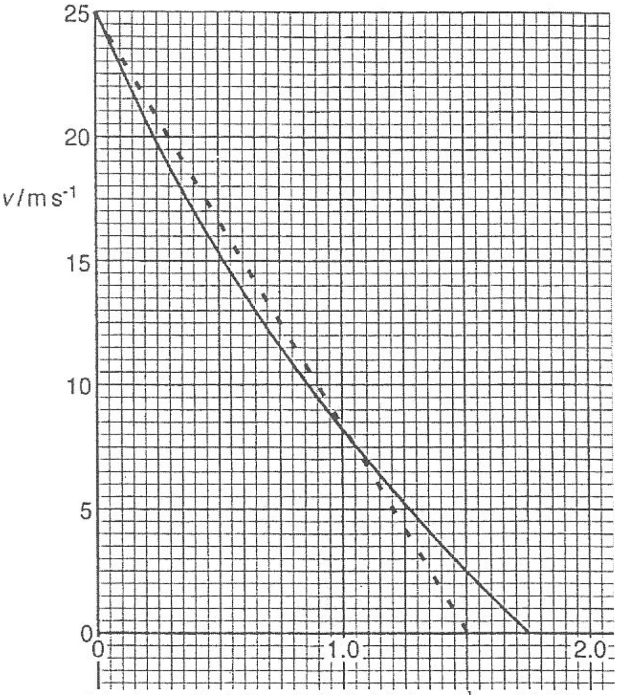
Solution

(a) The gradient of the graph is not constant, indicating the acceleration of the ball changes with speed. From Newton's second law of motion, it can be deduced that the resultant force also changes with speed. Since the resultant force comprises the ball's weight and air resistance, but weight is constant, it may be deduced that air resistance varies with speed.

(b) When the ball is at rest (v = 0) at t = 1.75 s, there is no air resistance hence the resultant force acting on the ball is its weight alone. The acceleration of the ball at this instance is the acceleration of free fall, which can be found from the gradient of the graph at t = 1.75 s.

(c) (i) The maximum height is the area under the graph between t = 0 and t = 1.75 s (when the ball is momentarily at rest at the top of its motion).

The area may be approximated by finding the area under the dotted line in the figure below, where the part of the graph which lies above the dotted line equalizes the part that lies below.



Maximum height ≈ ½(1.5 s)(25 m s^{-1}) = 18.75 m

height = <u>19 m</u>

(ii) Initial kinetic energy = ½mv^2 = ½$m(25)^2$ = 312.5m.

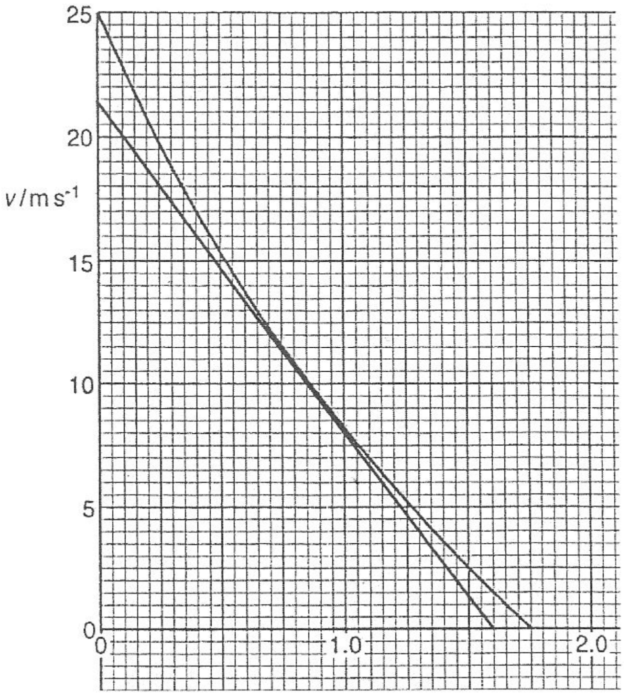
Gain in gravitational potential energy = mgh = m(9.81)(18.75) = 183.9m.

Energy lost due to air resistance = 312.5m – 183.9m = 128.6m.

Therefore, ratio = 128.6m / 312.5m = 0.4115

ratio = <u>0.41</u>

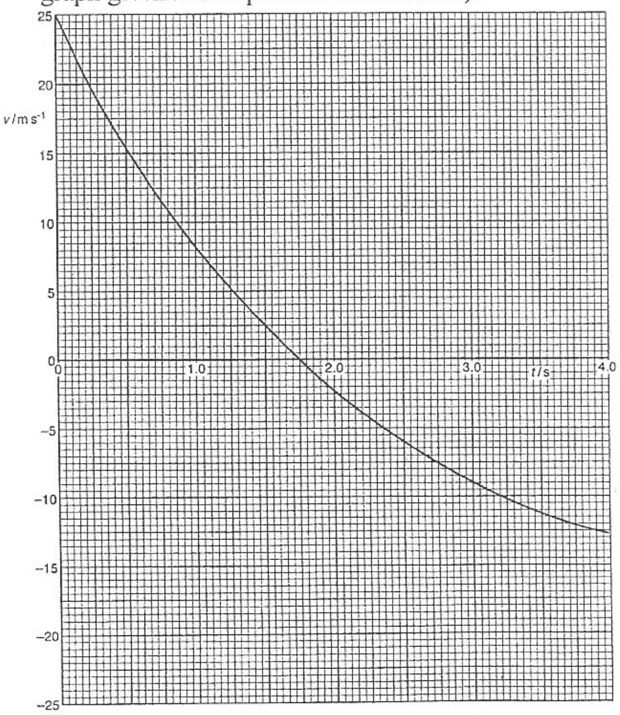
(d) (i) The gradient of the graph at v = 10 m s^{-1} is drawn below.



The gradient of the graph = (-21.5 m s^{-1}) / (1.6 s) = -13.44 ≈ -13 m s^{-2}.

(ii) Using Newton's second law of motion, the resultant force on the ball $F = ma$ = (0.350 kg)(-13 m s^{-2}) = -4.55 N. (The negative sign means that the resultant force is downwards).
The weight of the ball $W = mg$ = (0.350 kg)(-9.81 N kg^{-1}) = -3.43 N (The negative sign means that the weight is downwards).
Since the ball is on its way up, the air resistance and weight act in the same direction (i.e. downwards). Hence the air resistance = 4.55 – 3.43 = 1.12 N.
force = <u>1.1 N</u>

(e) The sketch is shown below (similar in shape to the graph given in the question of N03/III/1).



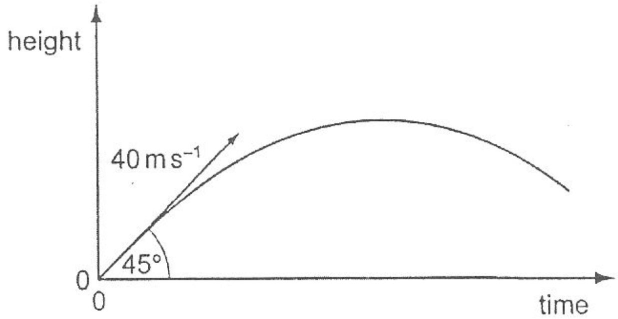
On its way up, with air resistance acting in concert with the force of gravity against its motion, the projectile experiences a greater decelerating force and hence deceleration (Newton's second law of motion), has a steeper gradient for its velocity-time graph, and is brought to rest in a shorter time. The gradient becomes progressively less steep as air resistance, and hence the total decelerating force as well as deceleration, decreases with speed on its way up.

On its way down, with air resistance acting opposite to the force of gravity, the projectile experiences a smaller downward accelerating force and hence acceleration, has a gentler gradient for its velocity-time graph, and takes a longer time to reach the ground. The gradient becomes progressively less steep as air resistance increases with speed on its way down, and hence the total accelerating force as well as acceleration decreases.

The total area under the graph above the time axis is equal to that below the time axis because the vertical distance travelled upwards is equal to the vertical distance travelled downwards back to the ground.

Non-linear Motion

22 An object is projected with velocity 40 m s^{-1} at an angle of 45° to the horizontal. Air resistance is negligible.



What is the speed of the object after 5.0 s?

A 21 m s^{-1} **B** 28 m s^{-1} **C** 35 m s^{-1} **D** 49 m s^{-1}

N09/I/5

Solution Answer: **C**

The horizontal velocity v_x remains unchanged due to the absence of air resistance.

$$v_x = u_x + a_x t = 40 \cos 45° + (0)(5.0) = 28 \text{ m s}^{-1}$$

The vertical velocity v_y can be found from:

$$v_y = u_y + a_y t = 40 \sin 45° + (-9.81)(5.0) = -21 \text{ m s}^{-1}$$

The resultant velocity

$$v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{((28)^2 + (-21)^2)} = 35 \text{ m s}^{-1}$$

Short Questions

23 A student throws a ball from point S to a friend at point F. The path of the ball is shown in Fig. 1.1.

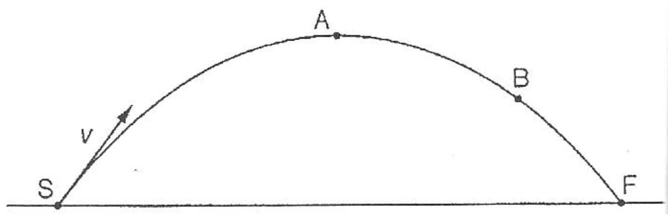


Fig. 1.1

The points S and F are on the same horizontal level. Air resistance is negligible.

The ball is thrown from point S with velocity v, represented by the vector arrow shown on Fig. 1.1.

On Fig. 1.1,

(i) draw arrows from point S to represent the initial horizontal and vertical components of the velocity v (label these components v_H and v_V respectively), [1]

(ii) draw arrows at A and at B to represent the horizontal and vertical components of the velocity of the ball at these two points. [3]

N04/II/1 (part)

Solution
The arrows are drawn in the figure below.

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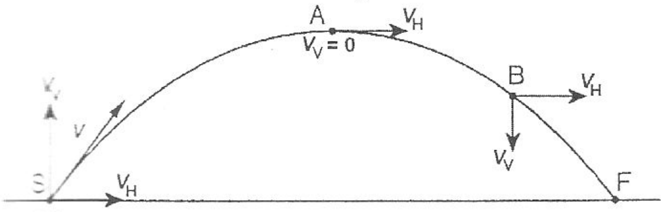


Note:
The magnitude of the horizontal component V_H remains constant throughout.
The magnitude of the vertical component V_V decreases to zero, then increases in the opposite direction.

24 A stone of mass 130 g is thrown horizontally from the top of a cliff of height 32 m, as illustrated in Fig. 1.1.

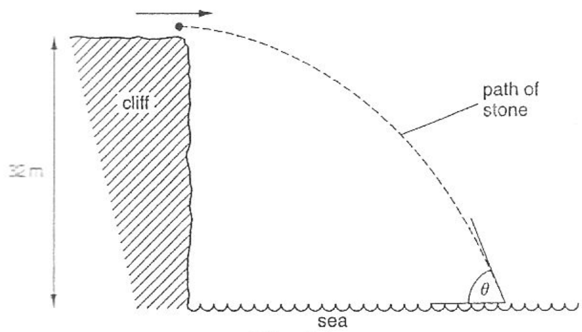


Fig. 1.1

Air resistance is negligible. The stone enters the sea with a speed of 34 m s^{-1}.

(a) Determine, for the stone as it hits the sea,
(i) the vertical component of the velocity of the stone,
vertical component of velocity = ... m s^{-1} [2]
(ii) the angle θ to the horizontal of the stone's path.
θ = ... ° [2]
N08/II/1 (part)

Solution
(a) (i) Use the kinematics equation, applied to the vertical direction,

$$v_y^2 = u_y^2 + 2a_y s_y$$

where v_y is the final vertical component of the stone's velocity, u_y the initial vertical component of the stone's velocity (0 m s^{-1} since the stone was thrown horizontally), a_y the vertical component of the stone's acceleration (g) and s_y the vertical component of the stone's displacement (32 m).

$$v_y^2 = u_y^2 + 2gs_y$$

$$v_y^2 = 0^2 + 2(9.81 \text{ m s}^{-2})(32 \text{ m}) = 627.84 \text{ m}^2 \text{ s}^{-2}$$

$$v_y = 25.06 = 25 \text{ m s}^{-1}$$

(ii) The vector diagram including the final velocity and its components is shown in Fig. 1.2 below.

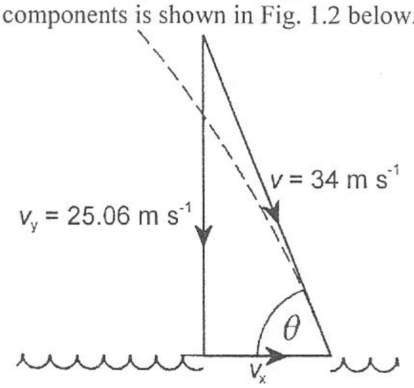


Fig. 1.2

$$\sin\theta = v_y/v = 25.06/34 = 0.7371$$

$$\theta = 47.5°$$

TOPIC 3 Dynamics

Newton's Laws of Motion

1 A trolley runs down a slope with a constant acceleration a. The mass of the trolley is now doubled and the trolley is allowed to run down the same slope. In both cases effects of friction and air resistance are negligible.

Which statement is correct for the second experiment?

A The accelerating force is the same.

B The acceleration is $\frac{1}{2}a$

C The acceleration is a.

D The acceleration is $2a$. N02/I/6

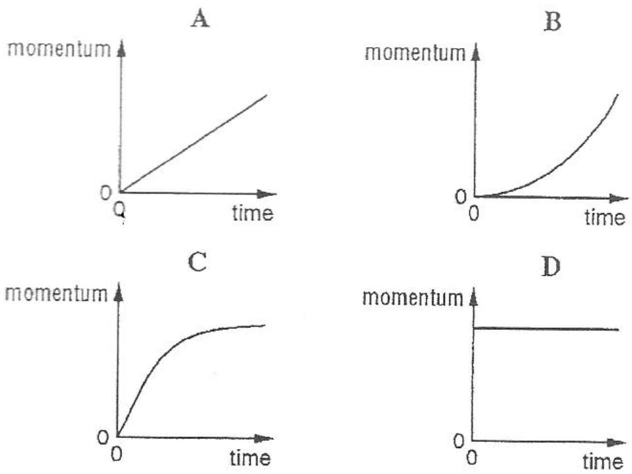
Solution Answer: **C**

When the mass of the trolley is doubled, its weight (mg) will also be doubled.

Hence the down-slope component of its weight ($mg \sin \theta$) will also be doubled (where θ is the angle the slope makes with respect to the horizontal).

Since the down-slope accelerating force is doubled and the mass is also doubled, the acceleration remains unchanged ($a = mg \sin \theta / m = g \sin \theta$) and is hence independent of the mass of the trolley m.

2 Which graph best shows the variation with time of the momentum of a body accelerated by a constant force?



N04/I/5

Solution Answer: **A**

According to Newton's second law of motion, the rate of change of momentum of a body is directly proportional to the resultant force acting on it.

Since the force is constant, the rate of change of its momentum is also constant.

Thus its momentum increases uniformly with time.

3 A force $2F$ acting on a particle of mass 10 kg produces an acceleration of 60 m s^{-2}.

A force $5F$ acting on a particle of mass M produces an acceleration of 50 m s^{-2}.

What is the mass M?

A 3.3 kg

B 4.8 kg

C 21 kg

D 30 kg N04/I/6

Solution Answer: **D**

Applying Newton's second law of motion to the first situation, $2F = (10 \text{ kg})(60 \text{ m s}^{-2})$ yields $F = 300$ N.

Applying the same equation to the second situation, $5(300 \text{ N}) = M(50 \text{ m s}^{-2})$ yields $M = 30$ kg.

4 A car of mass 1200 kg is accelerated by a resultant force o 3000 N for a time of 5.0 s.

What is the gain in momentum of the car?

A 2.5 kg ms^{-1} C 6.0×10^3 kg m s^{-1}

B 6.0×10^2 kg ms^{-1} D 1.5×10^4 kg ms^{-1}

N05/I/

Solution Answer: **D**

The gain in momentum (or impulse)

$\Delta p = Ft = (3000 \text{ N})(5.0 \text{ s}) = 1.5 \times 10^4$ N s.

5 A racing car of mass 500 kg, including driver but not fuel, decelerates from a speed of 50 m s^{-1} to 30 m s^{-1} when approaching a bend.

The brakes exert a fixed retarding force of 7000 N. The time for the car to decelerate when it is almost out of fuel is t_1. The time for it to decelerate when it has a full load of 130 kg of fuel is t_2.

What is the difference $(t_2 - t_1)$ in the times?

A 0.37 s B 0.56 s C 0.93 s D 1.43 s

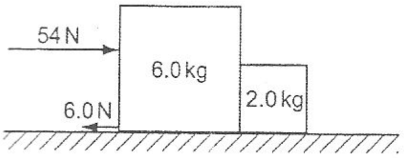
N07/I/5

Solution Answer: **A**

When the racing car is almost out of fuel, its deceleration $a = F/m = 7000 \text{ N} / 500 \text{ kg} = 14 \text{ m s}^{-2}$. The time for it to slow down $t_1 = v - u / a = (30 - 50) / (-14) = 1.43$ s.

When the racing car has a full load of fuel, its deceleration $a = F/m = 7000 \text{ N} / 630 \text{ kg} = 11.1 \text{ m s}^{-2}$. The time for it to slow down $t_2 = v - u / a = (30 - 50) / (-11.1) = 1.80$ s.
The difference $t_2 - t_1 = 1.80 - 1.43 = 0.37$ s.

6 A force of 54 N pushes two touching blocks of mass 6.0 kg and 2.0 kg along a flat surface. The frictional force between the blocks and the surface is 6.0 N.



What is the magnitude of the resultant force on the 6.0 kg mass?

A 12 N **B** 36 N **C** 45 N **D** 48 N

N10/I/7

Solution Answer: **B**

Considering the resultant force on the whole system and applying Newton's second law of motion,

$$(54\text{ N} - 6\text{ N}) = (6.0\text{ kg} + 2.0\text{ kg})\,a$$

The acceleration $a = 6.0\text{ m s}^{-2}$.

The resultant force on the 6.0 kg mass is

$$F = (6.0\text{ kg})(6.0\text{ m s}^{-2}) = 36\text{ N}$$

Short Questions

7 A steel ball of mass 250 kg is suspended from the jib of a crane, as illustrated in Fig. 3.1.

In order to demolish a wall, the ball is pulled away from the wall and then released. The ball swings down and hits the wall.

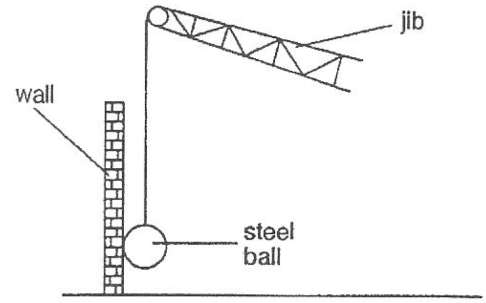


Fig. 3.1

The variation with time t of the speed v of the ball is shown in Fig. 3.2.

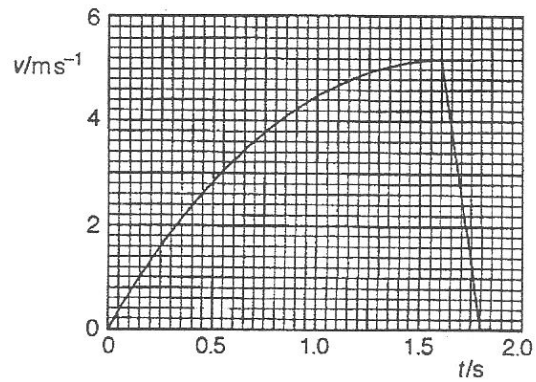


Fig. 3.2

(a) Using Fig. 3.2, determine

(i) the magnitude of the acceleration of the ball at time t = 0.8s,

acceleration = ... m s^{-2} [3]

(ii) the distance moved by the ball before it hits the wall, that is, from time t = 0 to t = 1.6 s.

distance = ... m [4]

(b) Calculate the magnitude of

(i) the change in momentum of the ball during its collision with the wall,

change in momentum = ... kg m s^{-1}

(ii) the average force exerted on the wall during the collision.

force = ... [4]

(c) When the ball hits the wall, 15% of the kinetic energy of the ball is converted to thermal energy in the ball. Calculate the mean temperature rise of the ball.

The specific heat capacity of steel is 450 J kg^{-1} K^{-1}.

temperature rise = ... K [4]

N02/II/3

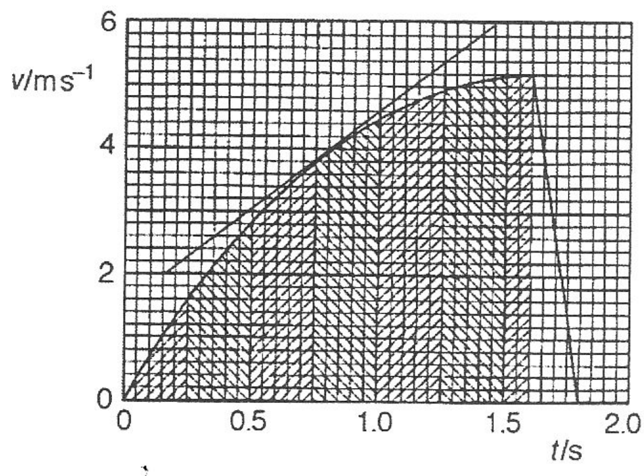
Solution

(a) (i) The acceleration can be found by measuring the gradient of the tangent of the speed-time graph at time $t = 0.8$ s.

The gradient
$= (6.0 \text{ m s}^{-1} - 2.0 \text{ m s}^{-1})/(1.45 \text{ s} - 0.15 \text{ s})$
$= 4.0 \text{ m s}^{-1} / 1.3 \text{ s}$
$= 3.1 \text{ m s}^{-2}$.

Refer to diagram below.

(ii) The distance can be found by measuring the area under the speed-time graph from $t = 0.0$ s to $t = 1.6$ s. Using the trapezium rule,
The area from $t = 0.0$ s to $t = 1.5$ s is A = (0.25 s / 2) [0 + 2(1.6) + 2(2.8) + 2(3.8) + 2(4.5) + 2(4.9) + 5.2] m s^{-1} = 5.05 m.
The area from $t = 1.5$ s to $t = 1.6$ s is A = 0.1 s x 5.2 m s^{-1} = 0.52 m.
The total distance moved = 5.05 m + 0.52 m = 5.57 = 5.6 m.
Refer to diagram below.



(b) (i) Magnitude of the change in momentum
$= |0 - mu| = |-(250 \text{ kg})(5.2 \text{ m s}^{-1})| = 1300 \text{ kg m s}^{-1}$.

(ii) Average force
= (change in momentum)/time
$= (1300 \text{ kg m s}^{-1}) / (0.20 \text{ s})$
= 6500 N.

(c) The kinetic energy of the ball just before impact
$= \frac{1}{2}mu^2 = \frac{1}{2}(250 \text{ kg})(5.2 \text{ m s}^{-1})^2 = 3380$ J.

15% of 3380 J = 507 J = Q (thermal energy)

$$Q = mc\Delta\theta$$

$$507 \text{ J} = (250 \text{ kg})(450 \text{ J kg}^{-1} \text{ K}^{-1})\, \Delta\theta$$

$$\Delta\theta = 4.507 \times 10^{-3} = 4.5 \times 10^{-3} \text{ K}$$

8 (a) State Newton's second law of motion. [1]

(b) A car of mass 750 kg is travelling at 25 m s^{-1} along a horizontal road. The brakes are applied and the car is brought to rest by an average resistive force F. The car has an average deceleration of 4.8 m s^{-2}.

(i) Show that the resistive force acting on the car is 3600 N. [1]

(ii) Calculate the distance travelled by the car during this deceleration.
distance = ... m [2]

(iii) Describe, in terms of Newton's third law, the horizontal forces acting on the tyres of the car and on the road. [2]

(c) The car in (b) now travels at 25 m s^{-1} down a slope where the angle to the horizontal is 10°. The car is brought to rest by applying the brakes. The same resistive force of 3600 N acts on the car.

(i) Explain why the distance the car travels before coming to rest is greater than in (b). [1]

(ii) Calculate the deceleration of the car.
deceleration = ... m s^{-2} [2]

N10/II/1

Solution

(a) Newton's second law of motion states that the rate of change of momentum of a body is directly proportional to the resultant force acting on it, and takes place in the direction of the resultant force.

(b) (i) Since the resistive force is the only force acting on the car while it is braking, the resistive force is also the resultant force $F = ma = (750 \text{ kg})(4.8 \text{ m s}^{-2}) = 3600$ N.

(ii) Using the kinematics equation:

$$v^2 = u^2 + 2as$$

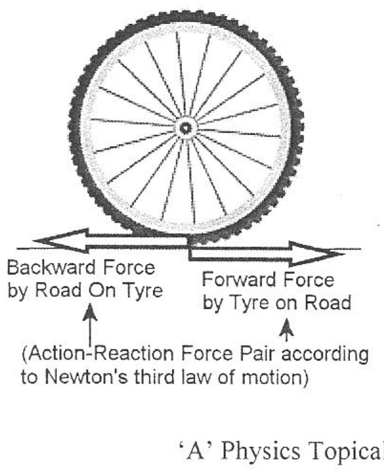
$$0 = (25)^2 + 2(-4.8)s$$

$$s = 65 \text{ m}$$

(iii) Friction exists between the tyres and the road surface.

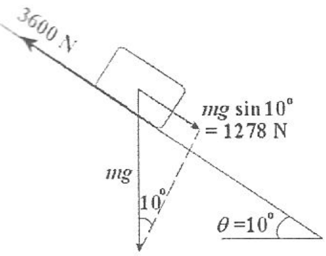
When the brakes are applied, the wheels stop rolling. Friction between the tyres and the road surface causes the tyres to exert a forward frictional force on the road.

According to Newton's third law of motion, the road surface will exert an equal and opposite (i.e. backward) frictional force on the tyres, causing the car to decelerate.



(ii) When the car is on a slope, the component of its gravitational force acting down the slope acts in the opposite direction to the resistive force hence reducing the resultant decelerating force. According to Newton's second law of motion, the magnitude of the deceleration thus decreases and the car travels longer before coming to rest.

(iii) The component of the car's gravitational force acting down the slope is $mg \sin \theta = (750)(9.81)\sin 10° = 1278$ N as shown in the diagram below.



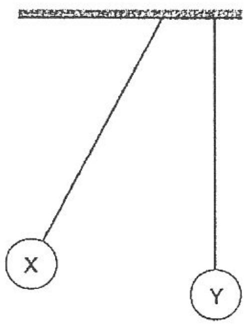
This reduces the magnitude of the resultant decelerating force to 3600 – 1278 = 2322 N. Applying Newton's second law of motion, acceleration $a = F/m = -2322/750 = -3.1$ m s^{-2}.
deceleration = <u>3.1 m s^{-2}</u>.
Using the kinematics equation, the distance (s) the car travels before coming to rest can be calculated (if required): $v^2 = u^2 + 2as$

$$0 = (25)^2 + 2(-3.1)s$$
$$s = 101 \text{ m}$$

Linear Momentum and its Conservation

9 Two steel balls X and Y are suspended on strings. Ball X is pulled to one side as shown.

After ball X is released, the balls collide.

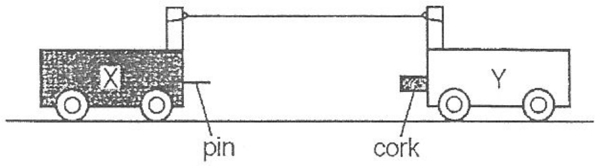


Which quantities must be conserved in the collision?

A kinetic energy, total energy and momentum
B kinetic energy and momentum only
C kinetic energy and total energy only
D total energy and momentum only N02/I/5

Solution Answer: **D**
Total energy and momentum <u>must</u> be conserved in accordance with the laws of conservation of energy and momentum respectively. However, the kinetic energy of the system is conserved only if the collision is elastic.
Whether that is so is not mentioned in this question, hence no conclusion can be drawn regarding the kinetic energy.

10 The diagram shows two trolleys X and Y held stationary and connected by an extended elastic cord. The mass of X is twice that of Y.



The trolleys are released at the same instant. They move towards each other and stick together on impact. Just before the collision, the speed of X is 20 cm s^{-1}.

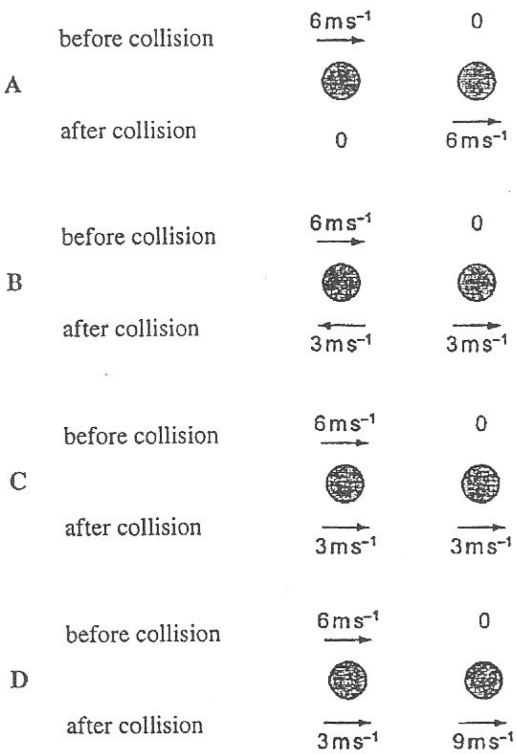
What is the speed of Y after the collision?

A zero C 7 cm s^{-1}
B 5 cm s^{-1} D 10 cm s^{-1} N03/I/4

Solution Answer: **A**
Considering the two trolleys as one system, the force of the elastic cord as well as the force of impact are both internal forces.
Since no net external force acts on the system, the total momentum of the system remains constant, i.e. zero.
Upon collision, the rightward momentum of X ($2m$ x 20 m s^{-1}) cancels out the leftward momentum of Y (m x 40 m s^{-1}), and both trolleys come to rest.

11 Two spheres of equal mass collide head-on. Before the collision, one sphere is stationary and the other is moving with speed 6 m s^{-1} directly towards the stationary sphere.

Which diagram represents an elastic collision?



N06/I/5

Solution Answer: **A**

In an elastic collision, the total kinetic energy of the system is conserved.

The total kinetic energy before collision is $\frac{1}{2}m(6)^2 = 18m$ J.

After the collision, the total kinetic energy for answer A is $\frac{1}{2}m(6)^2 = 18m$ J.

The total K.E. for answer B is $\frac{1}{2}m(3)^2 + \frac{1}{2}m(3)^2 = 9m$ J.

The total K.E. for answer C is $\frac{1}{2}m(3)^2 + \frac{1}{2}m(3)^2 = 9m$ J.

The total K.E. for answer B is $\frac{1}{2}m(3)^2 + \frac{1}{2}m(9)^2 = 45m$ J.

12 An object of mass M travelling to the right with velocity $2v$ collides with another object of mass $2M$ travelling to the left with velocity v. After the collision, the objects stick together.

Which line in the table shows the total momentum and the total kinetic energy of the two objects after the collision?

	momentum	kinetic energy
A	0	0
B	4*Mv*	0
C	0	$3Mv^2$
D	4*Mv*	$3Mv^2$

N07/I/6

Solution Answer: **A**

The total momentum of the system before collision is $(M)(2v) - (2M)(v) = 0$ hence, according to the law of conservation of linear momentum, the total momentum of the system after the collision must also be 0.

If the total momentum p of the system after the collision is 0, then its total kinetic energy K must also be 0 ($K = p^2/2m$).

13 A particle of mass m travelling with velocity u collides elastically and head-on with a stationary particle of mass M.

Which expression gives the velocity of the particle of mass M after the collision?

A u B $\dfrac{mu}{M}$ C $\dfrac{2mu}{M+m}$ D $\dfrac{(M-m)u}{M+m}$

N07/I/7

Solution Answer: **C**

For elastic collisions, the velocity of approach equals the velocity of separation.

$$u - 0 = v_M - v_m$$
$$v_m = v_M - u \ldots (1)$$

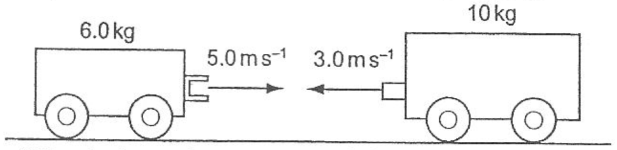
Applying the law of conservation of linear momentum,

$$mu + 0 = mv_m + Mv_M \ldots (2)$$

Substituting (1) into (2),

$$mu = m(v_M - u) + Mv_M$$
$$2mu = (m + M)v_M$$
$$v_M = 2mu / (m + M)$$

14 A trolley of mass 6.0 kg travelling at a speed of 5.0 m s^{-1} collides head-on and locks together with another trolley of mass 10 kg which is travelling in the opposite direction at a speed of 3.0 m s^{-1}. The collision lasts for 0.20 s.



What is the total momentum of the two trolleys before the collision and the average force acting on each trolley during this collision?

	total momentum before collision / kg m s^{-1}	average force on each trolley / N
A	0	300
B	60	150
C	0	150
D	60	300

N08/I/6

Solution Answer: **C**

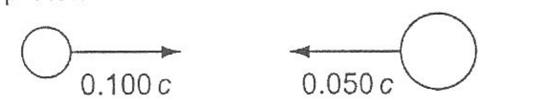
Total momentum of the system before collision = (6.0 kg)(5.0 m s^{-1}) – (10 kg)(3.0 m s^{-1}) = 30 Ns – 30 Ns = 0. According to the law of conservation of linear momentum, since no external net force acts on the system, the total momentum of the system after the collision must also be zero. This means both trolley will be brought to rest.

According to Newton's second law of motion, the resultant force acting on each trolley is its rate of change of linear momentum, F = (30 N s)/(0.20 s) = 150 N.

The force exerted by the 6.0 kg trolley on the 10 kg trolley should be equal and opposite to that by the 10 kg trolley on the 6.0 kg trolley (Newton's third law of motion).

Hence both trolleys experience a force of 150. The left trolley exerts a force of 150 N to bring the right trolley to rest, and vice versa.

15 A proton (mass 1u) travelling with velocity +0.100 c collides elastically head-on with a helium nucleus (mass 4u) travelling with velocity -0.050 c.



What are the velocities of each particle after the collision?

	proton	helium nucleus
A	–0.140 c	+0.010 c
B	+0.140 c	+0.010 c
C	+0.233 c	–0.083 c
D	–0.233 c	+0.083 c

N09/I/7

Answer: **A**

Solution
For an elastic collision, the relative velocity of approach equals the relative velocity of separation.

relative velocity of separation = velocity of approach

$v_2 - v_1 = u_1 - u_2 = 0.100\ c - (-0.050\ c) = 0.150\ c$

Applying the law of conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(1u)(0.100\ c) + (4u)(-0.050\ c) = (1u)\ v_1 + (4u)\ v_2$$

$$-0.1\ c = v_1 + 4\ v_2$$

Solving simultaneously,

$$v_2 = +0.010\ c$$
$$v_1 = -0.140\ c$$

6 A molecule of mass m travels with velocity $+u$ towards a stationary molecule of mass $4m$ and collides elastically with it.
What is the velocity of the molecule of mass m after the collision?

A $-\frac{1}{5}u$ B $-\frac{3}{5}u$ C $-\frac{4}{5}u$ D $-u$

N10/I/6

Answer: **B**

Solution
For elastic collisions, the relative velocity of approach equals relative velocity of separation, $u - 0 = v_2 - v_1$.
From conservation of linear momentum, $mu = mv_1 + 4mv_2$.
Substituting $v_2 = u + v_1$ into the second equation,

$$mu = mv_1 + 4m(u + v_1)$$

$$5v_1 = -3u$$

$$v_1 = -(3/5)u$$

Short Questions

1 A tritium nucleus moves towards a deuterium nucleus as illustrated in Fig. 1.1.

Fig. 1.1

The nuclei initially have the same speed v. The tritium nucleus consists of two neutrons and a proton. The deuterium nucleus consists of a neutron and a proton. The proton and the neutron have the same mass m.

(a) (i) State why the two nuclei will repel one another.
.... [1]

(ii) State what will happen if v is sufficiently high.
.... [1]

(iii) Explain why it is **not** possible for the nuclei to stop at the same instant.
....
....
.... [2]

(b) At one instant during the interaction between the nuclei, they are both travelling **in the same direction** with the same speed. Calculate this speed, in terms of v.
speed = ... [2]

(c) Fig. 1.2 is a velocity-time sketch graph showing how the velocity of each nucleus varies. The interaction between the nuclei is elastic.

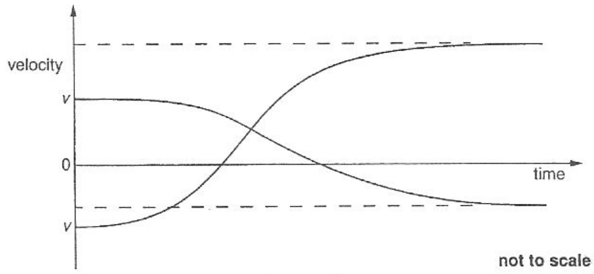


Fig. 1.2

(i) Label the graph to show
1. which curve is for the tritium nucleus,
2. the times at which each nucleus stops,
3. the time at which they are at their distance of closest approach. [3]

(ii) Determine the final speed of each nucleus in terms of v.
final speed of deuterium = ...
final speed of tritium = ... [4]

N07/III/1

Solution
(a) (i) Like charges repel.
Since both nuclei are positively charged, they repel.

(ii) Nuclear fusion will occur.
If v is sufficiently high, the nuclei will have sufficient kinetic energy to overcome the electrostatic repulsion.

(iii) The initial momentum of the system before collision is not zero because the nuclei are travelling in opposite directions with different magnitudes of momentum.
According to the law of conservation of linear momentum, since there is no external net force acting on the system, momentum must be conserved hence the momentum of the system at any instant during the interaction cannot be zero.

(b) Apply the law of conservation of linear momentum and let v' be their common velocity.

$$3mv - 2mv = (3m + 2m)v'$$
$$mv = 5mv'$$
$$v' = v/5$$

(c) (i) The labeling is shown in Fig. 1.3 below.

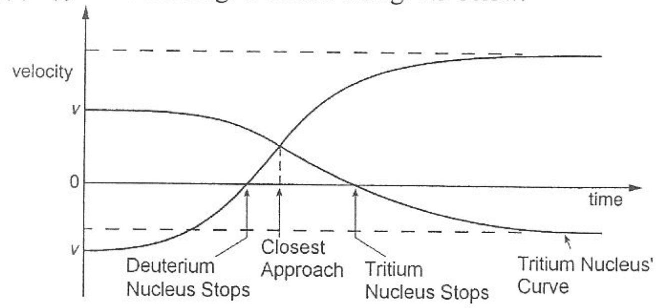


Fig. 1.3

The tritium nucleus, being the heavier of the two nuclei, experiences a smaller reversal in velocity after the collision. The distance of closest approach occurs when both nuclei are travelling at the same speed, just before they separate.

(ii) For an elastic collision, the relative velocity of approach equals the relative velocity of separation.

$$v - (-v) = v_D - v_T$$
$$v_D - v_T = 2v \ldots (1)$$

where v_D and v_T are the velocities of the deuterium nucleus and tritium nucleus after the collision, respectively.

From the law of conservation of linear momentum,

$$3mv - 2mv = 3mv_T + 2mv_D$$
$$3v_T + 2v_D = v \ldots (2)$$

3 x (1) + (2),

$$5v_D = 7v$$
$$v_D = 1.4v$$

Substitute into (1),

$$1.4v - v_T = 2v$$
$$v_T = -0.6v$$

Therefore, the final speed of the deuterium nucleus is 1.4v while the final speed of the tritium nucleus is 0.6v. *Check: This confirms the answer to (c)(i)1.*

18 Fig. 2.1 shows the path of a ball that is kicked off the ground towards a vertical wall.

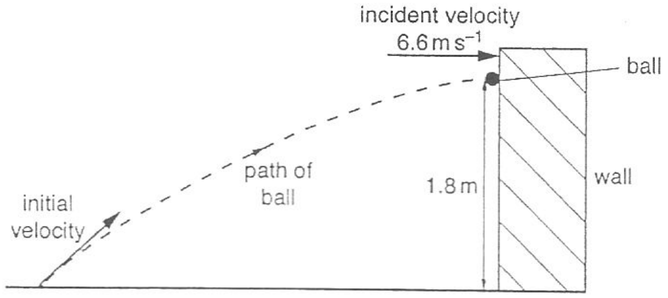


Fig. 2.1

The ball of mass 0.45 kg hits the wall when it reaches its maximum height of 1.8 m. The ball is incident with a horizontal velocity of 6.6 m s^{-1} and rebounds in a horizontal direction with a velocity of 5.2 m s^{-1}. The ball is in contact with the wall for 0.22 s.

(a) Calculate the initial vertical component of the ball's velocity.
vertical velocity = ... m s^{-1} [2]

(b) Calculate the average force acting on the ball due to its collision with the wall.
magnitude of the force = ... N
direction of force on the ball = ... [3]

(c) State and explain whether the collision with the wall is elastic. [1]

(d) Explain why the ball does not rebound to the point on the ground from where it was kicked. [2]

N09/II/2

Solution

(a) Using the kinematics equation on the vertical component of the ball's velocity, and taking upwards as the positive direction:

$$v_y^2 = u_y^2 + 2a_y s_y$$
$$0 = u_y^2 + 2(-9.81)(1.8)$$
$$u_y = 5.943 \text{ m s}^{-1}$$

vertical velocity = 5.9 m s^{-1}

(b) Applying Newton's second law of motion, and taking rightwards as the positive direction:

$$F = (mv - mu) / t = m(v - u) / t$$

$$= 0.45 (-5.2 - 6.6) / 0.22 = -24.14 \text{ N}$$

magnitude of the force = 24 N
direction of the force on the ball = towards the left

(c) Not elastic. The speed of the ball v has decreased from 6.6 m s^{-1} to 5.2 m s^{-1}. Since its kinetic energy E_k = ½mv^2 (where m is its mass), its kinetic energy has decreased.

Another way to explain will be that the relative velocity of approach (6.6 – 0 = 6.6 m s^{-1}) is greater than the relative velocity of separation (0 – (-5.2) = 5.2 m s^{-1}).

(d) In situations like this where air resistance is negligible, the time for the ball to rise is equal to that for it to fall. However, the ball's horizontal velocity has decreased after the rebound. In the same time interval that it took to rise, the ball will thus travel a shorter horizontal distance after the rebound, and land closer to the wall.

(a) (i) Define *linear momentum.* [1]

(ii) State the relationship between the change in linear momentum of an object, the constant force acting on the object, and the time for which the force acts. [1]

(b) In a collision between two bodies A and B, the force that A exerts on B varies with time in the way shown in Fig. 2.1.

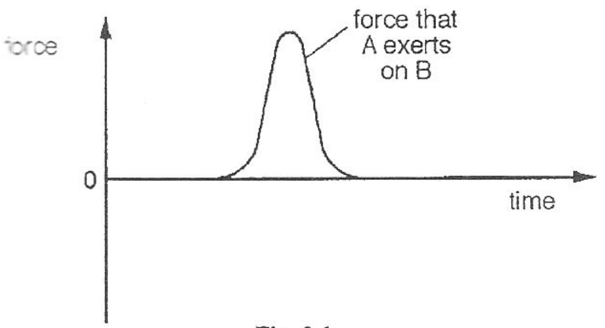


Fig. 2.1

(i) Copy Fig. 2.1 and show on your sketch a graph of the force that B exerts on A. [1]

(ii) Explain your answer to (i). [1]

(iii) Explain how your answer to (i) is consistent with the principle of conservation of momentum. [3]

(c) In a collision, when a truck of mass 12 000 kg runs into the back of a car of mass 1200 kg, a constant force of 72 000 N acts for 0.25 s. Calculate the change in velocity of

(i) the car, [3]

(ii) the truck. [2]

(d) Suggest **one** way in which the conditions in *(c)* are unrealistic. [1]

(e) Discuss how seat belts and air bags in a car ensure greater safety. [2]

(f) In order to reduce the number of road traffic accidents, many countries conduct research into improving road safety.

(i) One area of research concerns braking. State **three** factors that affect braking which might be considered by researchers. [3]

(ii) State **one** other aspect of car safety that could be researched, and suggest briefly how the research could be conducted. [2]

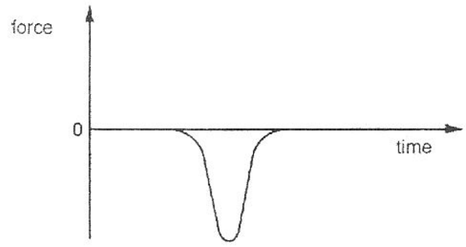
N04/III/2

Solution

(a) (i) The *linear momentum* of a body is the product of its mass and its velocity, and acts in the same direction as its velocity.

(ii) The change in linear momentum Δp of an object (impulse) is equal to the produce of the constant force F acting on it and the time t for which the force acts.
$\Delta p = Ft$.

(b) (i) The force that B exerts on A is shown in the figure below.



(ii) According to Newton's third law of motion, when body A exerts a force on body B, body B exerts an equal but opposite force on body A.

Hence the force that B exerts on A is equal in magnitude but opposite in direction to that exerted by A on B.

(iii) The force that A exerts on B is equal in magnitude to that by B on A and the time of contact over which the forces act is also the same for both bodies.

The change in A's linear momentum ($\int F\, dt$) is thus equal in magnitude but opposite to B's ($-\int F\, dt$).

Considering A and B as one system, their changes in linear momentum cancel out vectorially, thus the total momentum of the system remains unchanged.

Since their mutual contact forces are considered as internal forces and there are no external forces acting on the system, the fact that the linear momentum of the system remains unchanged in the absence of external forces is consistent with the law of conservation of linear momentum.

(c) (i) The impulse (i.e. change of momentum)
= constant force x time
= 72 000 N x 0.25 s
= 18 000 N s.

For the car, $m\,\Delta v$ = 18000 N s

Δv = (18 000 N s)/(1200 kg) = 15 m s^{-1}.

(ii) For the truck, $m\,\Delta v$ = -18000 N s

Δv = (-18 000 N s)/(12 000 kg) = -1.5 m s^{-1}.

(d) It is not possible for vehicles to experience a constant force during a collision.

The force varies as the bodies undergo deformation during a collision.

(e) By undergoing deformation, seat belts and air bags increase the time for the body to be brought to rest.

According to Newton's second law of motion, the resultant force (F) acting on a body is directly proportional to the rate of change of linear momentum (p) of the body ($F \propto dp/dt$). Hence an increase in the time taken reduces the rate of change of momentum, and hence the resultant force acting on the body.

(f) (i) Frictional force between the tire and the road (material engineering research).

Skidding due to the tires locking in abrupt braking situations (antilock braking system (ABS) research).

Speed of braking action (replacement of mechanical system with electronic brake-by-wire systems).

(ii) Shock absorption in the event of collision.

Research can be conducted into the efficacy of the crumple zone at the front and back of the car (including the bumper).

The material used should be stiff enough so that the car does not break apart, yet not so stiff as to result in an extremely large force of impact.

Force sensors placed on dummy drivers can be used to measure the force of injury in simulated collisions.

20 *(a)* Newton's third law of motion can be stated in the form

when object X exerts a force on object Y, then object Y exerts a force of the same type that is equal in magnitude and opposite in direction on object X.

(i) Explain what is meant by *of the same type* in this statement of the law. [1]

(ii) Object X is a book resting in equilibrium on a table (object Y). Draw labelled force diagrams to show the forces on X and on Y. Make it clear which forces are equal in magnitude and opposite in direction. [3]

(iii) Now suppose object X is a book that has fallen onto the table (object Y) and has just landed on it. For the instant of arrival, draw labelled force diagrams to show the forces on X and on Y. Make it clear which forces are equal in magnitude and opposite in direction, and which forces are different in magnitude from those in (ii). [3]

(b) In planning a railway timetable, three options are being considered for trains going through a certain town. The options are

A. a train of mass 6.0×10^5 kg that does not stop,

B. a train of mass 6.0×10^5 kg that can accelerate and decelerate at a rate of 0.50 m s^{-2}, which stops in the station for 200 s,

C. a train of mass 1.0×10^5 kg that can accelerate and decelerate at a rate of 2.00 m s^{-2}, which stops in the station for 200 s.

The steady speed of the trains between stations is 50 m s^{-1}. Velocity-time graphs of their movement are shown in Fig. 1.1 opposite.

(i) Copy and complete the last three columns of the following table. [9]

	train A	train B	train C
distance travelled in the 400 s that the trains have different speeds	20 000 m		
distance train is behind train A as a result of stopping	0	15 000 m	
time delay as a result of stopping	0	300 s	
kinetic energy of train at a speed of 50 m s^{-1}			1.25×10^8 J
stopping force exerted on train by the track	0		

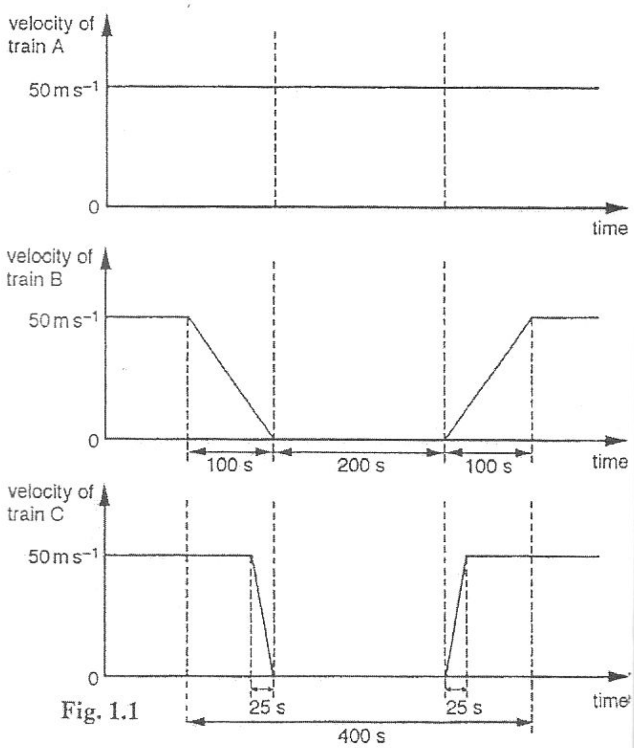


Fig. 1.1

(ii) Suggest two advantages and two disadvantages of using trains such as C rather than B. [4]

N05/III/1

Solution

1 (i) This means that the action and reaction forces obey the same set of physical laws and are grouped within the same category, e.g. gravitational, electrical, contact forces etc.

(ii) The labelled force diagram is in Fig. 1.2.

W_X is the weight of book X.
W_Y is the weight of table Y.

F_{XY} is the force that book X exerts on table Y.
F_{YX} is reaction force that table Y exerts on book X.

F_{XY} and F_{YX} are equal in magnitude and opposite in direction, and they are action-reaction forces according to Newton's third law of motion because they act on different bodies.

W_X and F_{YX} are equal in magnitude and opposite in direction, but they are not action-reaction forces because they both act on the same body, i.e. book X. They balance each other so that the resultant force acting on book X is zero, ensuring that the book is in equilibrium.

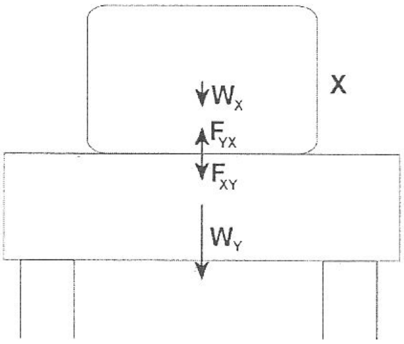


Fig. 1.2

(iii) The labelled force diagram is in Fig. 1.3.

W_X is the weight of book X.
W_Y is the weight of table Y.

F_{XY} is the force that book X exerts on table Y.
F_{YX} is reaction force that table Y exerts on book X.

F_{XY} and F_{YX} are equal in magnitude and opposite in direction, and they are action-reaction forces according to Newton's third law of motion because they act on different bodies. However, both the action and reaction forces F_{XY} and F_{YX} are greater in magnitude than their counterparts in (ii).

W_X and F_{YX} are <u>not</u> equal in magnitude anymore although still opposite in direction, they are not action-reaction forces anyway since they both act on the same body, i.e. book X. The resultant upward force ($F_{YX} - W_X$) is the decelerating force that brings the book to rest.

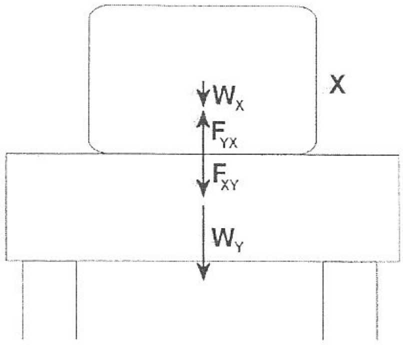


Fig. 1.3

(b) (i) The completed table is shown below.

	train A	train B	train C
distance travelled in the 400 s that the trains have different speeds	20 000 m	**5 000 m**	**8 750 m**
distance train is behind train A as a result of stopping	0	15 000 m	**11 250 m**
time delay as a result of stopping	0	300 s	**225 s**
kinetic energy of train at a speed of 50 m s^{-1}	**7.5 x 10^8 J**	**7.5 x 10^8 J**	1.25 x 10^8 J
stopping force exerted on train by the track	0	**3.0 x 10^5 N**	**2.0 x 10^5 N**

(ii) Two advantages:

1. Due to its smaller mass, train C is easier to accelerate and decelerate thus reaching the terminal station faster, especially if there are many intermediate stops during the journey.

2. The braking force required is less hence the wear and tear on the brake pads will also be lower.

Two disadvantages:

1. Lighter trains tend to have smaller engine power and are able to transport fewer loads per trip. They thus need to make more trips to transport the same amount of load.

2. As a proportion of its useful energy, the work done against air resistance will be greater for the lighter train, thus making it less energy efficient.

TOPIC 4 Forces

Hooke's Law

Long Questions

1 In the sport of clay pigeon shooting, a clay disc is launched into the air by a spring, and the contestant fires a shot at the moving disc. A launching device has been modified to project the disc vertically upwards. The spring in the launching device has a spring constant of 2000 N m^{-1} and the clay disc has mass 80 g. In use the spring obeys Hooke's law and the extension decreases from 13 cm to 7.0 cm.

(i) Sketch a graph of force against extension for the spring and shade the area which represents the loss of energy stored by the spring when launching the clay disc.

(ii) Calculate

1. the loss of elastic potential energy of the spring,

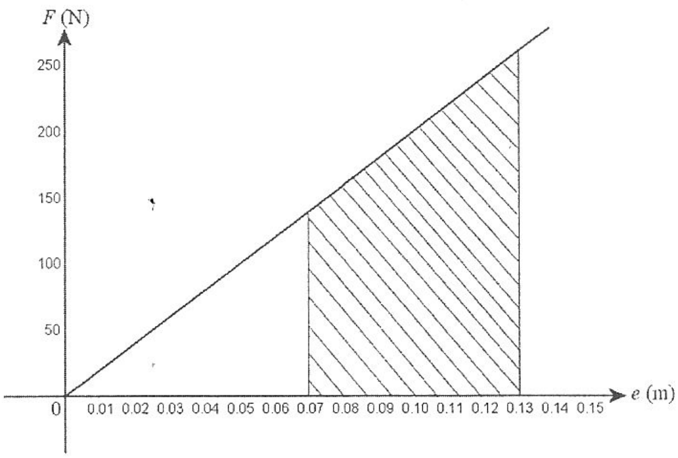
2. the initial speed of the clay disc, assuming all the energy lost by the spring becomes kinetic energy of the disc,

3. the height to which the disc will rise, assuming that air resistance is negligible. [8]

N02/III/1 (part)

Solution

(i) The graph of force F against extension e is shown in the figure below.



(ii) 1. The loss in elastic potential energy is the area under the graph
$= \frac{1}{2}$ (0.07 x 2000 + 0.13 x 2000) N x (0.13 – 0.07) m
= 12 J.

2. Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}(0.080 \text{ kg})v^2 = 12$ J.
$v = 17.32 = 17$ m s^{-1}.

3. Gain in gravitational potential energy = loss in kinetic energy.
$mgh = \frac{1}{2}mv^2 = 12$ J
(0.080 kg)(9.81 N kg^{-1}) h = 12 J
$h = 15.29 = 15$ m.

2 (a) Define *force*. [2]

(b) A light helical spring is suspended vertically from a fixed position, as shown in Fig. 6.1.

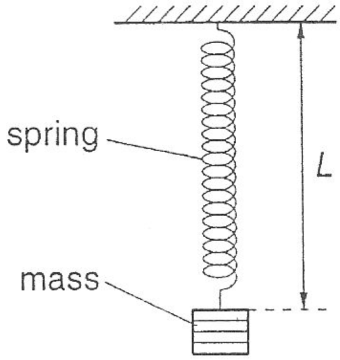


Fig. 6.1

Different masses are suspended from the spring. The weight W of the mass and the length L of the spring are noted.
The variation with weight W of the length L is shown in Fig. 6.2.

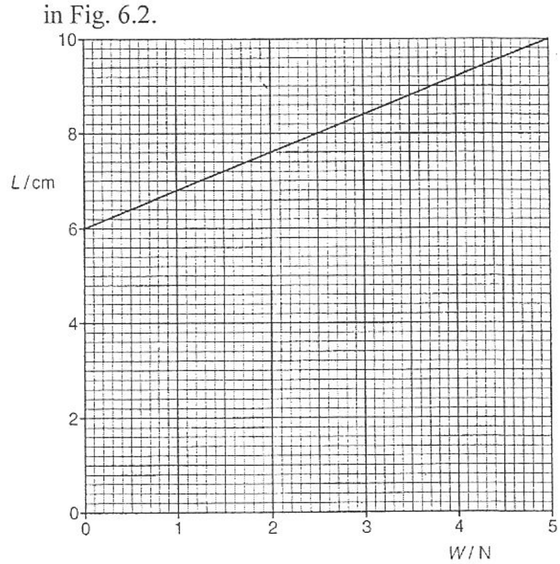


Fig. 6.2

(i) On Fig. 6.2, show clearly the area of the graph that represents the energy stored in the spring when the weight on the spring is increased from zero to 5.0 N. [1]

(ii) For a spring undergoing an elastic change, the force per unit extension of the spring is known as the force constant k.
Show that the energy E stored in the spring for an extension x of the spring is given by the expression

$$E = \tfrac{1}{2}kx^2.$$

[2]

(c) A mass of weight 4.0 N is suspended from the spring in (b).
When the mass is stationary, it is then pulled downwards a distance of 0.80 cm and held stationary.

(i) Determine the total length of the spring.
length = ... cm [1]

(ii) For the increase in extension of 0.80 cm, determine the magnitude of the change in
1. the gravitational potential energy of the mass,
change = …………….. J [2]
2. the elastic potential energy of the spring.
change = …………….. J [3]

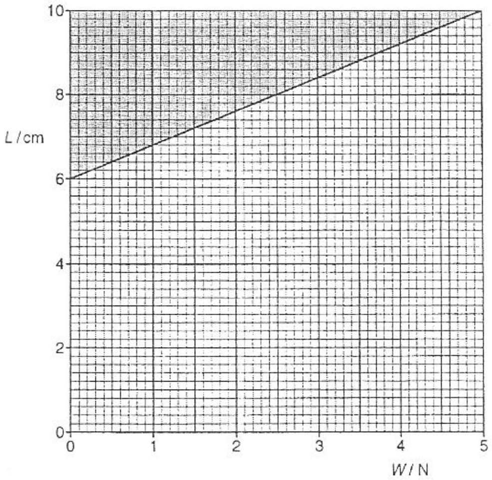
(iii) Use your answers in (ii) to show that the work done to cause the additional extension of 0.80 cm is 4.0 x 10^{-3} J. [1]

N10/III/6 (part)

Solution

The resultant force acting on a body is the rate of change of momentum of the body, and acts in the direction of the change in momentum. The unit for force is the newton, which is the force required to accelerate a mass of 1 kg at a rate of 1 m s^{-2} in the direction of the force.

(i) The energy stored is represented by the shaded area:



(ii) The energy E stored in a spring is the work done to stretch it, which is the sum of the spring's force (F) multiplied by incremental displacements (δx). This can be found by integration with limits from 0 to x,

$$E = \int F\,dx = \int kx\,dx = \tfrac{1}{2}kx^2$$

Alternatively, this is also the triangular area under the force-extension graph.

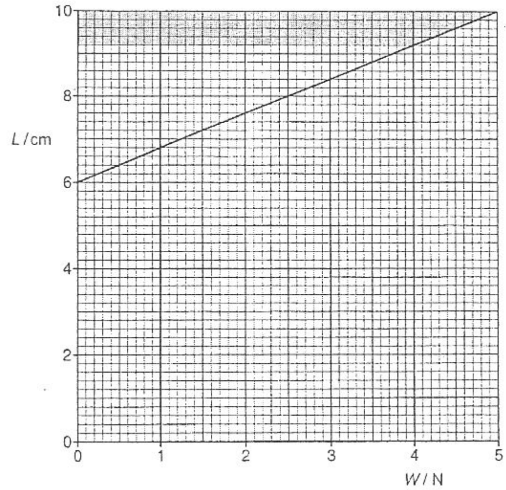
$$E = \tfrac{1}{2}Fx = \tfrac{1}{2}(kx)x = \tfrac{1}{2}kx^2$$

The spring constant for this spring
$k = (5.0\text{ N})/(0.04\text{ m}) = 125\text{ N m}^{-1}$.

(i) From the graph, the length of the spring is 9.2 cm when the weight is 4.0 N. Adding to this another 0.8 cm, the total length becomes 10 cm.
length = 10 cm

(ii) 1. The change in gravitational potential energy $= (mg)(h) = (4.0)(0.8 \times 10^{-2}) = 0.032$ J.

2. The change in elastic potential energy is the shaded area shown:



$= \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$
$= \tfrac{1}{2}(125)(0.04)^2 - \tfrac{1}{2}(125)(0.032)^2$
$= 0.036$ J

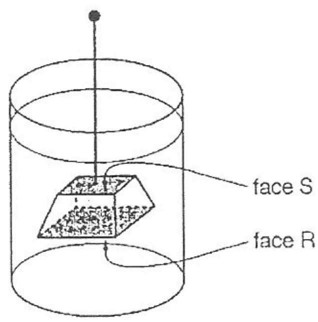
(iii) Work done to cause the additional 0.80 cm of extension = gain in elastic potential energy – loss in gravitational potential energy = 0.036 – 0.032 = 4.0 x 10^{-3} J.

Fluids

3 The diagram shows a block of copper suspended in water.

The block experiences an upthrust from the water.

Which statement is the basis of an explanation for this upthrust?



A Copper is more dense than water.
B The area of face R is greater than the area of face S.
C The density of water increases with depth.
D The pressure of water increases with depth. N02/I/7

Solution Answer: **D**

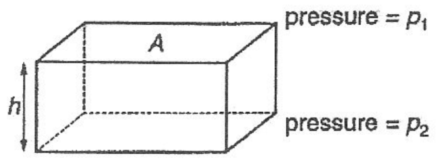
In Archimedes's principle, the upthrust is due to the pressure difference between the top and bottom surfaces of the immersed object.

Since pressure increases with depth, the upward force exerted by the liquid on the bottom surface R of the immersed object is greater than the downward force exerted by the liquid on the top surface S plus the slanted sides.

The difference between the upward and downward forces is the upthrust.

(Note: even if the object is turned upside down, the upthrust will still be upwards).

4 A solid block of material of density ρ, height h and horizontal surface area A is immersed in a liquid. The pressures of the liquid at the upper and lower surfaces are p_1 and p_2 respectively.



Which of the following is an expression for the upthrust on the block?

A $Ah\rho g$

B $Ah\rho g + p_1 A$

C $p_2 A$

D $p_2 A - p_1 A$

N03/I/6

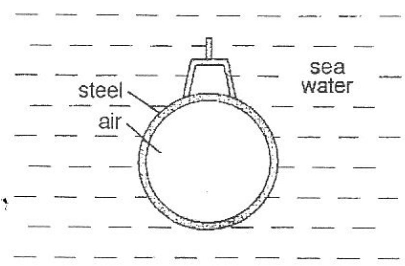
Solution Answer: **D**

In Archimedes's principle, the upthrust is due to the pressure difference between the top and bottom surfaces of the immersed object.

Since pressure increases with depth, the upward force (p_2A) exerted by the liquid on the bottom surface of the immersed object is greater than the downward force (p_1A) exerted by the liquid on the top surface.
The difference between the upward and downward forces is the upthrust.

(Note: ρ in this question refers to the density of the object and not the liquid so the upthrust cannot be $Ah\rho g$).

5 A submarine is in equilibrium in a fully submerged position.



What causes the upthrust on the submarine?

A The air exerts a greater upward force on the submarine than the weight of the steel.

B The air in the submarine is less dense than sea water.

C There is a difference in water pressure acting on the top and bottom of the submarine.

D The submarine displaces its own volume of sea water.

N04/I/7

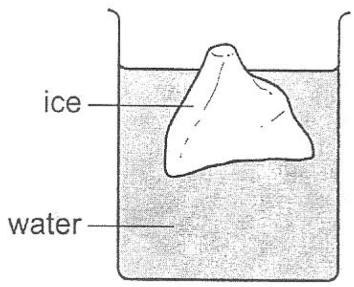
Solution Answer: **C**

In Archimedes's principle, the upthrust is due to the pressure difference between the top and bottom surfaces of the immersed object.

Since pressure increases with depth, the upward force exerted by the liquid on the bottom surface of the submarine is greater than the downward force exerted by the liquid on the top surface of the submarine.

The difference between the upward and downward forces is the upthrust.

6 A lump of ice floats in water as shown.



Which statement is correct?

A The lump of ice floats because the area of its lower surface is larger than the area of its upper surface.

B The pressure difference between the lower and the upper surfaces of the lump of ice give rise to an upthrust equal to its weight.

C The ice has a greater density than the water.

D The mass of water displaced by the ice is equal to the upthrust.

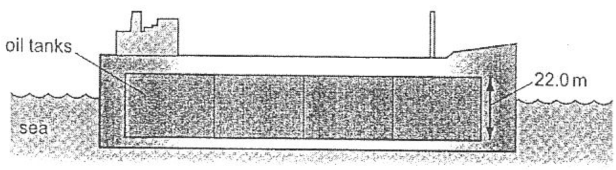
N07/I/8

Solution Answer: **B**

Upthrust is a result of pressure difference between the lower and upper surfaces.

Statement A is incorrect because the **effective** areas acted upon by both vertically downward and vertically upward forces are the same.
Statement C is incorrect as ice has a lower density than water.
Statement D is incorrect as mass is measured in kilograms while upthrust is measured in newtons. The equation is not homogeneous.

7 An oil tanker, with vertical sides, has an external cross-sectional area of 36 500 m^3 in the plane of the sea.



The tanker carries oil of density 930 kg m^{-3} in its tanks, which have a constant cross-sectional area of 34 000 m^2 and depth 22.0 m. Sea water has density 1030 kg m^{-3}.

By how much does the tanker rise in the water when it unloads its oil?

A 26.2 m B 22.7 m C 21.3 m D 18.5 m

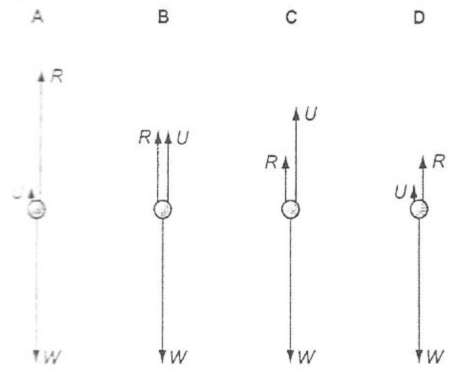
N09/I/10

Solution Answer: **D**

The weight of the oil unloaded is (34 000 m^3 x 22.0 m x 930 kg m^{-3})g = 6.956 x 10^8 g. The tanker will be lighter by this amount.
The reduction in upthrust needed to support the tanker is, according to Archimedes' Principle, equal to the reduction in weight of the sea water displaced.

$$(36\,500 \text{ m}^3 \times h \times 1030 \text{ kg m}^{-3})g = 6.956 \times 10^8\, g$$
$$h = 18.5 \text{ m}$$

8 A water droplet in a cloud is falling through air and is in equilibrium. Three forces act on it, its weight W, upthrust U and air resistance R.
Which diagram, showing these three forces to scale, is correct?



N10/I/8
Solution
Answer: **A**
Upthrust has a magnitude equal to the weight of air displaced.
Since the density of air is low, the upthrust should be very small compared to air resistance.
Since the droplet is in equilibrium, the resultant force acting on it is zero hence the sum of $U + R$ is equal to W.

Short Questions

9 (a) Explain why a body falling through air reaches a terminal velocity. [1]

(b) Explain why the terminal velocity of falling spheres of the same density increases with mass. [3]

N04/III/13

Solution
(a) Air resistance increases with speed.
As the body falls through air, its speed increases and so does the air resistance opposing its motion.
Since the body's weight is constant but the resistive force increases, the resultant force decreases.
According to Newton's second law of motion, the acceleration of a body is directly proportional to the resultant force acting on it ($a = F/m$) thus the acceleration also decreases.
Eventually, the speed reaches a value such that the air resistance plus upthrust is equal and opposite to the body's weight, and the resultant force becomes zero.
The acceleration becomes zero and the body reaches terminal velocity.

(b) The gravitational force acting on the body is proportional to its volume and the cube of its linear dimension r, $W = mg = (4/3)\pi r^3 \rho g$.
On the other hand, air resistance is proportional to the effective area facing the wind and the square of its linear dimension r, $F = Br^2\rho v^2$.
At terminal velocity, acceleration and resultant force is zero hence,

$$F_{drag} = W - U$$
$$Br^2\rho v^2 = (4/3)\pi r^3\rho g - U$$
$$v^2 = [(4/3)\pi r\rho g - U]/B$$
$$v \propto \sqrt{r}$$

For spheres of the same density, the greater its mass, the greater its dimension r and the higher its terminal velocity.

Long Questions

10 *(a)* A block of ice of mass 150 kg is floating in a fresh-water lake. The density of the water is 1000 kg m^{-3}. Determine

(i) the upthrust on the ice,

(ii) the weight of water displaced,

(iii) the volume of water displaced,

(iv) the volume of water produced when all the ice melts.

[6]

(b) Explain why, when ice floating in a jug of water melts, there is no change in the level of the water. [2]

(c) Fig. 10.1 shows an object that is not in equilibrium partially submerged in water.

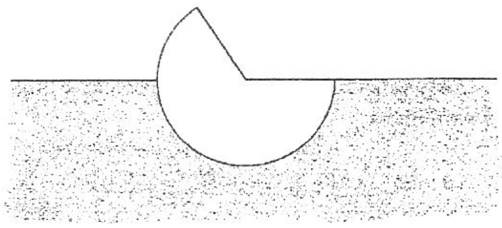


Fig. 10.1

The density of the object is uniform and is less than the density of water.

(i) Make a rough copy of Fig. 10.1. Draw arrows on your copy to show the weight of the object and the upthrust. Pay particular attention to the relative positions of the lines of action of the two forces.

(ii) Describe what will happen to the object and suggest its approximate final position after it comes to equilibrium. [4]

N02/III/10

Solution
(a) (i) For a floating body, the upthrust is equal in magnitude to the weight of the body.
Hence $U = W = mg$
$= (150 \text{ kg})(9.81 \text{ N kg}^{-1})$
$= 1472$
$= 1.5 \times 10^3$ N.

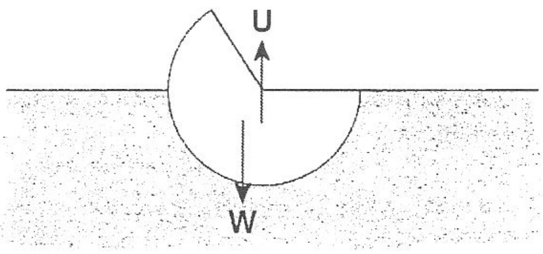
(ii) A floating body displaces an amount of fluid of a weight equal to its own weight (principle of floatation.
Hence the weight of water displaced is 1.5×10^3 N.

(iii) $V = W/\rho g$
$= (1472 \text{ N})/(1000 \text{ kg m}^{-3})(9.81 \text{ N kg}^{-1})$
$= 0.15 \text{ m}^3$.

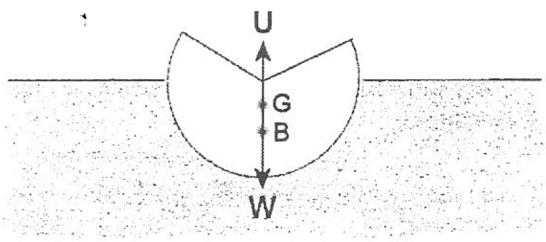
(iv) Since the weight of ice is exactly equal to the weight of the water it displaces, the mass of the ice is equal to the mass of water it displaces.
When the ice melts, it produces the same volume of water as the water it displaces, i.e. 0.15 m^3.

(b) Since the ice floats, in accordance with the principle of floatation, the weight of ice is exactly equal to the weight of the water it displaces.
Therefore, the mass of the ice is equal to the mass of water it displaces.
Thus when the ice melts, it will produce the same volume of water as the water it displaces.
The water formed by the melted ice will exactly replace the water displaced by the ice. Therefore there is no change in the level of the water.

(c) (i) The diagram is shown in the figure below. In the figure, W represents the weight of the object and U the upthrust exerted by the fluid on it.



(ii) The weight W and upthrust U are equal and opposite. Since their lines of action are not aligned, they form a couple exerting an anticlockwise torque on the object, causing it to rotate anticlockwise.
The object will overshoot the equilibrium position, with W's line of action moving to the right of that of U's. The couple then exerts a clockwise torque on the object.
This causes the object to oscillate clockwise and anticlockwise alternately, until resistive forces acting on the body bring it to its final equilibrium position, with the lines of action of W and U aligned and the centre of buoyancy B directly below the centre of gravity G.
This is shown in the figure below.



11 *(a)* Using definitions of pressure p and density ρ, derive the equation that gives the pressure at a depth h in a fluid. [4]

(b) A compressible object has mass 1.80 kg and volume V given by the equation

$$V = a - bp$$

where a is 0.0020 m^3, b is 3.2×10^{-10} m^{3}Pa^{-1} and p is the excess pressure above atmospheric pressure.

Seawater can be assumed to be incompressible and of density 1030 kg m^{-3}.

Calculate

(i) the volume of the object on the surface of the sea, [1]

(ii) the density of the object on the surface of the sea, [1]

(iii) the excess pressure from the water at which the object ceases to rise and starts to sink to the sea floor, [3]

(iv) the depth in the water at which this change from rising to sinking takes place. [2]

N03/III/10

Solution

(a) Pressure p is the force acting normally per unit area thus the pressure at a depth h will be given by $p = F/A = W/A$ where W is the weight of the column of fluid above the area.

The density ρ of the fluid is the ratio of its mass m to volume V hence mass $m = V\rho$, and weight $W = mg = V\rho g$.

Pressure $p = W/A = V\rho g/A$ but $V/A = h$ hence $p = h\rho g$.

(b) (i) At the surface of the sea, the excess pressure $p = 0$, hence $V = a = 0.0020$ m^3.

(ii) Density = mass/volume
= 1.80 kg / 0.0020 m^3
= 900 kg m^3.

(iii) The situation described occurs when the density of the object equals the density of sea water.

$$(1.80 \text{ kg})/(0.0020 \text{ m}^3 - 3.2 \times 10^{-10} p \text{ m}^3) = 1030 \text{ kg m}^{-3}$$

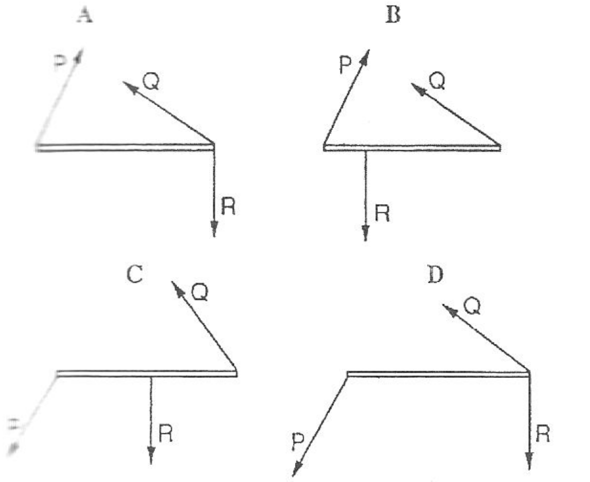
$$p = 7.888 \times 10^5 = 7.9 \times 10^5 \text{ Pa}$$

(iv) $h = p/\rho g$
$= 7.888 \times 10^5$ Pa/(1030 kg m^{-3} x 9.81 N kg^{-1})
= 78.07
= 78 m.

Equilibrium of Forces

12 A light rod is acted upon by three forces P, Q and R.

Which diagram could show the position and direction of each of the forces when the rod is in equilibrium?

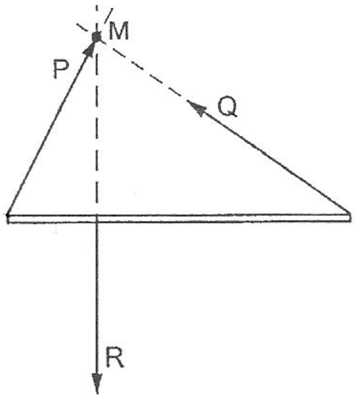


N05/I/7

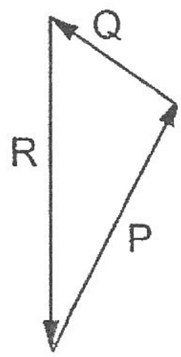
Solution Answer: **B**

For a system of non-collinear forces acting on a body which is in equilibrium, the lines of action of all the forces must meet at one point (M).

This is to satisfy the principle of moments: that a body in equilibrium has no resultant moment about *any* point, to ensure rotational equilibrium.

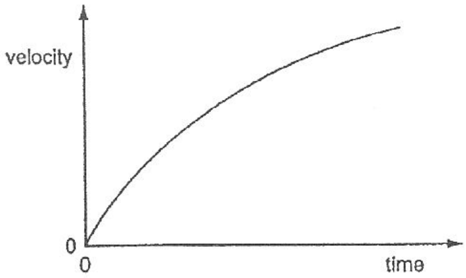


At the same time, a body in equilibrium should also have no resultant force acting on it, to ensure translational equilibrium.

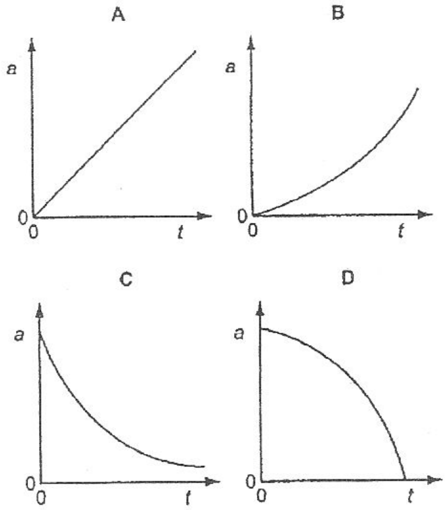


Only answer B satisfies both conditions.

13 A table-tennis ball is released in air and falls vertically. The graph shows how its velocity varies with time.



Which graph best illustrates the variation with time t of its acceleration a?

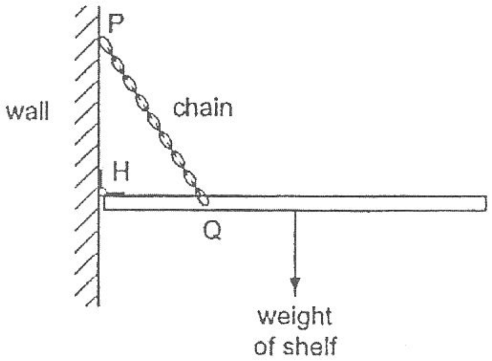


N06/I/3

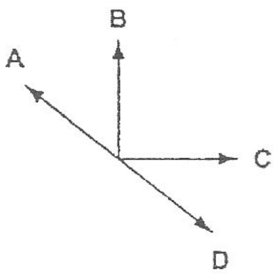
Solution Answer: **C**

Acceleration is the rate of change of velocity.
The acceleration-time graph can be obtained by measuring the gradient of velocity-time graph.
As the gradient of the velocity-time graph decreases at a decreasing rate, the acceleration decreases at a decreasing rate.

14 A hinged shelf is held horizontally against a wall by a chain PQ. The forces acting on the shelf are its weight, the force exerted by the chain and the force exerted by the hinge H.



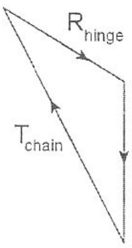
Which arrow could represent the direction of the force the hinge exerts on the shelf?



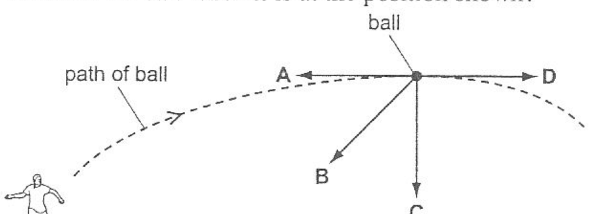
N06/I/6

Solution Answer: **D**

A body in equilibrium has no resultant force hence the three forces acting on it must form a closed vector triangle, as shown in the figure below.



15 The diagram shows a ball which has been thrown and is being acted on by air resistance.
Which labelled arrow shows the direction of the resultant force on the ball when it is at the position shown?



N07/I/4

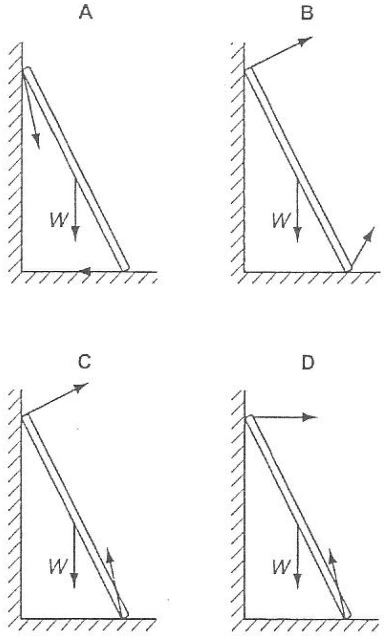
Solution Answer: **B**

There are two forces acting on the ball while it is in flight. The first is the force of gravity (acting vertically downwards) and the other is the air resistance (acting towards the left, against the direction of motion). The resultant of these two forces would be pointing down and to the left.
On the other hand if there had been no air resistance then C would have been the answer.

16 A ladder of weight *W* rests against a vertical wall.

Friction between the ladder and the ground and also between the ladder and the wall prevents the ladder from slipping.

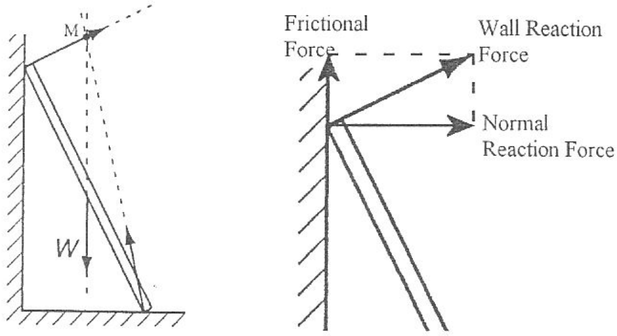
Which diagram shows the directions of the forces on the ladder?



N08/I/7

Solution Answer: **C**

For a system of non-parallel forces to be in a state of equilibrium, their lines of action must meet at one point (Labelled M) below. This can only be satisfied by either answer C or D.



Since it is mentioned that there is friction at the wall, the reaction force at the wall cannot be normal to the wall as it must contain a frictional component acting parallel to the wall. Therefore, the answer is C.

Answer D is for the case in which the wall is frictionless.

17 The resistive force R on a moving car can be given by the equation $R = P + Qv^2$ where v is the speed of the car and P and Q are constants.

(a) State suitable units for P and Q. [2]

(b) When the speed of the car is very close to zero the resistive force is 200 N; when the speed is 30 m s^{-1} the resistive force is 2000 N. Calculate the values of P and Q. [3]

(c) Use your answers to *(b)* in order to find the resistive force at 40 m s^{-1}. [1]

(d) Suggest factors which affect P and Q and explain how there can be a resistive force when the car is stationary. [4] N02/III/12

Solution

(a) P is in newton (N) since P has the same unit as force R.
Q is in kg m^{-1} since Qv^2 has the same unit as force R.

(b) When $v \approx 0$, $P = R = 200$ N.
Hence $P = 200$ N.

When $v = 30$ m s^{-1}, $P + Qv^2 = R = 2000$ N.
200 N + Q (30 m s^{-1})^2 = 2000 N
$Q = 2$ kg m^{-1}.

(c) R = 200 N + (2 kg m^{-1})(40 m s^{-1})^2 = 3400 N.

(d) P is independent of speed hence it is due to friction.
P is dependent on the coefficient of friction μ (determined by the roughness of contact surfaces) and the magnitude of the contact force R between the surfaces ($P = \mu R$).

Q is dependent on speed hence it is due to air resistance.
Q is dependent on the density ρ of the fluid medium (in this case air), the effective frontal area A_{eff} of the car and its drag coefficient C_D ($Q = \frac{1}{2}\rho C_D A_{eff}$).

Static friction is present even when the body is stationary.

18 (a) Drag is sometimes referred to as fluid friction. Describe a way in which drag and friction between solids are similar, and a way in which they differ. [3]

(b) The drag force F is given by the equation

$$F = Br^2\rho v^2$$

where B is a dimensionless quantity.

Apply base units to confirm the form of the equation. [2]

(c) A one-tenth scale model of a new type of car is tested in a wind tunnel. The drag force is found to be F when the air speed is set at v. Suggest and explain what the drag force will be when the full-size car travels at speed v through the air. [3]

N03/III/12 (part)

Solution

(a) Similarity: both forces are resistive forces which oppose motion and decelerate the object, resulting in dissipation of heat energy.
Difference: Frictional forces are independent of speed (once the object is moving) whereas drag force increases with speed (proportional to speed under laminar flow condition; and proportional to the square of speed under turbulent flow condition).

(b) On the left-hand-side of the equation, the SI base unit for force F is the newton (N) which can be reduced to base units of kg m s^{-2} (based on the equation $F = ma$).
On the right-hand-side of the equation, the SI base unit for r^2 is m^2; the SI base units of ρ is kg m^{-3} and the SI base units of v^2 is m^{2}s^{-2}.
Since B is dimensionless, $Br^2\rho v^2$ multiplied together have SI base units of kg m s^{-2}, which is exactly the same as that on the left-hand-side of the equation.
Hence the equation is homogeneous.

(c) A full-size car will have linear dimensions ten times that of the model, hence the variable r becomes $10\ r$, and r^2 becomes $100\ r^2$. Therefore, the force becomes $100\ F$.

19 *(a)* All forces outside the nucleus are caused by objects being in one or more of gravitational, electrical and magnetic fields.

(i) For each of these fields, state the physical property on which the force acts. [3]

(ii) State the direction of the force relative to field direction for each of the fields. [3]

(b) A car travelling along a flat road may be considered to have three forces acting on it. These are represented in Fig. 1.1.

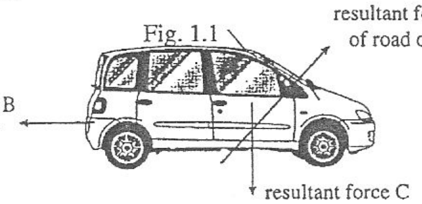


Fig. 1.1

(i) Explain why it is necessary to regard forces A and C as resultant forces. [2]

(ii) Force A has a magnitude of 8200 N and is at an angle of 28° to the vertical. Force B is horizontal and has magnitude 1500 N.

Calculate

1. the weight and the mass of the car, [2]
2. the resultant force on the car, [2]
3. the acceleration of the car. [1]

(iii) For the car, motion is impossible without friction. Discuss what is meant by *friction* and the direction in which it acts on the car. In your answer, suggest another example where friction is useful. [5]

(c) Describe a situation in which motion is produced without friction being required. [2]

N06/III/1

Solution

(a) (i) Gravitational field acts on mass.
Electrical field acts on electric charge.
Magnetic field acts on moving electric charge.

(ii) The gravitational force acting on a mass is always in the same direction as the gravitational field.
The electrical force acting on an electric charge may be in the same direction as the electrical field (for a positive charge) or in the opposite direction (for a negative charge).
The magnetic force acting on a moving electric charge acts at right angles to the direction of the magnetic field (direction determined by Fleming's left hand rule).

(b) (i) Forces A and C are not single forces but made up of component forces.
Force A is the resultant of the vertical normal reaction force exerted by the road on the four wheels, and the horizontal forward accelerating force due to friction between the wheels and the road.
Force C is the resultant force of the weight of every part of the car from the back to the front, but is considered to act at a location known as the centre of gravity – the point at which the entire weight of the car appears to act.

(ii) 1. The weight W of the car is balanced by the vertical component of the resultant force A,

$$W = A \cos 28^\circ$$
$$= 8200 \text{ N} \cos 28^\circ = 7240 = 7200 \text{ N}$$

The mass of the car

$$m = W/g$$
$$= 7240 \text{ N} / 9.81 \text{ N kg}^{-1} = 738 = 740 \text{ kg}$$

2. The resultant force on the car is the difference between the forward (horizontal) component of A and the resultant force B of the air.

$$\text{Resultant } F = A \sin 28^\circ - B$$
$$= 8200 \text{ N} \sin 28^\circ - 1500$$
$$= 3850 - 1500 = 2350 \text{ N}$$

3. The acceleration $a = F / m$

$$= (2350 \text{ N})/738 \text{ kg} = 3.184 = 3.2 \text{ m s}^{-2}.$$

(iii) *Friction* is a type force that exists between contact surfaces, and which resists relative motion between the two surfaces or the tendency to such motion.

In the case of the car, the frictional force acts in the forward direction of the car (i.e. towards the right).

There is friction at the contact point between the tires and the road. As the wheels rotate clockwise, the tires exert a leftward frictional force on the road surface.

By Newton's third law of motion, the road surface will exert an equal but opposite rightward frictional force on each of the rotating tires. This rightward force is the forward force which accelerates the car.

Friction is also useful in braking. When the brakes are stepped on, there is a tendency for the wheels to stop rotating and rub against the road surface.

In this instance, the frictional force acts in the opposite direction to the car's motion. This produces a deceleration which brings the car to rest.

(c) Rockets in space which eject gas to accelerate themselves produce motion without friction being required.

The principles of their motion are the law of conservation of momentum and Newton's third law of motion – the backward thrust on the gas produces an equal but opposite thrust on the rocket.

Turning Effects of Forces

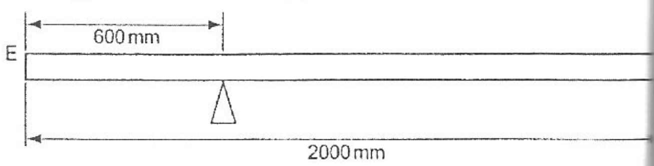
20 What is **not** true of two forces that give rise to a couple?

A They act in opposite directions.
B They both act at the same point.
C They both act on the same body.
D They both have the same magnitude. N03/I/5

Solution Answer: B

Answer B is **not** true as the two forces in a couple must be separated by a perpendicular distance apart, in order to produce a torque.
If both forces act at the same point, the resultant torque will be zero.

21 A uniform plank of weight 60 N is 2000 mm long. It rests o a support that is 600 mm from end E.



At what distance from E must a 160 N weight be placed order to balance the plank?

A 150 mm
B 225 mm
C 375 mm
D 450 mm N05/

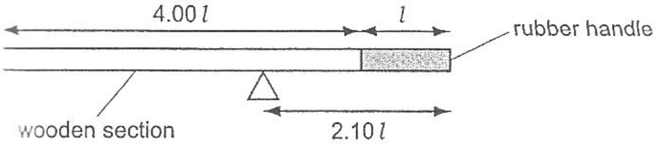
Solution Answer: I

A body in equilibrium has no resultant moment hence t anticlockwise moment exerted by the 160 N weight about t pivot must be equal to the clockwise moment exerted by t plank's weight about the same pivot.
The plank's entire weight acts through the centre of grav which is at the centre of the plank, 400 mm to the right of t pivot. Let d be the distance from the 160 N weight to pivot.

$$(160 \text{ N})\, d = (60 \text{ N})(400 \text{ mm})$$
$$d = 150 \text{ mm}$$

The distance from E is thus 600 mm – 150 mm = 450 mm.

22 A uniform rod has a wooden section and a solid rubber handle, as shown.



The length of the handle is l and the length of the wooden section is 4.00 l. The rod balances a distance 2.10 l from the rubber end.

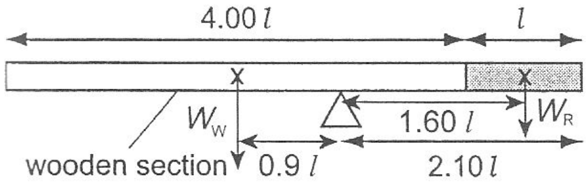
What is the ratio $\dfrac{\text{density of rubber}}{\text{density of wood}}$?

A 1.71 **B** 2.25 **C** 2.50 **D** 3.27

N08/I/8

Solution Answer: **B**

The weight of the wooden section W_W and that of the rubber handle W_R act from their respective centres of gravity, as shown below.



Applying the principle of moments,

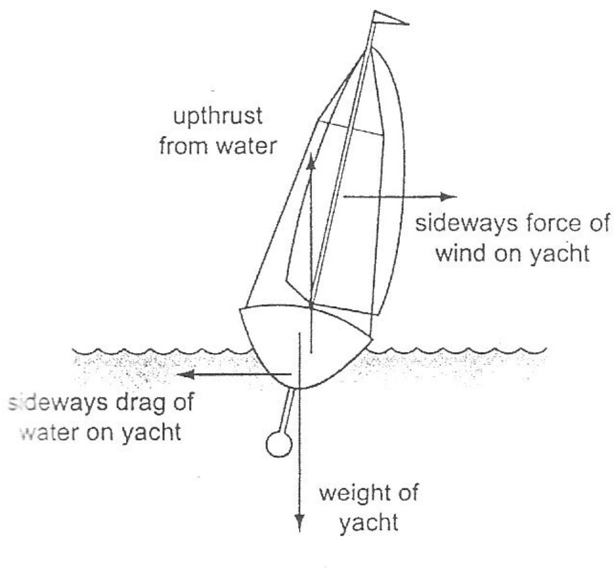
$$W_W (0.9\, l) = W_R (1.60\, l)$$

$$\rho_W (4.00\, l\, A\, g)(0.9\, l) = \rho_R (l\, A\, g)(1.60\, l)$$

$$\rho_W (3.60) = \rho_R (1.60)$$

$$\rho_R / \rho_W = 3.60 / 1.60 = 2.25$$

23 A yacht that is in equilibrium has two vertical and two horizontal forces acting on it.



Which statement about the forces is **not** correct?

A The sideways drag from the water on the yacht is equal and opposite to the sideways force of the wind on the yacht.

B The resultant of all four forces is zero.

C The torque provided by the vertical forces is the same as the torque provided by the horizontal forces.

D The upthrust from the water is equal and opposite to the weight of the yacht.

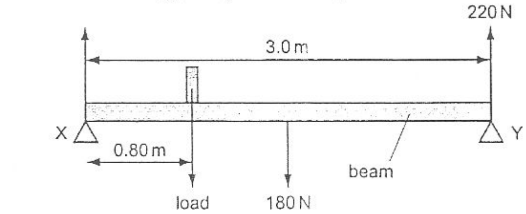
N09/I/8

Solution Answer: **C**

Torque, just like force, is a vector quantity which has both magnitude and direction.

The correct version of statement C should be "The torque provided by the vertical forces is **equal and opposite to** the torque provided by the horizontal forces."

24 A uniform beam in a roof structure has a weight of 180 N. It is supported in two places X and Y, a distance 3.0 m apart. A load is placed on the beam a distance of 0.80 m from X. The support provided by Y is 220 N.



What is the value of the load?

A 270 N **B** 490 N **C** 520 N **D** 830 N

N10/I/9

Solution Answer: **B**

Taking the pivot at X and applying the principle of moments,
sum of clockwise moments = sum of anticlockwise moments.

(load)(0.80 m) + (180 N)(1.5 m) = (220 N)(3.0 m)

load = 490 N

Short Questions

25 Fig. 3.1 shows a force diagram that represents a boat that is being lifted by two ropes so that the boat remains horizontal and travels vertically upwards at a constant speed after leaving the water.

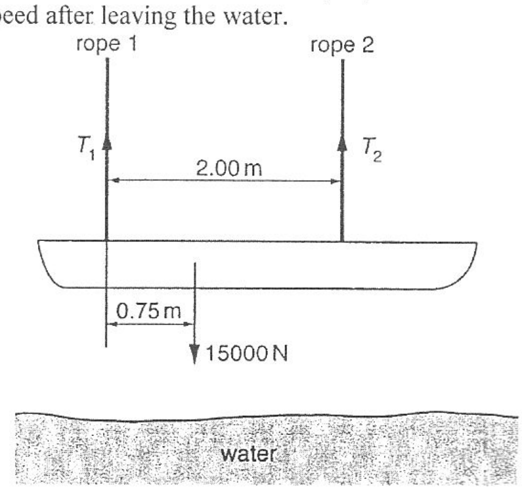


Fig. 3.1

The weight of the boat is 15000 N and the tensions in the ropes 1 and 2 are T_1 and T_2 respectively.

(a) The position of the centre of gravity of the boat is not at its midpoint. Suggest what this implies about the distribution of mass in the boat.

....
.... [1]

(b) Explain two conditions required for the boat to be in a state of equilibrium while it is moving upwards.
1. ....
2. ....
[2]

(c) Use the principle of moments to determine the tensions in the two ropes.
T_1 = ... N
T_2 = ... N [4]

N09/II/3

Solution

(a) The mass is not uniformly distributed throughout the boat. The mass of the boat is more concentrated on its left side.

(b) 1. No resultant force acts on the boat, so that the boat is in translational equilibrium.

2. No resultant moment acts on the boat about any point, so that the boat is in rotational equilibrium.

(c) Applying condition 1,

$$T_1 + T_2 = 15000 \text{ N}$$

Applying condition 2, and taking moments about the point where rope 1 is attached to the boat,

sum of clockwise moments = sum of anticlockwise moments

$$(15000 \text{ N})(0.75 \text{ m}) = T_2 (2.00 \text{ m})$$

$$T_2 = 5625 \text{ N}$$

$$T_1 = 15000 - 5625 \text{ N} = 9375 \text{ N}$$

T_1 = 9400 N
T_2 = 5600 N

Long Questions

26 *(a)* Define the terms *moment of a force* and *torque of a couple*. For each of the terms draw a sketch to illustrate its meaning. [4]

(b) State the two conditions necessary for a body to be in equilibrium. [2]

(c) During the construction of many modern bridges, sections are added from both banks until the two halves meet at the centre. Fig 1.1 shows a new section S, of weight 3.0×10^5 N, after it has been attached to an existing part B of a bridge.

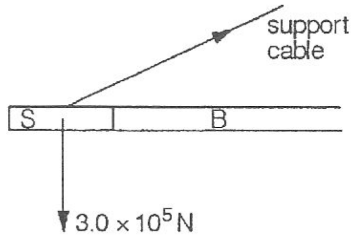


Fig. 1.1

The support cable which keeps section S in equilibrium is at an angle of 25° to the horizontal. The existing part B of the bridge provides a horizontal force on S.

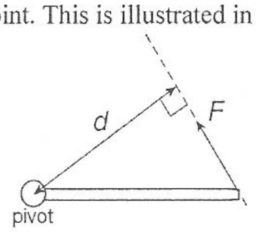
(i) Draw a labelled vector diagram showing the three forces on S.

(ii) Use your diagram to determine the tension in the cable and the horizontal force which B exerts on S. [6]

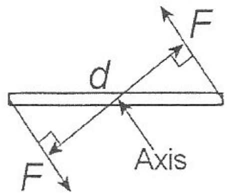
N02/III/1 (part)

Solution

(a) The *moment of a force* is the turning effect of a force about a point and is the product of the force F and the perpendicular distance d from the line of action of the force to that point. This is illustrated in the figure below.

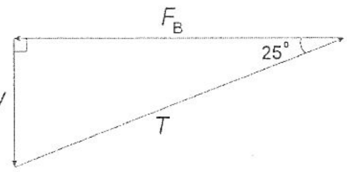


The *torque of a couple* is the turning effect of a pair of equal and antiparallel forces known as a couple about an axis, and is the product of the force F of one of the couple and the perpendicular distance d between their lines of action. This is illustrated in the figure below.



(b) A body in equilibrium should have:
- No resultant force acting on it; and
- No resultant moment about any point.

(c) (i) A labelled diagram is shown in the figure below. In the diagram, T is the tension in the cable, W the weight of the section S and F_B the force exerted by B on S.



(ii) $T = W / \sin 25° = 3.0 \times 10^5 \text{ N} / \sin 25°$
$= 7.099 \times 10^5 = 7.1 \times 10^5$ N.
$F_B = W / \tan 25° = 3.0 \times 10^5 \text{ N} / \tan 25°$
$= 6.434 \times 10^5 = 6.4 \times 10^5$ N.

27 (a) Hydrostatic pressure p is given by the expression

$$p = \rho g h.$$

(i) Explain what is meant by *hydrostatic pressure*. [1]

(ii) Derive this expression from the definitions of pressure and density. Explain what you are doing at each stage in your derivation. [3]

(b) (i) An object of mass m and density d is surrounded by air of density ρ. Show that the resultant force F acting downward on the object is given by

$$F = mg\left(1 - \frac{\rho}{d}\right).$$ [3]

(ii) A chemist uses an accurate balance to weigh a sample as shown in Fig. 10.1.

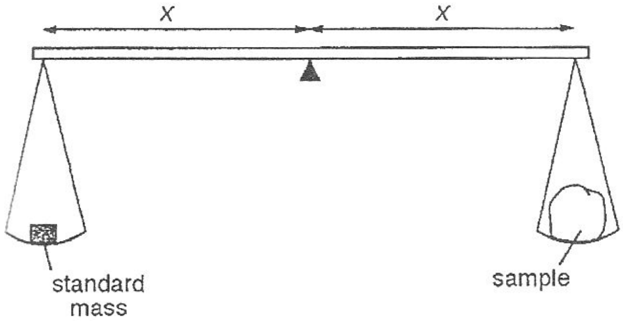


Fig. 10.1

The chemist ignores the effect of upthrust and records the mass of the sample as 0.17851 kg. The density of the sample is 940.0 kg m^{-3}, the density of the standard mass is 8493 kg m^{-3}, and the density of air is 1.29 kg m^{-3}. Calculate the actual mass of the sample. [5]

(iii) Calculate the percentage error in the recorded mass. [1]

N04/III/10

Solution

(a) (i) Hydrostatic pressure is the pressure at a given depth in a static fluid resulting from the weight of fluid above that depth acting per unit area.

(ii) Pressure p is the force acting normally per unit area thus the pressure at a depth h will be given by $p = F/A = W/A$ where W is the weight of the column of fluid above the area.

The density ρ of the fluid is the ratio of its mass m to volume V hence mass $m = V\rho$, and weight $W = mg = V\rho g$.

Pressure $p = W/A = V\rho g/A$ but $V/A = h$ hence $p = h\rho g$.

(b) (i) The resultant force $F = W - U$ where W is the weight of the object and U the upthrust due to the fluid (air) displaced.

$W = mg$ while $U = V\rho g$ where V is the volume of the air displaced (equal to the volume of the object since it is completely immersed in air) and ρ the density of air.

However, the volume V of the object = m/d where m is its mass and d its density hence $U = (m/d)\rho g$. Therefore,

$$F = W - U = mg - (m/d)\rho g = mg(1 - \rho/d)$$

(iii) Applying the principle of moments,

$$m_{st}\, g\,(1 - 1.29/8493)\, x = m_s\, g\,(1 - 1.29/940.0)\, x$$

where m_{st} is the standard mass of 0.17851 kg mistakenly thought to be the exact mass of m_s (because the chemist ignored the effect of upthrust).

Eliminating g and x from both sides of the equation,

$$0.17851 \text{ kg } (1 - 1.29/8493) = m_s\,(1 - 1.29/940)$$

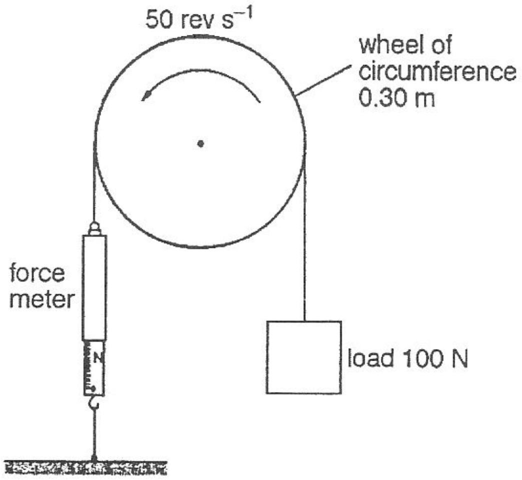
$$m_s = 0.178728162 = 0.17873 \text{ kg}.$$

(iv) The error is 0.178728162 kg – 0.17851 kg
$= 2.1816 \times 10^{-4}$ kg.

The percentage error
$= (2.1816 \times 10^{-4} \text{ kg} / 0.178728162 \text{ kg}) \times 100\%$
$= 0.1221$
$= 0.12\%$.

TOPIC 5 Work, Energy and Power

1 The diagram shows a wheel of circumference 0.30 m. A rope is fastened at one end to a force meter (spring balance). The rope passes over the wheel and supports a freely hanging load of 100 N. The wheel is driven by an electric motor at a constant rate of 50 revolutions per second. When the wheel is turning at this rate, the force meter reads 20 N.



What is the output power of the motor?

A 0.3 kW
B 1.2 kW
C 1.5 kW
D 1.8 kW

N02/I/8

Solution Answer: **B**

The net force exerted on the wheel is the difference between the load and the force meter reading, i.e. 80 N.

The linear speed of the circumference of the wheel
= 50 rev s^{-1} x 0.30 m rev^{-1} = 15 m s^{-1}.

Power = Force x Speed = 80 N x 15 m s^{-1} = 1200 W.

2 An aircraft moving through air at velocity v experiences a resistive force F given by the expression

$$F = kv^2,$$

where k is a constant.

What is the power required to keep the aircraft moving at this constant velocity?

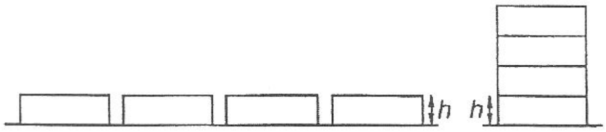
A kv
B kv^2
C kv^3
D kv^4

N03/I/7

Solution Answer: **C**

Power = Force x Speed = $(kv^2)(v) = kv^3$.

3 Initially, four identical uniform blocks, each of mass m and thickness h, are spread on a table.



How much work is done on the blocks in stacking them on top of one another?

A 2 mgh
B 3 mgh
C 4 mgh
D 6 mgh

N04/I/8

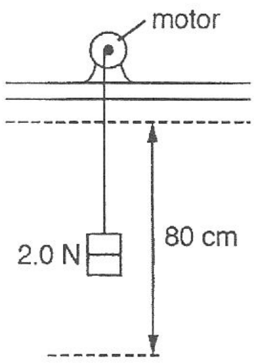
Solution Answer: **D**

Work done = gain in gravitational potential energy (P.E.).

The gain in P.E. for the first block is zero (not raised), the second block is mgh, the third block is 2 mgh and the fourth block is 3 mgh.

The total gain in P.E. is 6 mgh.

4 A small electric motor is used to raise a weight of 2.0 N at constant speed through a vertical height of 80 cm in 4.0 s.



The efficiency of the motor is 20 %.

What is the electrical power supplied to the motor?

A 0.080 W
B 0.80 W
C 2.0 W
D 200 W

N04/I/

Solution Answer: C

Power output = rate of gain in gravitational potential energy
= $mgh/t = Wh/t$ = (2.0 N)(0.80 m)/(4.0 s) = 0.40 W.

Electrical power supplied
= power output / (efficiency/100)
= power output / 0.20
= 0.40 W / 0.20
= 2.0 W.

5 Which expression may be used to calculate power?

A charge × potential difference
B force × distance moved in the direction of the force
C velocity × force in the direction of the velocity
D work done × time taken

N05/I/8

Solution Answer: **C**

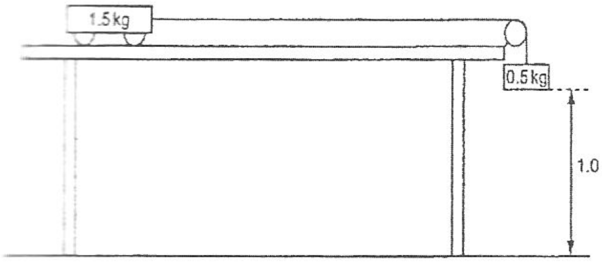
Since work done by a constant force = force x distance moved in the direction of the force, dividing both sides of the equation by time, power exerted by a constant force = force x velocity in the direction of the force (Answer C).

Answer A (charge x potential difference) is the work done by the charge passing through two points with a potential difference.

Answer B (force x distance moved) is the work done by the force.

Answer D (work done x time taken) has no physical meaning.

6 The diagram shows a trolley being pulled from rest along a horizontal table by a falling mass. The trolley mass is 1.5 kg and the falling mass is 0.50 kg. The mass falls through 1.0 m.



What is the maximum kinetic energy of the trolley?

A 3.7 J B 4.9 J C 15 J D 20 J

N05/I/9

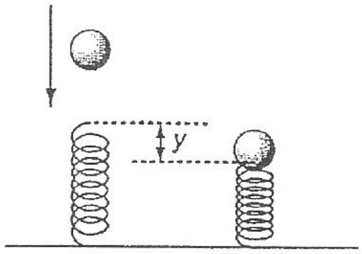
Solution Answer: **A**

Gain in kinetic energy of the *whole system*
= loss in gravitational potential energy of the 0.50 kg mass
= mgh
= (0.50 kg)(9.81 N kg^{-1})(1.0 m)
= 4.905 J.

Maximum kinetic energy of the trolley
= [(1.5 kg)/(1.5 kg + 0.50 kg)] x 4.905 J
= (0.75 x 4.905 J) =3.679 = 3.7 J.

7 A ball of mass m falls freely from rest. When it has reached a speed v, it strikes a vertical spring.

The spring is compressed by a distance y before the ball moves upwards again.



Assume that all the energy the ball loses becomes elastic potential energy in the spring.

What is the average force exerted by the spring during its compression?

A $\frac{mv^2}{2y}$ C $\frac{mv^2}{y}$

B $\frac{m}{2y}(v^2 - 2gy)$ D $\frac{m}{2y}(v^2 + 2gy)$

N06/I/7

Solution Answer: **D**

Applying the law of conservation of energy, the work done against elastic force equals the sum of the loss in kinetic energy and loss in gravitational potential energy:

$$Fy = ½mv^2 + mgy$$

$$F = (½mv^2 + mgy) / y = (m/2y)(v^2 + 2gy)$$

8 Which word equation is **not** required to derive power as the product of force and velocity?

A force = mass × acceleration

B power = $\frac{\text{work done}}{\text{time}}$

C velocity = $\frac{\text{displacement}}{\text{time}}$

D work done = force × displacement in the direction of the force

N06/I/8

Solution Answer: **A**

The first word equation is **not** required.
The other word equations are required, as follows:
Power = work done / time
= (force x displacement) / time = force x velocity.

9 A car of mass 1.2 x 10^3 kg travels along a horizontal road at a speed of 10 m s^{-1}. It then accelerates at 0.20 m s^{-2}. At the time it begins to accelerate, the total resistive force acting on the car is 160 N.

What total output power is developed by the car as it begins the acceleration?

A 0.80 kW **B** 1.6 kW **C** 2.4 kW **D** 4.0 kW

N07/I/10

Solution Answer: **D**

Apply Newton's 2nd law of motion

$$F_{engine} - F_{resist} = ma$$
$$F_{engine} - 160\text{ N} = (1.2 \times 10^3\text{ kg})(0.20\text{ m s}^{-2})$$
$$F_{engine} - 160\text{ N} = 240\text{ N}$$
$$F_{engine} = 400\text{ N}$$

Power developed

$$P = F_{engine}v = (400\text{ N})(10\text{ m s}^{-1}) = 4000\text{ W}$$

10 A bungee jumper has 24 kJ of gravitational potential energy at the top of his jump. He is attached to an elastic rope which starts to stretch after a short time of free fall. The values of gravitational potential energy, elastic potential energy and kinetic energy are given for the top and the bottom of the jump.

	gravitational potential energy / kJ	elastic potential energy / kJ	kinetic energy / kJ
top	24	0	0
bottom	0	24	0

Which row of the table below shows possible values of these three energies when the jumper is half-way down? Losses of energy through air resistance are negligible.

	gravitational potential energy / kJ	elastic potential energy / kJ	kinetic energy / kJ
A	12	10	2
B	12	8	4
C	8	8	8
D	12	2	10

N07/I/11

Solution Answer: **D**

Gravitational potential energy (GPE) is directly proportional to distance travelled (GPE = mgh) hence when the jumper is half-way down, his GPE is half of 24 kJ = 12 kJ.

Elastic potential energy (EPE) is proportional to the square of extension x (EPE = $\frac{1}{2}kx^2$). In the extreme case whereby the elastic rope stretches immediately the moment he jumps, the EPE when he is half-way down will be $(\frac{1}{2})^{1/2}(24\text{ kJ}) = 6$ kJ. Since the rope stretches after a short time of free fall, the EPE stored in it would not have reached 6 kJ yet when the jumper is half-way down.

The only possible answer is D.

11 The driving force F of a car of mass m causes the car to accelerate. In a time t it travels a distance s and its speed increases from u to v.

What is the useful work done by the car engine?

A $\frac{Fs}{t}$ B $m(v-u)$ C Ft D $\frac{m(v^2-u^2)}{2}$

N08/I/9

Solution Answer: **D**

Useful work done is equal to the gain in the kinetic energy of the car = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$.

It can also be noted that the other three answers (A, B and C) do not even have the units of energy.

Answer A is power (which has unit of watt W), while answers B and C are impulse (which has unit of kg m s^{-1}).

12 A driving force of 200 N is needed for a car of mass 800 kg to travel along a level road at a speed of 20 m s^{-1}.

What power is required to maintain the car at this speed up a gradient in which the car rises 1 m for each 8 m of travel along the road?

A 6.0 kW B 7.2 kW C 20 kW D 24 kW

N09/I/12

Solution Answer: **D**

The power required to maintain the car at this speed along a level road is $P = Fv$ = 200 N x 20 m s^{-1} = 4000 W or 4.0 kW.

If the car rises 1 m for each 8 m along the road, it will rise 2.5 m for each 20 m within a time of 1 s.

The rate of gain in its gravitational potential energy is mgh = (800 kg)(9.81 N kg^{-1})(2.5 m) = 19,620 J s^{-1} ≈ 20 kW.

The total power required is 4.0 kW + 20 kW = 24 kW.

13 A speed-boat with two engines, each of power output 36 kW, can travel at a maximum speed of 12 m s^{-1}. The total drag D on the boat is related to the speed v of the boat by the equation shown.

$$D \propto v^2$$

What is the maximum speed of the boat when only one engine is working?

A 3.0 m s^{-1} B 6.0 m s^{-1} C 8.5 m s^{-1} D 9.5 m s^{-1}

N09/I/13

Solution Answer: **D**

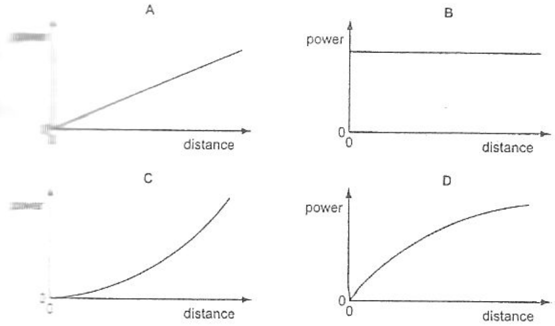
At a constant speed (with no resultant force), the engine force = drag force.

Hence Power $P = Fv = Dv = kv^3$ where k is a constant of proportionality.

Since $P \propto v^3$, then $v \propto P^{(1/3)}$.

When P is halved, v becomes $(\frac{1}{2})^{(1/3)}$ or 0.794 of its initial value, i.e. 0.794 x 12 = 9.5 m s^{-1}.

14 A vehicle starts from rest and accelerates uniformly. Which graph shows how the power output of the vehicle varies with distance travelled?



N10/I/11

Solution Answer: **D**

Power output $P = Fv$ where force F is constant since according to Newton's second law of motion $F = ma$ and acceleration a is uniform. From the kinematics equation $v^2 = u^2 + 2as$, since the vehicle started from rest, $u = 0$ hence $v^2 = 2as$ or $v = \sqrt{2as}$. Substituting this expression for v into the power equation, power $P = (ma)(\sqrt{(2as)}) = \sqrt{(2m^2a^3s)}$. As m and a are constants, $P \propto \sqrt{s}$.

Short Questions

15 Energy plays a key part in all branches of Physics. Discuss the energy changes which take place in the following systems. In each case you need to refer to the different forms of energy involved.

(a) A mass suspended from a spring and oscillating vertically.

(b) A nucleus undergoing radioactive decay.

N07/III/4

Solution

(a) When the mass is at the top of its oscillation, it has maximum gravitational potential energy (GPE) (Since GPE = mgh and the height h is maximum), minimum elastic potential energy (EPE) (Since EPE = $\frac{1}{2}kx^2$ and the extension x is minimum) and zero kinetic energy (KE) (Since KE = $\frac{1}{2}mv^2$ and velocity v is zero).

As the mass accelerates downward towards its equilibrium position, it losses GPE (decreasing height h), gains EPE (increasing extension x) and gains KE (increasing velocity v). Loss in GPE = Gain in KE + Gain in EPE.

As the mass passes its equilibrium position, it has lost half its total GPE (taking its lowermost position as the zero reference for GPE). Of this GPE lost, half is converted into KE (maximum at this position) and the other half stored as EPE (greater extension x).

After the equilibrium position, the tension in the spring decelerates the mass. The mass slows down and loses KE (decreasing v), continues to lose GPE (decreasing height h), but gains EPE (increasing extension x). Gain in EPE = Loss in GPE + Loss in KE.

Finally at the lowermost position, its GPE = 0 (taking the lowermost position as the zero reference for GPE and the height h = 0), its KE is zero (v = 0) and its EPE is maximum (maximum extension x). Its entire GPE lost is converted into EPE.

In the case where resistive forces (e.g. air resistance and dissipative forces within the spring) are present, the oscillating system will experience damping and progressively lose total energy due to work done against these resistive forces. This energy will be dissipated as heat, which then raises the internal energy of the spring and the surrounding.

(b) An unstable nucleus can undergo radioactive decay to emit an alpha particle, beta particle or gamma radiation.

In alpha and beta decay, the stationary radioactive nucleus disintegrates into two parts: a daughter nucleus and either an alpha or beta particle.

The products (daughter nucleus plus either alpha or beta particle) have a total rest mass lower than that of the radioactive nucleus. The difference Δm is released as energy according to Einstein's mass-energy equivalence equation $E = \Delta mc^2$ where c is the speed of light in free space. Most of this energy is released as the kinetic energy of the products, with the remainder as electromagnetic radiation mainly in the form of gamma radiation.

As mass decreases, binding energy increases hence the products have a higher binding energy and are more stable than the radioactive nucleus.

The gamma radiation comes from the de-excitation of the daughter nucleus formed in alpha or beta decay. The daughter nucleus, which is in an excited state, de-excites and falls to a lower energy state. The energy difference is emitted as gamma radiation (radiation energy).

The energy of the gamma ray photon emitted is equal to the difference in the quantised energy levels within the nucleus, which are considerably larger than those of the atomic-orbital electrons system.

16 A stone of mass 130 g is thrown horizontally from the top of a cliff of height 32 m, as illustrated in Fig. 1.1.

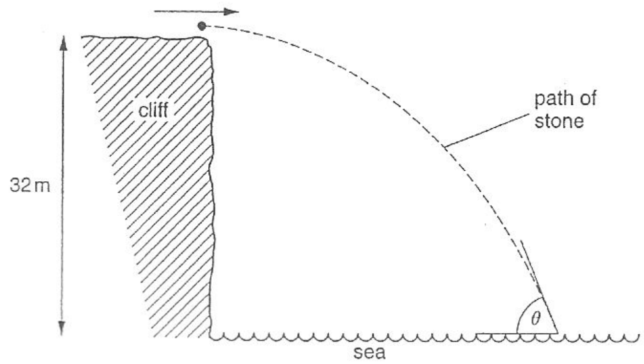


Fig. 1.1

Air resistance is negligible. The stone enters the sea with a speed of 34 m s^{-1}.

(a) Determine, for the stone as it hits the sea,

(i) the vertical component of the velocity of the stone,

vertical component of velocity = ... m s^{-1} [2]

(ii) the angle θ to the horizontal of the stone's path.

θ = ... ° [2]

(b) On hitting the sea, the speed of the stone is reduced from 34 m s^{-1} to 2.0 m s^{-1} in a time of 0.95 s. Use momentum considerations to determine the average force on the stone during this time.

force = ... N [2]

(c) Use energy considerations to suggest why, if the stone causes a large splash on hitting the sea, it will be slowed down in a shorter distance than when no splash is produced.

....

....

.... [2]

N08/II/1

Solution

(a) (i) Use the kinematics equation, applied to the vertical direction,

$$v_y^2 = u_y^2 + 2a_y s_y$$

where v_y is the final vertical component of the stone's velocity, u_y the initial vertical component of the stone's velocity (0 m s^{-1} since the stone was thrown horizontally), a_y the vertical component of the stone's acceleration (g) and s_y the vertical component of the stone's displacement (32 m).

$$v_y^2 = u_y^2 + 2gs_y$$

$$v_y^2 = 0^2 + 2(9.81 \text{ m s}^{-2})(32 \text{ m}) = 627.84 \text{ m}^2 \text{ s}^{-2}$$

$$v_y = 25.06 = 25 \text{ m s}^{-1}$$

(ii) The vector diagram including the final velocity and its components is shown in Fig. 1.2 below.

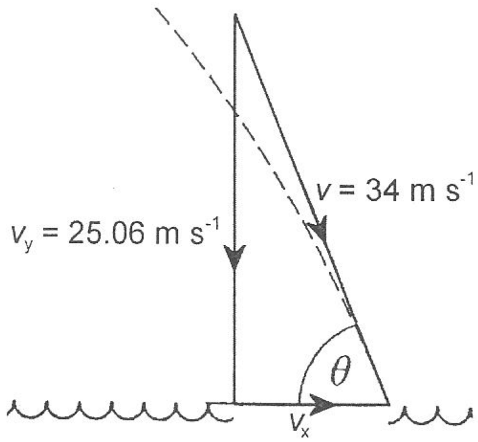


Fig. 1.2

$$\sin \theta = v_y/v = 25.06/34 = 0.7371$$

$$\theta = 47.5°$$

(b) According to Newton's second law of motion, the resultant force F acting on a body is equal to the rate of change of its momentum.

$$F = (mv - mu)/t$$

where m is the mass of the body, v and u its final and initial velocities and t the time taken for the change in velocity to take place.

$$F = [(0.130 \text{ kg x } 2.0 \text{ m s}^{-1}) - (0.130 \text{ kg x } 34 \text{ m s}^{-1})]/0.95 \text{ s}$$

$$= [0.26 - 4.42] \text{ kg m s}^{-1} / 0.95 \text{ s} = -4.379 = -4.4 \text{ N}$$

The negative sign denotes that this is a decelerating force which opposes the direction of the stone's motion.

The magnitude of the force is 4.4 N.

(c) If the stone causes a large splash, part of its kinetic energy is converted into the kinetic energy of the water spray; part into thermal energy due to collision with the water molecules; and part into sound energy.

With less kinetic energy, the stone will thus be able to do less work against the resistive force acting against its motion.

Since work done against resistive force = average resistive force x distance moved against the force, the stone will be slowed down in a shorter distance.

(b) Fig. 1.3 shows the variation with displacement d of the force F applied to an object. F and d are in the same direction.

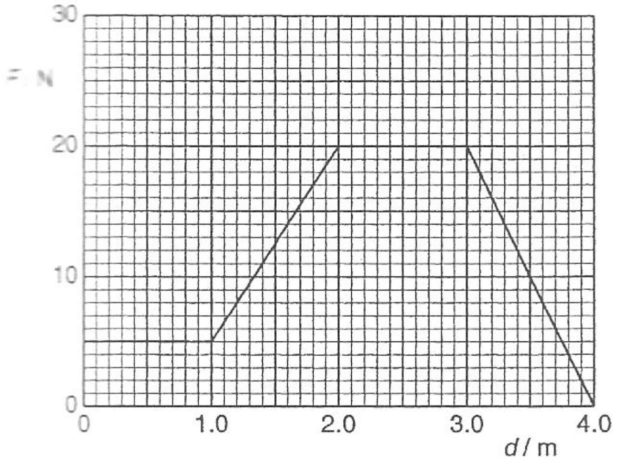


Fig. 1.3

On Fig. 1.4, draw a graph showing the variation with d of the work done.

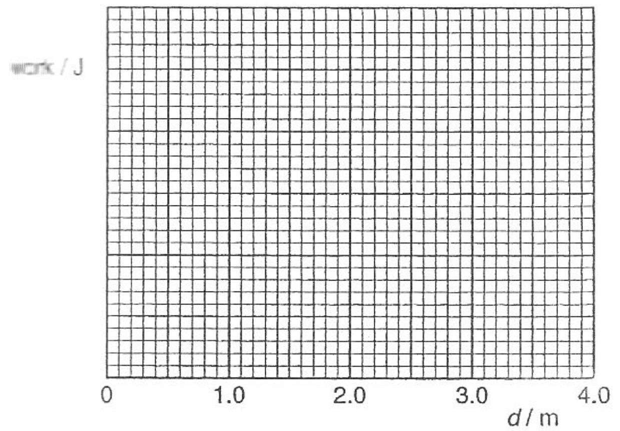


Fig. 1.4 [4]

N08/III/1(b)

Solution

(b) The graph showing variation with distance d of the work done is shown if Fig. 1.7 below.

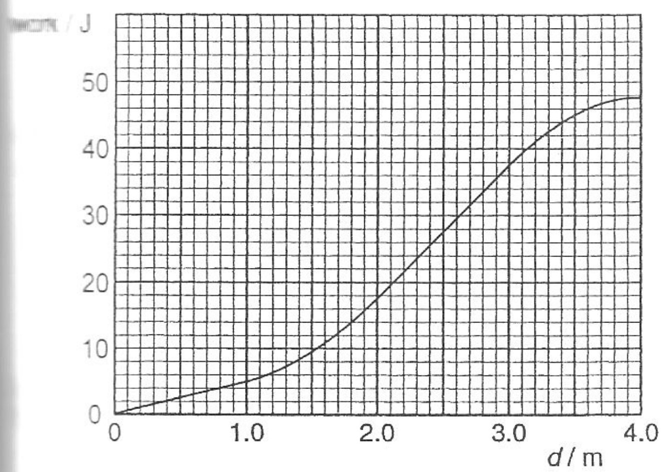


Fig. 1.7

Work done $W = \int F\, dd$ (i.e. area under the force distance graph). The total area under the graph is 47.5 J, hence the final work done at the end of 4.0 m is 47.5 J. The critical points are tabulated below.

Distance d / m	Work Done / J
1.0	5.0
2.0	17.5
3.0	37.5
4.0	47.5

Conversely, force $F = dW/dd$ hence force F is the gradient of the Work Done vs Distance graph.

TOPIC 6 Motion in a Circle

1 An object is travelling in a circle of radius r with angular velocity ω and speed v.

Which expression gives the centripetal acceleration?

A $r\omega$ C $\frac{v}{r}$

B $v\omega$ D $\frac{v}{r^2}$ N03/I/8

Solution Answer: **B**

The centripetal acceleration $a = v^2/r = v(v/r) = v\omega$.

2 The maximum safe speed of a car rounding an unbanked corner is 20 m s^{-1} when the road is dry. The maximum frictional force between the road surface and the wheels of the car is halved when the road is wet.

What is the maximum safe speed for the car to round the corner when the road is wet?

A $\frac{20}{4}$ m s^{-1}

B $\frac{20}{2\sqrt{2}}$ m s^{-1}

C $\frac{20}{2}$ m s^{-1}

D $\frac{20}{\sqrt{2}}$ m s^{-1} N03/I/9

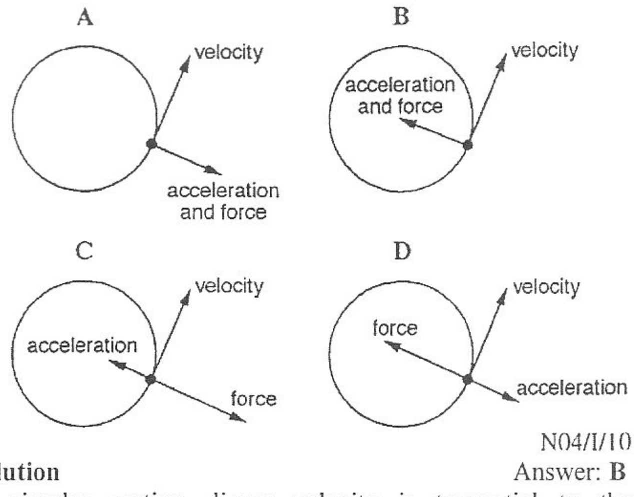
Solution Answer: **D**

The frictional force (F) provides the centripetal force: $F = mv^2/r$.

Therefore $F \propto v^2$ and $v \propto \sqrt{F}$ hence if F is halved, v will become $v/\sqrt{2}$.

3 A child on a roundabout is travelling in a horizontal circle with a constant speed. The child's velocity, acceleration and the resultant horizontal force on the child are all vectors.

Which diagram shows the correct directions for these vectors?



N04/I/10

Solution Answer: **B**

In circular motion, linear velocity is tangential to the circumference of the circle, while the centripetal acceleration and centripetal force are both directed towards the centre of the circular motion.

4 A body rotates with uniform speed in a circle of radius r and takes time T to complete one revolution.

What are the magnitudes of the angular velocity ω, the linear velocity v, and the acceleration a?

	angular velocity ω	linear velocity v	acceleration a
A	$\frac{1}{T}$	$\frac{4\pi r}{T}$	$\frac{2\pi r}{T^2}$
B	$\frac{2\pi}{T}$	$\frac{2\pi r}{T}$	$\frac{2\pi r}{T^2}$
C	$\frac{2\pi}{T}$	$\frac{2\pi r}{T}$	$\frac{4\pi^2 r}{T^2}$
D	$\frac{2\pi}{T}$	$\frac{4\pi r}{T}$	$\frac{4\pi^2 r}{T^2}$

N05/I/1

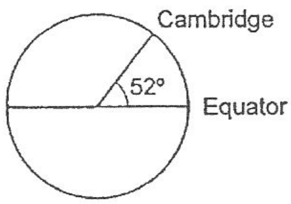
Solution Answer: C

Angular velocity $\omega = 2\pi/T$.

Linear velocity $v = r\omega = r(2\pi/T) = 2\pi r/T$.

Centripetal acceleration $a = r\omega^2 = r(2\pi/T)^2 = 4\pi^2 r/T^2$.

5 Singapore is on the Equator. Cambridge is at a latitude of 52° N, as shown in the diagram.



A student at Singapore has a centripetal acceleration a_s because of the Earth's rotation about its axis. The centripetal acceleration of another student at Cambridge is a_c.

What are the magnitudes of the centripetal accelerations?

(radius of Earth = 6.4×10^6 m; angular velocity of Earth about axis = 7.3×10^{-5} rad s^{-1}.)

	a_s/ms^{-2}	a_c/ms^{-2}
A	3.4×10^{-2}	2.1×10^{-2}
B	3.4×10^{-2}	2.7×10^{-2}
C	3.4×10^{-2}	3.4×10^{-2}
D	4.7×10^{2}	4.7×10^{2}

N06/I/

Solution Answer: A

The centripetal acceleration at Singapore is:

$a_s = r\omega^2 = (6.4 \times 10^6 \text{ m})(7.3 \times 10^{-5} \text{ rad s}^{-1})^2 = 0.034 \text{ m s}^{-2}$.

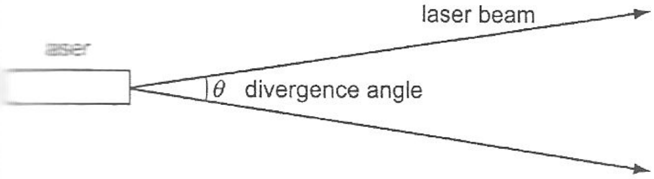
The radius of rotation at Cambridge is:

$r \cos 52° = (6.4 \times 10^6 \text{ m}) \cos 52° = 3.940 \times 10^6$ m.

The centripetal acceleration at Cambridge is:

$a_c = r\omega^2 = (3.940 \times 10^6 \text{ m})(7.3 \times 10^{-5} \text{ rad s}^{-1})^2 = 0.021 \text{ m s}^{-2}$.

An astronomer points a powerful laser beam at the Moon, a distance R away. The beam has a very small divergence angle θ, as shown in the diagram (which exaggerates greatly the size of the angle).



The astronomer looks up a value for the distance R. This value is in kilometres. He measures the angle θ in degrees.

What is the diameter, in metres, of the circle of light the laser produces on the Moon?

A $1.75 \times 10^{-5}\, R\,\theta$

B $17.5\, R\,\theta$

C $1000\, R\,\theta$

D $5.73 \times 10^{4}\, R\,\theta$

N07/I/12

Solution Answer: **B**

The angle in radians is $(\pi/180)\theta = 0.0175\theta$. The distance in metres is $1000R$.

Since the angle is small, the small angle formula

$$s = 1000R(0.0175\theta) = 17.5\, R\,\theta$$

can be applied where s is the arc length (equal to the diameter of the circle of light).

An oscillating pendulum bob of mass m and weight W is supported by a string of length r.

What is the relationship between W and tension T in the string at the instant when the string is vertical and the bob is moving with speed v?

A $T = W$

B $T - W = \dfrac{v^2}{r}$

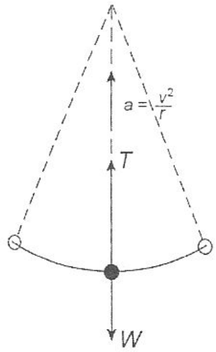
C $T - W = m\dfrac{v^2}{r}$

D $W - T = m\dfrac{v^2}{r}$

N07/I/13

Solution Answer: **C**

Refer to the figure below.



The tension T is pointing upwards (towards the centre of the circle) while the weight W is pointing downwards. The resultant force $T - W$ provides the centripetal force mv^2/r. Hence,

$$T - W = mv^2/r$$

8 A part in an engine is rotating in a circle of radius 8.0 cm at 3000 revolutions per minute.

What is its centripetal acceleration?

A $25\ \mathrm{m\,s^{-2}}$

B $7900\ \mathrm{m\,s^{-2}}$

C $3.1 \times 10^{4}\ \mathrm{m\,s^{-2}}$

D $7.2 \times 10^{7}\ \mathrm{m\,s^{-2}}$

N08/I/11

Solution Answer: **B**

Centripetal acceleration

$$a = r\omega^2 = r(2\pi f)^2 = (0.080\ \mathrm{m})(2\pi\ 3000/60)^2 = 7900\ \mathrm{m\,s^{-2}}$$

9 A pendulum bob of mass 1.27 kg is supported by a string so that the radius of its path is 0.600 m. It is moving with velocity $0.575\ \mathrm{m\,s^{-1}}$ horizontally at the centre of its motion when the string is vertical.

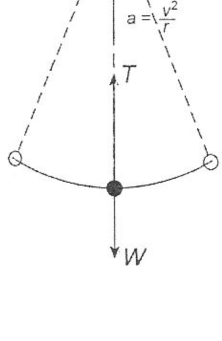
What is the tension in the string at this instant?

A 11.8 N **B** 12.5 N **C** 13.2 N **D** 13.7 N

N08/I/12

Solution Answer: **C**

Refer to the figure below.



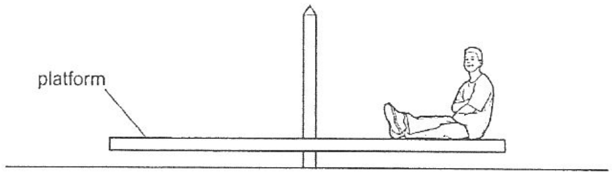
The tension T is pointing upwards (towards the centre of the circle) while the weight W is pointing downwards. The resultant force $T - W$ provides the centripetal force mv^2/r. Hence,

$$T - W = mv^2/r$$

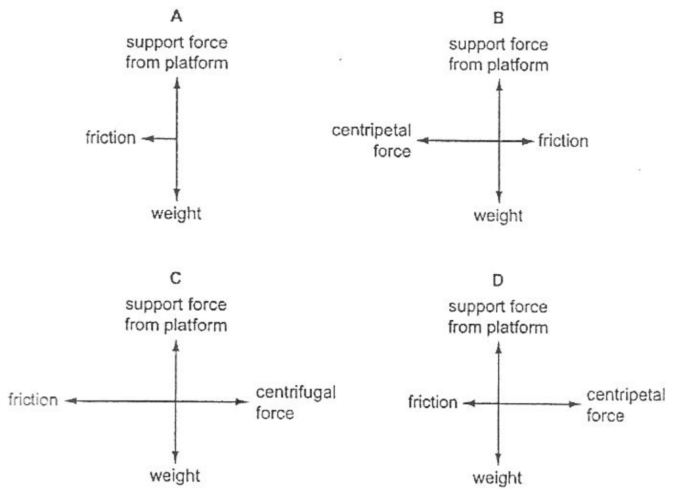
$$T = W + mv^2/r = mg + mv^2/r = m(g + v^2/r)$$

$$= (1.27 \text{ kg})[9.81 \text{ N kg}^{-1} + (0.575 \text{ m s}^{-1})^2/0.600 \text{ m}] = 13.2 \text{ N}$$

10 The diagram shows a child sitting on a playground turntable, which is turning with constant angular velocity.



Which diagram shows the forces acting on the child when in the position shown?



N09/I/14

Solution Answer: **A**

Friction provides the centripetal force required to keep the child in circular motion. There is no need to draw the centripetal force separately in the force diagram.

The centripetal force (which is provided by friction) acts towards the centre of the turntable.

11 What is the linear speed of a point on the Earth's equator as a result of the Earth's rotation about its axis? (radius of the Earth = 6.38×10^6 m)

A 2.16×10^{-3} m s^{-1}
B 4.31×10^{-3} m s^{-1}
C 232 m s^{-1}
D 464 m s^{-1}

N09/I/15

Solution Answer: **D**

The linear speed $v = r\omega$ where r is the radius of rotation, and $\omega = (2\pi/T)$ the angular velocity of the Earth's rotation about its axis.

$$v = r\omega = r\,(2\pi/T)$$

$$(6.38 \times 10^6 \text{ m})(2\pi / (24 \times 60 \times 60 \text{ s})) = 464 \text{ m s}^{-1}$$

12 A stone rotates in a horizontal circle with constant angular velocity ω.
What changes occur in the linear speed and in the centripetal acceleration of the stone as the radius of the circle increases?

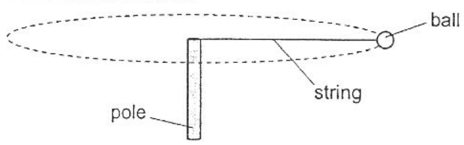
	linear speed	centripetal acceleration
A	constant	decrease
B	constant	increase
C	increase	constant
D	increase	increase

N10/I/12

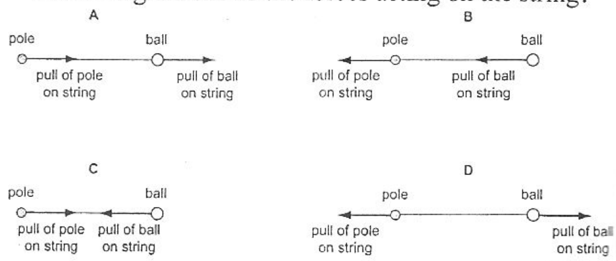
Solution Answer: **D**

The linear speed $v = r\omega$ and the centripetal acceleration $a = r\omega^2$ where r is the radius of the circle. Hence both v and a are directly proportional to r if ω is constant.

13 A tennis ball is attached to a light string that is fixed to a pole. The tennis ball travels round the pole in a horizontal circle at a constant speed that is high enough for the string to be almost horizontal.

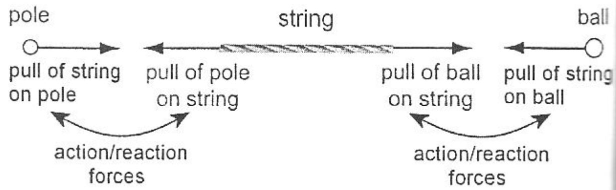


Which diagram shows the forces acting on the string?



N10/I/1

Solution Answer: D



Short Questions

1 (a) Distinguish between vector and scalar quantities. [1]

(b) Define *acceleration.* [1]

(c) Fig. 1.1. shows the path of a car as it travels on a curved path.

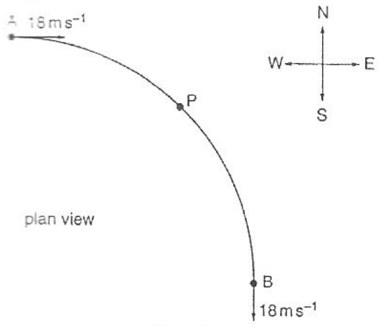


Fig. 1.1

The car travels at a constant speed of 18 m s^{-1}. At A the direction of the car is from west to east, and at B the direction is from north to south. The time taken for the car to travel from A to B is 4.4 s.

(i) Determine the magnitude of the change in velocity of the car.
change in velocity = ... m s^{-1} [2]

(ii) Calculate the acceleration of the car between A and B.
acceleration = ... m s^{-2} [1]

(iii) The path of the car is part of a circle. On Fig. 1.1, draw an arrow to show the direction of the resultant force that acts on the car as it passes through point P. [1]

(iv) Explain how a resultant force acts on the car to cause the acceleration, but there is no change to the kinetic energy of the car.

....
....
.... [2]

N09/II/1

Solution

(a) Vector quantities have both magnitude and direction. Scalar quantities have only magnitude but no direction.

(b) The *acceleration* of an object is the rate of change of its velocity.

(c) (i) The vector diagram relating the initial velocity, final velocity and change in velocity is shown Fig. 1.2 below.

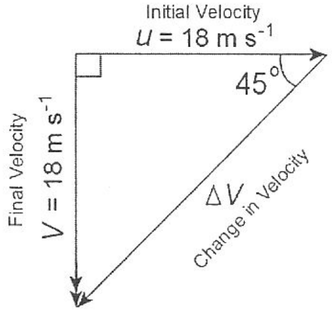


Fig. 1.2

initial velocity + change in velocity = final velocity

$$u + \Delta v = v$$

The magnitude of the change can be found from Pythagoras' theorem as

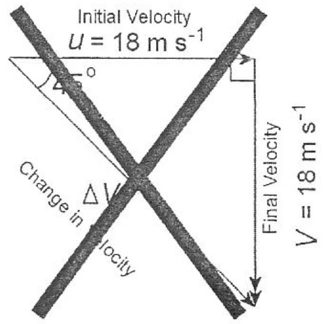
$$\sqrt{(u^2 + v^2)} = \sqrt{(18^2 + 18^2)} = 25.46 \text{ m s}^{-1}$$

change in velocity = <u>25 m s^{-1}</u>

*Note: in the working, the direction of the change is 45° South of West and **<u>NOT</u>** 45° South of East.*

This question is similar to the one in June 1989 Paper II Question 8.

There is a very high tendency amongst new JC1 students to draw the following diagram, which is wrong (although the magnitude will be the same as the correct answer, the direction is wrong).



(ii) Acceleration = change in velocity / time
$a = (25.46 \text{ m s}^{-1} / 4.4 \text{ s}) = 5.786 \text{ m s}^{-2}$

acceleration = <u>5.8 m s^{-2}</u>

(iii) The arrow is shown in Fig. 1.3.

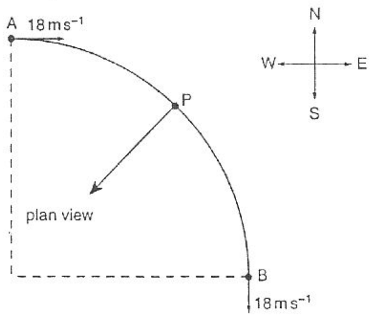


Fig. 1.3

(iv) The resultant force is perpendicular to the velocity. From Newton's second law of motion, the direction of the acceleration is the same as that of the resultant force, and hence also perpendicular to the velocity. This centripetal acceleration thus changes only the direction, but not the magnitude of the velocity. Since the kinetic energy of a body $E_k = ½mv^2$ where m is its mass and v its speed, its kinetic energy remains unchanged if its speed is unchanged.

Long Questions

15 *(a)* An object rotates in a vertical circle. Which of the following quantities are constant when the object has constant speed?

PERIOD, FREQUENCY, ANGULAR VELOCITY, VELOCITY, ACCELERATION, KINETIC ENERGY, POTENTIAL ENERGY. [4]

(b) Fig. 2.1 shows a ride in an amusement park.

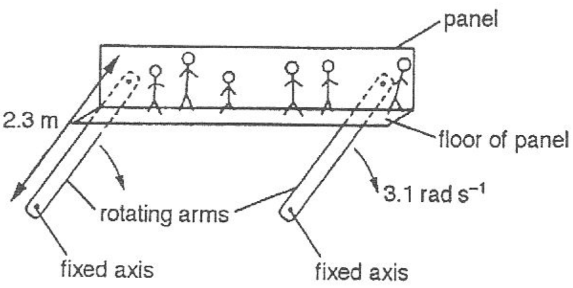


Fig. 2.1 *(a)*

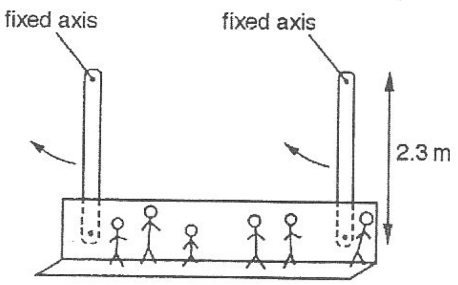


Fig. 2.1 *(b)*

The passengers are strapped firmly to a vertical panel. The floor of the panel stays horizontal while it is being moved by two arms rotating with angular velocity 3.1 rad s^{-1}. Each passenger rotates in a vertical circle of radius 2.3 m. Calculate

(i) the time taken for one revolution of the arms,

(ii) the linear speed of each passenger,

(iii) the centripetal acceleration of each passenger. [5]

(c) Consider a person of mass 60 kg on the ride described in *(b)* at the instant when the panel is at the bottom of the path, as shown in Fig. 2.1 *(b)*.

(i) State the direction of the acceleration of this person.

(ii) Calculate the resultant force necessary to cause this acceleration.

(iii) Draw a force diagram showing the weight W of the person and the force P which the panel exerts on the person.

(iv) Calculate P. [7]

(d) Using energy considerations, suggest why it is difficult to drive a fairground ride such as described in *(b)* at a constant speed. [4]

N02/III/2

Solution

(a) The following are constant: PERIOD, FREQUENCY, ANGULAR VELOCITY, KINETIC ENERGY.

(b) (i) Period $T = 2\pi/\omega = 2\pi/(3.1 \text{ rad s}^{-1}) = 2.027 = 2.0$ s.

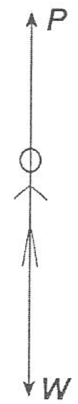
(ii) Linear speed
$v = r\omega = (2.3 \text{ m})(3.1 \text{ rad s}^{-1}) = 7.13 = 7.1 \text{ m s}^{-1}$.

(iii) Centripetal acceleration
$a = r\omega^2 = (2.3 \text{ m})(3.1 \text{ rad s}^{-1})^2 = 22.10 = 22 \text{ m s}^{-2}$.

(c) (i) Upwards.

(ii) Resultant force
$F = ma = (60 \text{ kg})(22.10 \text{ m s}^{-2}) = 1326 \text{ N} = 1300 \text{ N}$.

(iii) The force diagram is shown in the figure below.



(iv) $P - W$ = centripetal force = 1326 N
$P - mg = 1326$ N
$P - (60 \text{ kg})(9.81 \text{ N kg}^{-1}) = 1326$ N
$P = 1915 = 1900$ N.

(v) As the panel rises and falls, it gains and lose gravitational potential energy (G.P.E.) respectively. There is a tendency for the platform to lose kineti energy and slow down on the way up, and gai kinetic energy and speed up on its way down.
To avoid this, the motor's power output has to b increased on its way up and reduced on its way dow.
To complicate matters, the rate of gain or lose i G.P.E. is not uniform throughout the motion, varyi from zero when the arms are vertical, to maximu when they are horizontal.
To maintain a constant speed would require ve precise control of motor power output usi computerized controllers.

16 (a) An object travelling in a circle of radius r at constant speed v is accelerating. By drawing a vector diagram to show the combination of vectors, explain how this is possible. [2]

(b) An object P moves at a constant speed v through an arc of a circle of radius r. The arc subtends an angle 0.010 rad at the centre of the circle, as shown in Fig. 3.1.

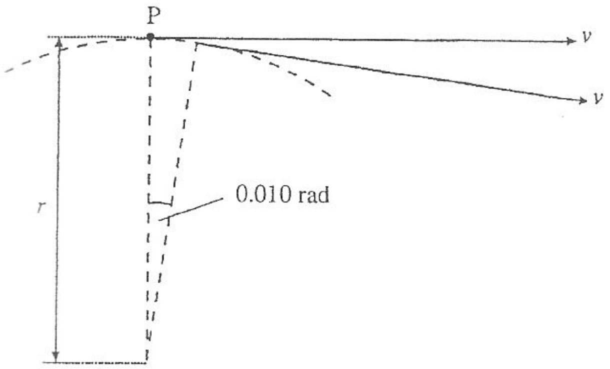


Fig. 3.1 (not to scale)

(i) Determine, in terms of v, the magnitude of the change in velocity. [1]

(ii) State the direction of the acceleration of P. [1]

(iii) Deduce, in terms of r and v, the time taken for P to travel 0.010 rad. [2]

(iv) Hence show that the magnitude of the acceleration of P is $\frac{v^2}{r}$. [1]

(c) A theme park ride is illustrated in Fig. 3.2. The carriages accelerate down the slope and then loop the loop on a circular section of track. The radius of the circular section of track is 8.6 m.

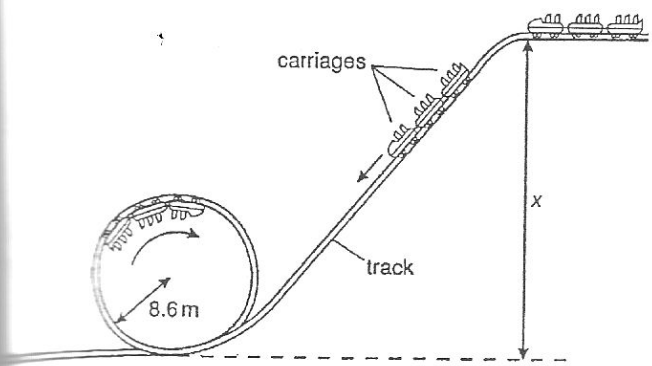


Fig. 3.2

(i) Find the minimum speed of the carriages at the top of the circular track so that the carriages remain in contact with the track. [2]

(ii) In practice, it is essential for designers to build in a considerable safety margin. Each carriage and its passengers has total mass 800 kg. At the top of the loop, the carriage travels at 17 m s^{-1}. Calculate the force that the track exerts on the carriage at the top of the loop. [3]

(d) For the carriage in *(c)* to have a speed of 17 m s^{-1}, at the top of the loop, it must have fallen from a height of at least x.

(i) Deduce a value for x. [3]

(ii) State an assumption that you made in making this deduction. [1]

*(e) Forces acting on bodies which travel in a circle are responsible for the following. Suggest an explanation for each.

(i) A string may snap if it is attached to an object and the object is spun around a vertical pole as shown in Fig. 3.3. [2]

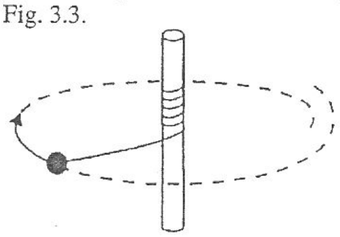


Fig. 3.3

(ii) The rings of Saturn are centred on the centre of the planet, as shown in Fig. 3 4. [2]

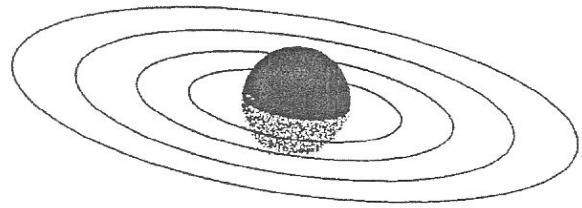


Fig. 3.4

N06/III/3

Solution

(a) The vector diagram is shown in Fig. 3.5 below.

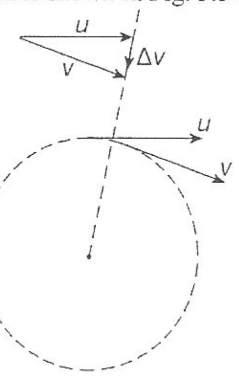


Fig. 3.5

In Fig. 3.5, u represents the object's velocity at one instant and v its velocity an instant later.

Velocity vectors u and v can be arranged to form a vector triangle where it can be seen that there is a change in velocity Δv even though there is no difference between the magnitude of u and v.

$$\Delta v = v - u$$

It can be further seen that the direction of this change in velocity Δv is towards the centre of the circle, resulting in a centripetal acceleration towards the centre of the circle.

(b) (i) Refer to Fig. 3.6 below.

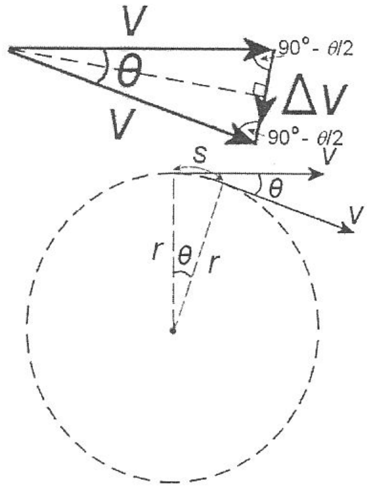


Fig. 3.6

Applying the sine rule

$$v / \sin (90^\circ - \theta/2) = \Delta v / \sin \theta$$

If θ is small, then $\sin (90^\circ - \theta/2) \approx \sin (90^\circ) = 1$ and $\sin \theta \approx \theta$ if θ is expressed in radians.

$$v = \Delta v / \sin \theta$$

$$\Delta v = v\theta = 0.010\ v$$

(ii) The direction of the acceleration is towards the centre of the circle.

(iii) The tangential displacement $s = r\theta$.
The time taken $t = s/v = r\theta/v = 0.010\ r/v$.

(iv) Acceleration $a = \Delta v/t = (0.010\ v) / (0.010\ r/v) = v^2/r$.

(c) (i) At the top of the track, the downward reaction force exerted by the track on the carriages and the weight of the carriages provide the centripetal force required for their circular motion (Fig. 3.7).

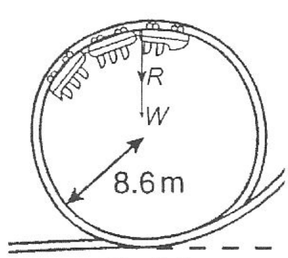


Fig. 3.7

$$R + W = mv^2/r$$
$$R + mg = mv^2/r$$
$$R = mv^2/r - mg$$

For the carriages to remain in contact with the track, the contact force $R \geq 0$ hence,

$$mv^2/r - mg \geq 0$$
$$v^2/r \geq g$$
$$v^2 \geq rg$$
$$v \geq \sqrt{(rg)}$$

In the limiting condition when $v = \sqrt{(rg)}$, the carriages are just about to fall off the track since $R = 0$. Hence the minimum speed required for the carriages to remain in contact with the track is $\sqrt{(rg)} = \sqrt{(8.6 \text{ m} \times 9.81 \text{ m s}^{-2})} = 9.185 = 9.2 \text{ m s}^{-1}$.

(ii) Applying the same equation as in (i),

$$R = mv^2/r - mg$$
$$= (800 \text{ kg})(17 \text{ m s}^{-1})^2/(8.6 \text{ m}) - (800 \text{ kg})(9.81 \text{ N kg}^{-1})$$
$$= 1.904 \times 10^4 = 1.9 \times 10^4 \text{ N}$$

(d) (i) Apply the law of conservation of energy,

Loss in gravitational potential energy = gain in kinetic energy,

$$mg(x - 2(8.6 \text{ m})) = \tfrac{1}{2}mv^2$$
$$(9.81 \text{ m s}^{-2})(x - 2(8.6 \text{ m})) = \tfrac{1}{2}(17 \text{ m s}^{-1})^2$$
$$x = 31.93 = 32 \text{ m}$$

(ii) It is assumed that the track is frictionless so that all the loss in gravitational potential energy is converted into kinetic energy.

(e) (i) As the string wraps round the pole, the radius of the circular orbit becomes progressively smaller.

Assuming no loss in kinetic energy of the object due to air resistance, its speed will remain constant.

The tension of the string, which provides the centripetal force for the object's circular motion, is given by: $T = mv^2/r$

With a constant mass m and speed v but a progressively smaller orbital radius r, the tension becomes progressively greater. The string may snap if this tension exceeds the string's breaking strength.

(ii) The gravitational force acting on the rings provide the centripetal force for their circular motion about the planet.

Since the gravitational force acts towards the centre of the planet while the centripetal force acts towards the centre of the orbit, the plane of the ring's orbit must contain the centre of the planet.

Any asteroid which orbits about a circle not centred on the centre of the planet will experience a force which moves it towards the equatorial plane shown in Fig. 3.4.

17 A small ball of mass m is fixed to one end of a light rigid rod. The ball is made to move at constant speed around the circumference of a vertical circle with centre at C, as shown in Fig. 2.1.

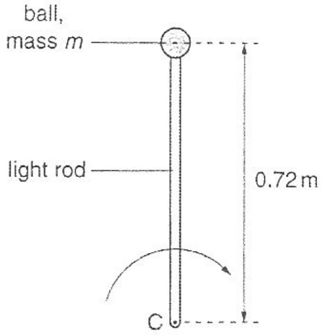


Fig. 2.1

When the rod is vertical with the ball above C, the tension T in the rod is given by

$$T = 2\ mg$$

where g is the acceleration of free fall.

(a) (i) Explain why the centripetal force on the ball is greater than 2 mg. [1]

(ii) State, in terms of mg, the magnitude of the centripetal force.
centripetal force = ... [1]

(iii) Determine the magnitude of the tension, in terms of mg, in the rod when the rod is vertical, with the ball below point C.
tension = ... [1]

(b) The distance from the centre of the ball to point C is 0.72 m.
Use your answer in (a)(ii) to determine, for the ball,

(i) the angular speed,
angular speed = ... rad s^{-1} [3]

(ii) the linear speed.
linear speed = ... m s^{-1} [2]

(c) The ball has a constant angular speed.

(i) Explain why work has to be done for the ball to move from the position where it is vertically above point C to the position where it is vertically below C. [2]

(ii) Calculate the work done in (i) for a ball of mass 240 g.
work done = ... J [2]

N10/III/2

Solution

(a) (i) The tension in the rod alone (2 mg) is not sufficient to provide the centripetal force required to keep the ball in circular motion. The balance has to be provided by the gravitational force acting on the ball (mg).

(ii) The centripetal force required is the resultant force = $T - mg = 2\ mg + mg = 3\ mg$.

(iii) When the ball is below C, the tension T and the gravitational force mg are in opposite directions. However, the centripetal force required is unchanged as the circular motion is uniform.

$$T - mg = 3\ mg$$
$$T = 4\ mg$$

(b) (i) Centripetal force $mr\omega^2 = 3\ mg$

$$r\omega^2 = 3\ g$$
$$(0.72)\ \omega^2 = 3\ g = 3\ (9.81) = 29.43$$
$$\omega = 6.393 = 6.4 \text{ rad s}^{-1}$$

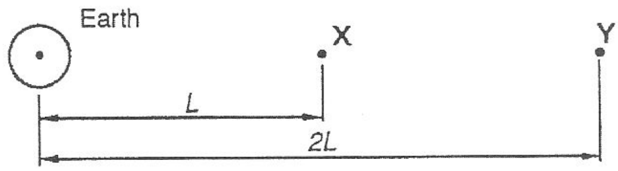
(ii) Linear speed $v = r\omega = (0.72)(6.393) = 4.6 \text{ m s}^{-1}$

(c) (i) Although there is no change in the ball's kinetic energy due to its uniform speed, there is a loss in its gravitational potential energy.
The rigid rod thus has to exert an upward force on the ball, against the force of gravity, to prevent this loss in gravitational potential energy from transforming into a gain in kinetic energy. Work has to be done.
Since the direction of the exerted force is opposite to the displacement of the ball, work done is negative.

(ii) Taking downwards as the positive direction for the ball's displacement,
Work done $W = Fd = (-mg)(h)$
$= (-0.240 \times 9.81)(2 \times 0.72) = -3.4$ J.

TOPIC 7 Gravitational Field

1 The diagram shows two points **X** and **Y** at distances L and $2L$, respectively, from the centre of the Earth. The gravitational potential at **X** is -8 kJ kg^{-1}.



What is the gain in gravitational potential energy of a 1 kg mass when it is moved from **X** to **Y**?

A -4 kJ C $+4$ kJ

B -2 kJ D $+8$ kJ N02/I/10

Solution Answer: **C**

The magnitude of the gravitational potential Φ is inversely proportional to the distance r from the centre of the Earth ($\Phi = -GM/r$).

The G.P.E. of a 1 kg mass at X is $U = m\Phi = (1\text{ kg})(-8\text{ kJ kg}^{-1}) = -8$ kJ.

The G.P.E. of a 1 kg mass at Y will be $U = (1\text{ kg})(-4\text{ kJ kg}^{-1}) = -4$ kJ.

The gain in gravitational potential energy = -4 kJ – (-8 kJ) = + 4 kJ.

2 The gravitational field strength outside a uniform sphere of mass M is the same as that due to a point mass M at the centre of the sphere.

The Earth may be taken to be a uniform sphere of radius r. The gravitational field strength at its surface is g.

What is the gravitational field strength at a height h above the surface?

A $\dfrac{gr^2}{(r+h)^2}$ C $\dfrac{g(r-h)}{r}$

B $\dfrac{gr}{(r+h)}$ D $\dfrac{g(r-h)^2}{r^2}$ N04/I/11

Solution Answer: **A**

A height of h above the surface means a distance of $r + h$ from the centre of the Earth.

Since the gravitational field strength is inversely proportional to the square of the distance from the centre of the Earth ($g = GM/r^2$), the gravitational field strength at height h above the surface is $gr^2/(r + h)^2$.

3 Which statement about a geostationary satellite is true?

A It can remain vertically above any chosen fixed point on the Earth.

B Its linear speed is equal to the speed of a point on the Earth's equator.

C It has the same angular velocity as the Earth's rotation on its axis.

D It is always travelling from east to west. N05/I/11

Solution Answer: **C**

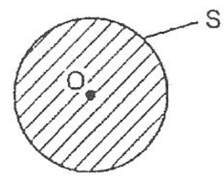
Since a geostationary satellite is always vertically above a fixed point on the equator, it follows the Earth's rotation with the same period (24 hours) and hence the same angular velocity (Answer C).

Answer A is wrong because the geostationary satellite must be above the equator and not any chosen fixed point.

Answer B is wrong because linear speed $v = r\omega$. The satellite, which has a larger radius of orbit than a point on the Earth's surface, will have a larger linear speed (although its angular speed ω is the same as that of a point on the Earth's surface).

Answer D is wrong because geostationary satellites follow the Earth's rotation from west to east, and not from east to west.

4 An astronomical gas cloud has mass M and radius R. The gravitational potential on its surface S is $-\frac{GM}{R}$ and at its centre O it is $-\frac{3GM}{2R}$



A unit mass is moved slowly by means of an external force from the surface S to the centre O.

What is the work done on the mass by the external force?

A $-\dfrac{5GM}{2R}$ B $-\dfrac{GM}{2R}$ C $\dfrac{GM}{2R}$ D $\dfrac{5GM}{2R}$

N06/I/1

Solution Answer: **B**

Work done by external force = change in gravitational potential energy

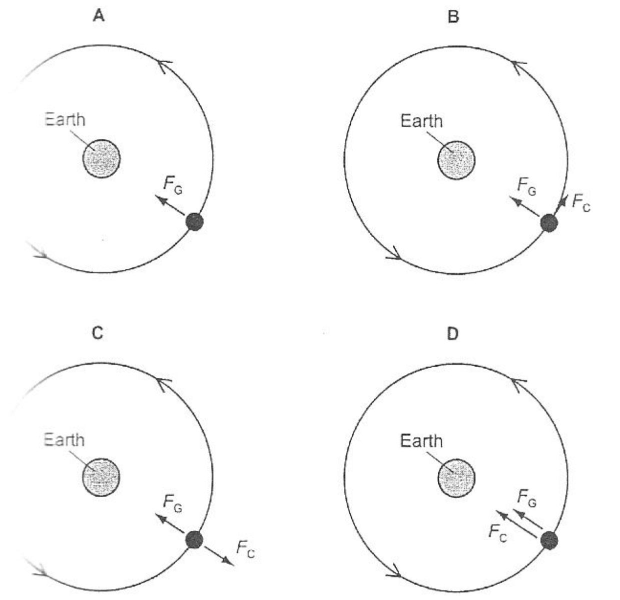
$= (-3GM/2R) - (-GM/R) = -GM/2R$.

Due to the attractive nature of the gravitational force, the work done by the external force is negative as positive work is done by the gravitational force.

5 A satellite is orbiting the Earth.

The gravitational force on the satellite is F_G. The centripetal force required to maintain the satellite in orbit is F_C.

Which diagram shows the force, or forces, acting on the orbiting satellite?



N07/I/14

Solution Answer: **A**

There is only once force acting on the satellite and that is the gravitational force F_G.

The centripetal force F_C is a requirement to maintain the satellite in circular motion, and should not be included in the free body diagram of the satellite.

F_G provides F_C.

A model of a black hole is a point mass of 6 x 10^{24} kg.
What is the force on a point mass of 1 kg at a distance of 2 cm from this black hole?

A 1 x 10^{14} N **B** 2 x 10^{14} N **C** 2 x 10^{16} N **D** 1 x 10^{18} N

N07/I/15

Solution Answer: **D**

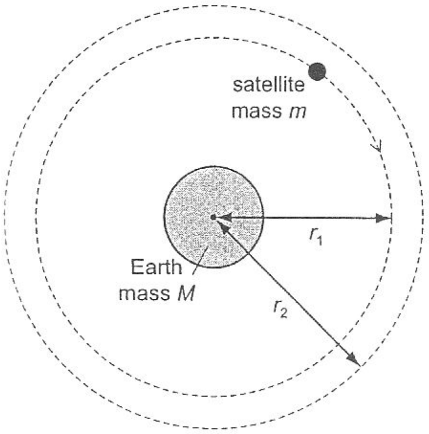
Using Newton's law of gravitation,

$$F = GMm/r^2.$$

$= (6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2})(6 \times 10^{24}\ \text{kg})(1\ \text{kg})/(0.02\ \text{m})^2$

$= 1 \times 10^{18}$ N

A satellite of mass m is moved from a circular orbit of radius r_1 around the Earth to a new circular orbit of radius r_2 as shown.



The mass of the Earth is M and the gravitational constant is G.
What is the increase in the potential energy of the satellite?

A $GM(\frac{1}{r_2} - \frac{1}{r_1})$

B $GM(\frac{1}{r_1} - \frac{1}{r_2})$

C $GMm(\frac{1}{r_2} - \frac{1}{r_1})$

D $GMm(\frac{1}{r_1} - \frac{1}{r_2})$

N08/I/13

Solution Answer: **D**

The gravitational potential energy (G.P.E.) at r_2 is greater than that at r_1.

Increase in G.P.E = G.P.E. at r_2 – G.P.E. at r_1

$= -GMm/r_2 - (-GMm/r_1) = GMm/r_1 - GMm/r_2$

$= GMm(1/r_1 - 1/r_2)$

8 The radius of the Earth's orbit about the Sun is 1.50 x 10^{11} m. The Earth takes 365 days to orbit the Sun.

What is the mass of the Sun?

A 6.40 x 10^{29} kg
B 2.01 x 10^{30} kg
C 1.16 x 10^{33} kg
D 3.31 x 10^{33} kg

N08/I/14

Solution Answer: **B**

The gravitational force that the Sun (mass M) exerts on the Earth (mass m) provides the centripetal force required for the Earth's circular motion about the Sun,

$$GMm/r^2 = mr\omega^2$$
$$GM/r^3 = (2\pi/T)^2$$
$$M = (4\pi^2 r^3)/(T^2 G)$$

$= (4\pi^2 \times (1.50 \times 10^{11})^3)/((365 \times 24 \times 60 \times 60\ \text{s})^2 \times 6.67 \times 10^{-11})$

$= 2.01 \times 10^{30}$ kg

9 An object in a space capsule orbiting the Earth seems to be floating.

Which statement describes the forces acting on the object?

A There are no forces on the object.
B The centrifugal force on the object is equal and opposite to its weight.
C The centripetal force on the object is equal and opposite to its weight.
D The weight of the object is the only force acting on it.

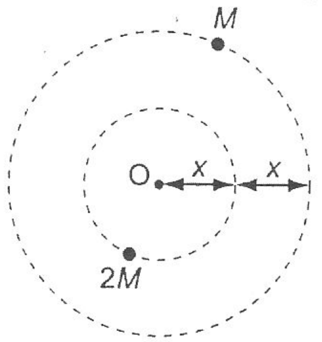
N09/I/6

Solution Answer: **D**

The gravitational force (true weight) acting on the object is the only force acting on it, and is just sufficient to provide the centripetal force to keep the object in circular motion.

The object thus seems to be weightless (apparent weight = 0).

10 Two stars of mass M and $2M$, a distance $3x$ apart, rotate in circles about their common centre of mass O.



The gravitational force acting on the stars can be written as $\dfrac{kGM^2}{x^2}$.

What is the value of k?

N09/I/16

A 0.22 **B** 0.50 **C** 0.67 **D** 2.0

Solution Answer: **A**

The gravitational force of attraction between the two stars can be found from Newton's law of gravitation as

$$G(M)(2M) / (3x)^2 = (2/9)\ GM^2 / x^2$$

Hence $k = 2/9 = 0.22$.

11 A satellite orbits a planet at a distance r from its centre. Its gravitational potential energy is -3.2 MJ.

Another identical satellite orbits the planet at a distance $2r$ from its centre.

What is the sum of the kinetic energy and the gravitational potential energy of this second satellite?

A -0.40 MJ **B** -0.80 MJ **C** -1.6 MJ **D** -6.4 MJ

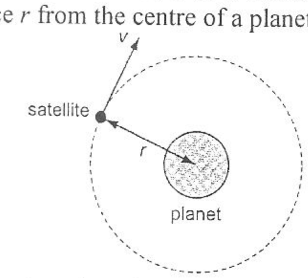
N09/I/17

Solution Answer: **B**

At any given location, the gravitational potential energy of an orbiting satellite $U = -GM/r$ whereas its total energy (sum of kinetic and potential energy) $T = -GMm/2r$ (half the magnitude of the potential energy).

For a satellite at twice the distance, the total energy
$T = -GMm/[2(2r)] = -GMm/[4r] = (-3.2\text{ MJ})/4 = -0.80$ MJ

12 A satellite of mass m moves in a circular orbit at speed [illegible] and distance r from the centre of a planet of mass M.



What expression gives the total energy of the satellite?

A $m\left(\frac{v^2}{r}-\frac{GM}{r}\right)$ **B** $m\left(\frac{v^2}{2}-\frac{GM}{r}\right)$ **C** $m\left(\frac{v^2}{r}+\frac{GM}{r}\right)$ **D** $m\left(\frac{v^2}{2}+\frac{GM}{r}\right)$

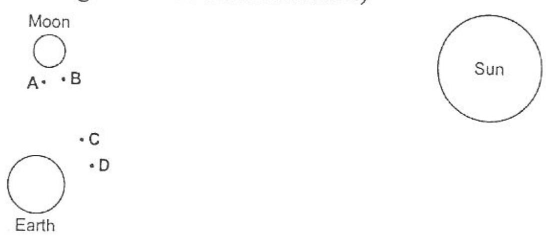
N10/I/1[illegible]

Solution Answer: [illegible]

Total energy = sum of kinetic energy ($\frac{1}{2}mv^2$) and gravitational potential energy ($-GMm/r$).

13 The neutral point in the gravitational field between the Sun, the Earth and the Moon is the point at which the resultant gravitational field due to the three bodies is zero. The mass of the Earth is about 80 times the mass of the Moon.

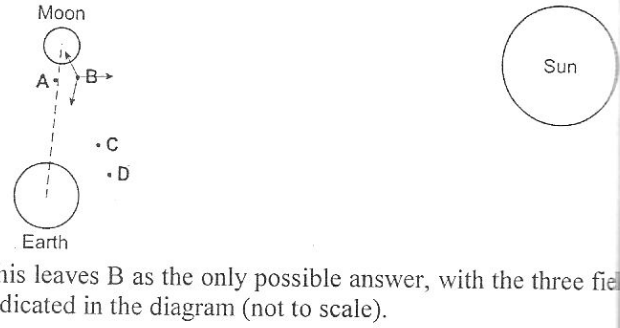
At what position is it possible for the neutral point to be? (The diagram is not drawn to scale.)



N10/I/1[illegible]

Solution Answer: **B**

Since gravitational field strength g is inversely proportional to the square of the distance r from the centre of each body ($g = GM/r^2$), the neutral point just considering the Earth-Moon system alone should be closer to the Moon than to the Earth. This leaves either answers A or B as possible answers. Since gravitational force must be attractive in nature, the direction of the gravitational field strength due to each body must be towards the centre of that body. Hence there is no way for the contribution due to the Sun to cause the neutral point to be located to the left side of the line joining the centres of the Earth and the Moon (see dashed line in the diagram below).



This leaves B as the only possible answer, with the three fiel[illegible] indicated in the diagram (not to scale).

Short Questions

14 Towards the end of the eighteenth century, Cavendish successfully measured the force of attraction between two spheres. Hence, he became one of the first scientists to determine a value for the gravitational constant G.

(a) Write down an equation representing Newton's law of gravitation, explaining any symbols used. [3]

(b) The acceleration of free fall g at the Earth's surface is given by the expression

$$g = \frac{GM}{r^2},$$

where M is the mass of the Earth and r is its radius.

Calculate a value for the mass of the Earth, given that $r = 6.40 \times 10^6$ m.

mass = ... kg [2]

(c) Some textbooks describe Cavendish's experiment as a means of 'weighing the Earth'.

Suggest why this statement is incorrect. [2] N02/II/1

Solution

(a) The equation is $F = GMm/r^2$ where G is the gravitational constant.
Newton's law of gravitation states that the gravitational force of attraction F between two point masses M and m is directly proportional to the product of their masses, and inversely proportional to the square of their separation r.

(b) The gravitational field strength at the surface of the Earth is 9.81 N kg^{-1}.
Using this value for g in the equation,

$$g = GM/r^2$$

$$9.81 \text{ N kg}^{-1} = (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})M/(6.40 \times 10^6 \text{ m})^2$$

$$M = 6.02 \times 10^{24} \text{ kg}$$

(c) In order to weigh an object of mass m, that object must be present in a gravitational field g, so that its weight can be calculated from the equation $W = mg$.
The weight of an object thus varies depending on the strength of the external gravitational field in which it is located.
In this case, the Earth is not present in an external gravitational field generated by some other body. Therefore it is not possible to find the weight of the Earth. The value obtained here is in fact its mass, which is a measure of the amount of matter in it and is independent of the strength of the gravitational field in which it is located (which is zero in this case).

15 A satellite orbits the Earth in a circular path, as illustrated in Fig. 3.1.

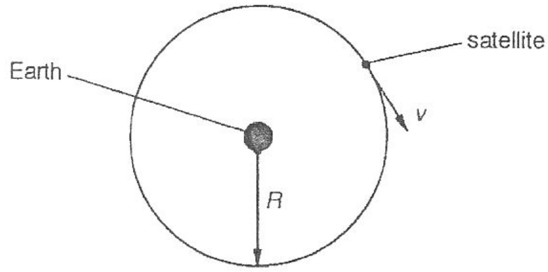


Fig. 3.1

Both the Earth and the satellite may be considered to be point masses with their masses concentrated at their centres.

The satellite has speed v and the radius of its orbit about the Earth is R.

(a) (i) Show that speed v is given by the expression

$$v^2 = \frac{GM}{R},$$

where M is the mass of the Earth and G is the gravitational constant. [2]

(ii) The mass of the satellite is m. Determine an expression for the kinetic energy E_k of the satellite in terms of G, M, m and R. [2]

(b) (i) State an expression, in terms of G, M, m and R, for the gravitational potential energy E_p of the satellite. [1]

(ii) Hence show that the total energy E_t of the satellite is given by

$$E_t = -\frac{GMm}{2R}.$$

[2]

(c) As the satellite orbits the Earth, it gradually loses energy because of air resistance.

(i) State whether the total energy E_t becomes more or less negative. [1]

(ii) Hence state and explain the effect of this change on

1. the radius of the orbit, [2]

2. the speed of the satellite. [2]

N04/II/3

Solution

(a) (i) The gravitational force provides the centripetal force.

$$GMm/R^2 = mv^2/R$$

$$v^2 = GM/R$$

(ii) Using the same equation $GMm/R^2 = mv^2/R$,
Kinetic energy $E_k = ½mv^2 = ½GMm/R$.

(b) (i) Gravitational potential energy $E_p = -GMm/R$.

(ii) Total energy
$E_t = E_k + E_p = ½GMm/R + (-GMm/R) = -½GMm/R$.

(c) (i) More negative (since E_t decreases).

(ii) 1. Decreases.
As E_t becomes more negative, the magnitude of the total energy $|½GMm/R|$ increases implying that the radius of orbit R decreases.

2. Increases.
As speed v is related to orbital radius R by the equation $v^2 = GM / R$, v is inversely proportional to $\sqrt{R}$ hence as R decreases, v increases. The speed becomes greater.

16 The Earth may be assumed to be a uniform sphere of radius R and mass M. At its surface, the gravitational field strength is g. The gravitational field above the surface is the same as that due to a point mass M situated at the centre of the Earth.

(a) Explain what is mean by a *gravitational field.* [2]

(b) A satellite orbits the Earth at a height 0.30 R above its surface. Show that the gravitational field strength at this height is 0.59 g. [2]

(c) A person in the satellite in *(b)* experiences 'weightlessness' although the gravitational field strength is not zero.

(i) Explain why the person seems to be weightless. [2]

(ii) Show that the angular speed of the satellite about the Earth is approximately 8.3 x 10^{-4} rad s^{-1}. The radius R of the Earth is 6.4 x 10^6 m. [2]

(iii) Calculate the time, in hours, for one complete orbit of the satellite.

time = ... hours [2]

N05/II/3

Solution

(a) A gravitational field is a region of space where a mass experiences a gravitational force due to the presence of another mass.

The strength of the gravitational field at a point is defined as the gravitational force per unit mass on a small point mass placed at that point, measured in N kg^{-1}.

(b) Gravitational field strength due to the Earth $g = GM/R^2$ where G is the gravitational constant, M the mass of the Earth and R the distance from the centre of the Earth. Hence, g is inversely proportional to R^2.

A distance of 0.3 R from the surface of the Earth is equivalent to a distance of 1.3 R from the centre of the Earth.

Let g' be the gravitational field strength at this height,

$$g'/g = (R / 1.3R)^2$$

$$g' = 0.59\ g$$

(c) (i) The gravitational force acting on the person is just sufficient to provide the centripetal force required to keep him in circular motion.

Hence there is no contact force between the person and the floor of the satellite on which he is standing.

The person thus seems to be weightless.

(ii) The radius of orbit
= 1.3 R
= 1.3 x 6.4 x 10^6 m
= 8.32 x 10^6 m.
Since the gravitational force acting on the person is just sufficient to provide the centripetal force required to keep him in circular motion, the gravitational field strength is just sufficient to cause his centripetal acceleration.

$$g' = r\omega^2$$

$$0.59\ g = r\omega^2$$

$$0.59\ (9.81\ \text{N kg}^{-1}) = (8.32 \times 10^6\ \text{m})\ \omega^2$$

$$\omega = 8.341 \times 10^{-4} = 8.3 \times 10^{-4}\ \text{rad s}^{-1}$$

(iii) Angular velocity $\omega = 2\pi / T$ where T is the period.

$$T = 2\pi / \omega$$

$$= 2\pi / 8.341 \times 10^{-4}\ \text{rad s}^{-1}$$

$$= 7533\ \text{s}$$

$$= 2.1\ \text{hours.}$$

17 A satellite of mass m orbits a planet of mass M and radius R_p. The radius of the orbit is R.
The satellite and the planet may be considered to be point masses with their masses concentrated at their centres. They may be assumed to be isolated in space.

(a) (i) Derive an expression, in terms of M, m and R, for the kinetic energy of the satellite. Explain your working. [2]

(ii) Show that, for the satellite in orbit, the ratio

$$\frac{\text{gravitational potential energy of satellite}}{\text{kinetic energy of satellite}}$$

is equal to -2. [1]

(b) The variation with orbital radius R of the gravitational potential energy of the satellite is shown in Fig. 3.1.

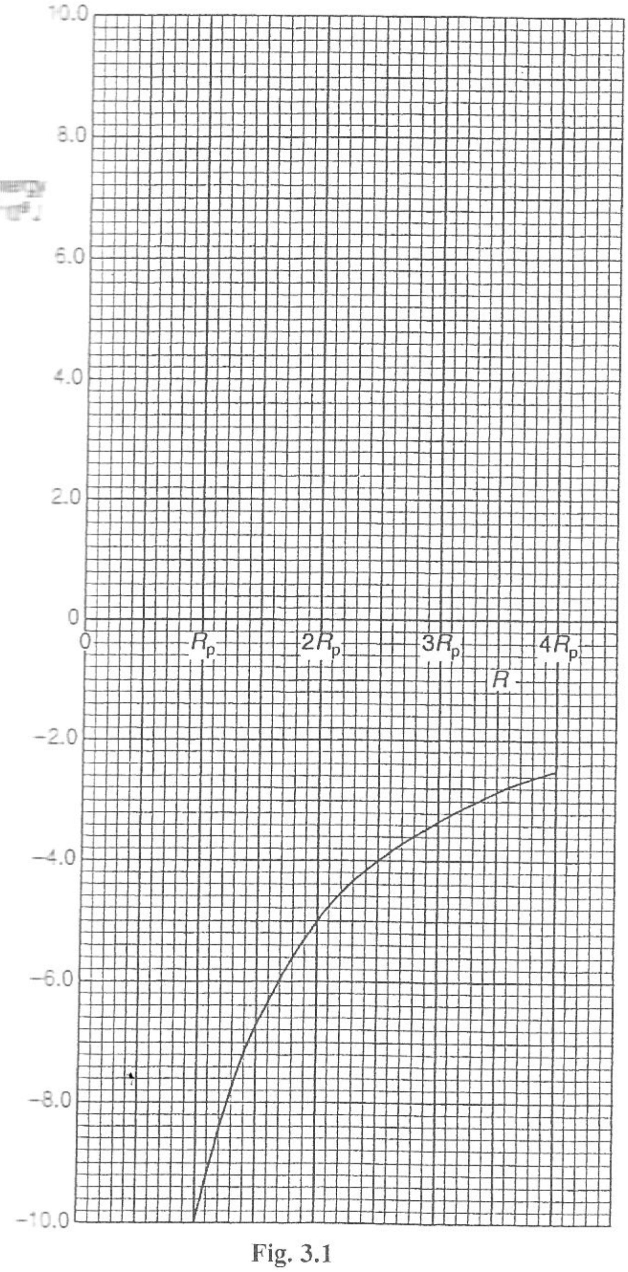


Fig. 3.1

(i) On Fig. 3.1, draw the variation with orbital radius of the kinetic energy of the satellite. Your line should extend from $R = 1.5R_P$ to $R = 4R_P$. [2]

(ii) The mass m of the satellite is 1600 kg.
The radius of the orbit of the satellite is changed from $R = 4R_P$ to $R = 2R_P$.
Use Fig. 3.1 to determine the change in orbital speed of the satellite.

change in speed = ... m s^{-1} [5]

N08/II/3

Solution

(a) The gravitational force of attraction (GMm/R^2) that the Earth exerts on the satellite provides the centripetal force (mv^2/R) required for the satellite's circular motion about the Earth.

$$GMm/R^2 = mv^2/R$$

where G is the gravitational constant.

$$GMm/R = mv^2$$

Kinetic energy $E_K = ½mv^2 = ½GMm/R$

(ii) Gravitational potential energy $E_P = -GMm/R$.

Therefore, $E_P/E_K = (-GMm/R)/(½GMm/R) = -2$.

(b) (i) The line is shown in Fig. 3.2.

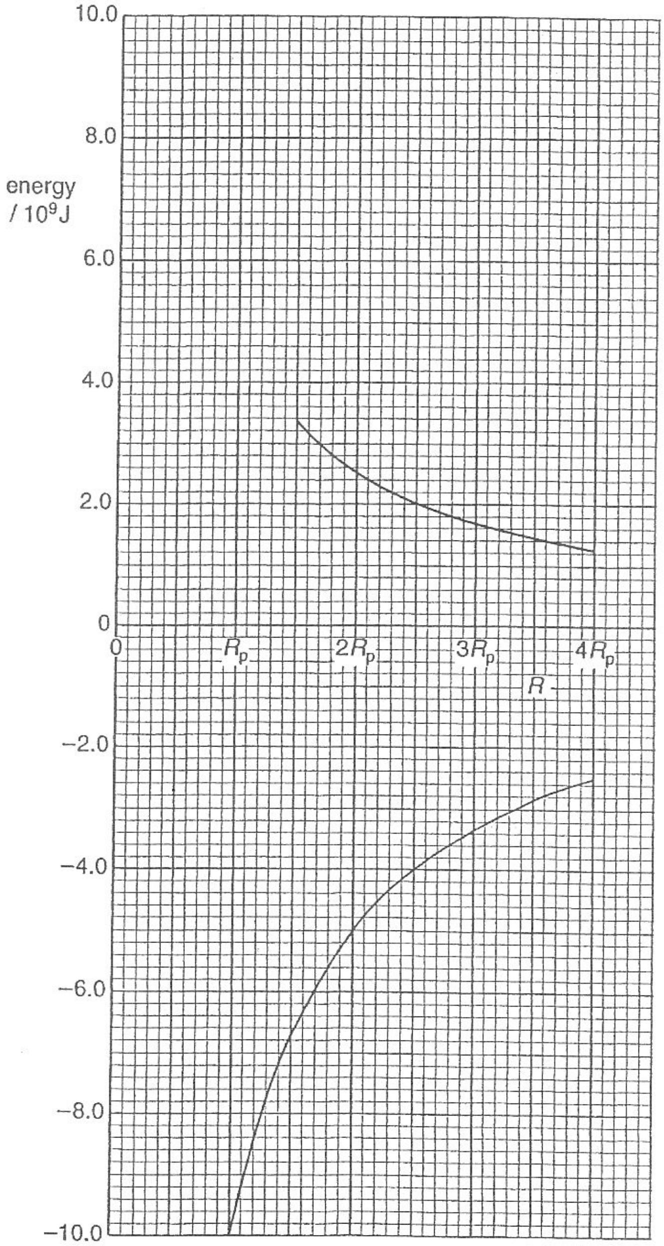


Fig. 3.2

(ii) When $R = 4R_P$, the gravitational potential energy of the satellite E_P = -2.5 x 10^9 J. Since E_P/E_K = -2, its kinetic energy E_K = 1.25 x 10^9 J.

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(1600 \text{ kg})v^2 = 1.25 \times 10^9 \text{ J}$$
$$v = 1250 \text{ m s}^{-1}$$

When $R = 2R_P$, the gravitational potential energy of the satellite E_P = -5 x 10^9 J. Since E_P/E_K = -2, its kinetic energy E_K = 2.5 x 10^9 J.

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(1600 \text{ kg})v^2 = 2.5 \times 10^9 \text{ J}$$
$$v = 1770 \text{ m s}^{-1}$$

The change in speed = 1770 – 1250 = 520 m s^{-1}.

18 (c) Measurements are made of the Earth's gravitational field strength g for different distances r from the centre of the Earth. Fig. 1.5 shows the variation with lg r of lg g.

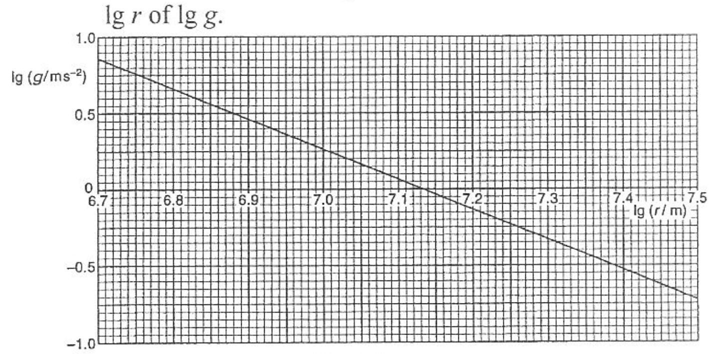


Fig. 1.5

(i) Determine the gradient of the graph. Show your working.
gradient = ... [3]

(ii) Summarise what is suggested by the value of the gradient of this graph.
....
.... [2]

N08/III/1(c)

Solution

(c) (i) Gradient = (-0.72 – 0.86) / (7.5 – 6.7) = -1.58 / 0.8
Gradient = -2.0

(ii) The equation is of the form:

$$\lg g = -2 \lg r + \log c$$

where log c is the y-intercept of this graph.

$$\lg g = \lg r^{-2} + \log c$$

$$\lg g = \lg c\, r^{-2}$$

$$g = c\, r^{-2}$$

The gradient of the graph (-2) shows that gravitational field strength varies according to the inverse square law with distance r from the centre of the Earth.

Long Questions

19 (*a*) Write down an equation expressing Newton's law of gravitation. Define your symbols. [2]

(*b*) Use the equation in (*a*) to derive a value for g, the acceleration due to gravity, at the Earth's surface.

mass of Earth = 5.98×10^{24} kg

mean radius of Earth = 6.37×10^6 m [3]

(*c*) A geostationary satellite has to be placed above the equator.

(i) State what is meant by *geostationary*. [1]

(ii) State the direction of rotation of the satellite around the Earth's axis. [1]

(iii) Explain why the satellite must be above the equator. [2]

(*d*) A geostationary satellite is in orbit at a distance of 4.23×10^7 m from the centre of the Earth. Calculate

(i) the Earth's gravitational field strength at this distance from the centre of the Earth, [1]

(ii) the speed of the satellite, [3]

(iii) the acceleration of the satellite. [2]

(*e*) Under the heading *Data* there is the entry

acceleration of free fall, g = 9.81 m s^{-2}.

Compare and comment on small differences between this value, the value you obtained in part (*b*), and the value of 9.79 m s^{-2} (which is the value obtained by making accurate measurements in Singapore, near the equator). [5]

N03/III/2

Solution

(a) Newton's law of gravitation states that the gravitational force of attraction F between two point masses M and m is directly proportional to the product of their masses, and inversely proportional to the square of their separation r.

The expression is $F = GMm/r^2$ where G is the gravitational constant.

(b) The gravitational force acting on a mass m at the Earth's surface is:

$F = GMm/R^2$ = (6.67 x 10^{-11} N m^2 kg^{-2})(5.98 x 10^{24} kg)m / (6.37 x 10^6 m)^2 = 9.830m N = 9.8m N.

This force causes the mass m to acceleration, according to Newton's 2nd law of motion, $F = ma$.

The acceleration due to gravity $a = F/m$ = 9.830m N / m = 9.830 = 9.8 m s^{-2}.

(i) A *geostationary* satellite will always be directly above the same location on the Earth's equator.
Such satellites will thus have a period equal to the Earth's rotational period about its own axis, i.e. 24 hours.

(ii) A geostationary satellite rotates from west to east around the Earth's axis, following the Earth's own direction of rotation.

(iii) The gravitational force acting on the satellite provides the centripetal force for its circular orbit.

Since the gravitational force acts towards the centre of the Earth, the plane of the satellite's orbit must include the centre of the Earth.

Additionally, since the satellite must be permanently above a fixed point on the Earth's equator and follow the Earth's rotation, its plane of orbit must also include the equator.

(i) Gravitational field strength
$g = GM/r^2$
$= (6.67 \times 10^{-11}\ N\ m^2\ kg^{-2})(5.98 \times 10^{24}\ kg)/(4.23 \times 10^7\ m)^2$
$= 0.22292$
$= 0.223\ N\ kg^{-1}$.

(ii) Speed
$v = r\omega$
$= r(2\pi/T)$
$= (4.23 \times 10^7\ m)(2\pi/(24 \times 60 \times 60\ s))$
$= 3076.1$
$= 3080\ m\ s^{-1}$.

(iii) Acceleration is numerically equal to the gravitational field strength, $a = 0.223\ m\ s^{-2}$.

The value obtained in part (b), $9.83\ m\ s^{-2}$, does not take into account the centripetal acceleration due to the Earth's rotation about its own axis.

The value $9.83\ m\ s^{-2}$ is the acceleration of free fall at the north or south poles, which are not rotating.

At the equator where Singapore is located, due to the Earth's rotation about its own axis, the gravitational field strength has to provide for both a body's acceleration of free fall and its centripetal acceleration.

As a result, the acceleration of free fall is slightly less in magnitude than the gravitational field strength.
The value of $9.81\ m\ s^{-2}$ is an average between the poles ($9.83\ m\ s^{-2}$) and the equator ($9.79\ m\ s^{-2}$).

20 (a) A car engine has a maximum rate of rotation of 5000 revolutions per minute. Express this angular velocity in radians per second.

angular velocity = ... rad s^{-1} [2]

(b) The Earth spins on its axis with a period of one day.

(i) Show that the angular velocity of a point on the Earth's surface is 7.27×10^{-5} rad s^{-1}. [1]

(ii) Calculate the centripetal acceleration of a point on the Earth's equator. The radius of the Earth's equator is 6.38×10^6 m.

centripetal acceleration = ... m s^{-2} [2]

(c) The acceleration of free fall *g* at the equator is not equal to the acceleration of free fall at the poles. Explain

(i) why they are different,

....
....
.... [2]

(ii) why the difference is small.

....
....
.... [1]

(d) (i) State Newton's law of gravitation.

....
.... [1]

(ii) The mass *M* of the Earth may be considered to be concentrated at its centre. The radius of the Earth is *R*. Derive, in terms of *M* and *R*, the equation relating the Earth's gravitational field strength *g* to the gravitational constant *G*. [2]

(e) (i) Calculate how far a satellite needs to be from the centre of the Earth for its angular velocity to be equal to the angular velocity of the Earth.

distance = ... m [3]

(ii) State two circumstances under which a satellite at this distance will be a geostationary satellite.
1. ....
....
2. ....
.... [2]

(f) Many systems, such as the Global Positioning System, use several satellites in low orbits that pass over the Earth's poles. Suggest two advantages of these low polar orbits and two advantages of geostationary orbits.
polar orbits: advantage 1
polar orbits: advantage 2
geostationary orbits: advantage 1
geostationary orbits: advantage 2 [4]
N07/III/5

Solution

(a) 5000 revolutions per minute = 5000 / 60 = 83.33 revolutions per second.

Angular velocity $\omega = 2\pi f = 2\pi(83.33 \text{ Hz}) = 524 \text{ rad s}^{-1}$.

(b) (i) The period of the Earth's rotation about its axis is

$$T = 24 \times 60 \times 60 \text{ s} = 86{,}400 \text{ s}$$

Its angular velocity $\omega = 2\pi/T = 7.27 \times 10^{-5} \text{ rad s}^{-1}$.

(ii) Centripetal acceleration

$$a = r\omega^2 = (6.38 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad s}^{-1})^2$$

$$= 3.37 \times 10^{-2} \text{ m s}^{-2}.$$

(c) (i) The equator undergoes circular motion about the Earth's axis, whereas the poles do not. At the equator, part of the gravitational force has to provide the centripetal force required to keep any mass located there in circular motion about the Earth's north-south axis. Since part of the acceleration due to gravity has to provide the centripetal acceleration, the acceleration of free fall is lower.

(ii) The Earth's angular velocity is low and its equatorial centripetal acceleration (0.0337 m s^{-2}) is significantly less than its acceleration due to gravity. Hence the difference is small.

(d) (i) Newton's law of gravitation states that the gravitational force of attraction F between two point masses M and m is directly proportional to the product of their masses, and inversely proportional to the square of their separation r.

(ii) Consider a point mass m located at the surface of the Earth. The distance between the centre of the Earth and the centre of m is R (the radius of the Earth).

According to Newton's law of gravitation, the force of attraction between the Earth and the mass m is

$$F = GMm/R^2$$

The gravitational field strength g at a point in a gravitational field is the gravitational force F per unit mass on a small point mass placed at that point.

$$g = F/m = (GMm/R^2)/m = GM/R^2$$

(e) (i) Rearranging the field equation in (d)(ii),

$$GM = gR^2 \ldots (1)$$

Let r be the distance from the centre of the Earth to the satellite. The gravitational field strength provides the centripetal acceleration hence

$$GM/r^2 = r\omega^2$$

$$r = (GM/\omega^2)^{1/3} \ldots (2)$$

Substitute (1) into (2),

$$r = (gR^2/\omega^2)^{1/3}$$

$$= (9.81 \times (6.38 \times 10^6 \text{ m})^2 / (7.27 \times 10^{-5})^2)^{1/3}$$

$$= 4.23 \times 10^7 \text{ m}$$

(ii) 1. The satellite lies on a plane containing the Earth's equator and is always directly above a point on the equator.

2. The satellite rotates from west to east, following the Earth's direction of rotation.

(f) Polar advantage 1: Satellites in low orbits are closer to the Earth hence their signals are received more strongly (*They also have better resolutions if used as photographic satellites: 50 m per pixel versus 2.5 km per pixel for geostationary satellites*).

Polar advantage 2: Satellites can fly over and serve every part of the world equally (*instead of being limited by the Earth's curvature to between 70 N and 70 S, with progressively weaker signal strength the further the receiver is from the equator*).

Geostationary advantage 1: Continuous transmission between the ground station and the satellite, since they are always at the same spot above the equator (*and the ability to photograph the entire Earth simultaneously, instead of bits and pieces as in the case of polar satellites which are above different places at different times*).

Geostationary advantage 2: Less energy required to launch and maintain (*When launching, they can make use of the momentum due to the Earth's west-east rotation to fling them into orbit; and while orbiting, they need less fuel to maintain their circular motion since atmospheric resistance decreases with altitude. Theoretically no fuel is required but space is not a perfect vacuum*).

21 (a) (i) Defined gravitational field strength. [1]

(ii) State Newton's law of gravitation and hence using your definition in (i), show that the gravitational field strength g at a distance R from a point mass M is given by

$$g = \frac{GM}{R^2}.$$ [3]

(b) A neutron star has mass 5.2×10^{30} kg and radius 1.7×10^{4} m.

(i) Calculate the mean density of the star.
density = ... kg m^{-3} [3]

(ii) Suggest, with a reason, whether the density is likely to vary with distance from the centre of the star. [3]

(c) The mass of the star in (b) may be considered to be a point mass at its centre.

(i) Calculate the gravitational field strength at the surface of the star.
field strength = ... N kg^{-1} [2]

(ii) Determine the centripetal acceleration of a particle moving in a circular path of radius 1.7×10^{4} m and with a period of rotation of 0.21 s.
acceleration = ... m s^{-2} [3]

(iii) The star rotates about its axis with a period of 0.21 s.
Use your answers in (i) and (ii) to suggest whether particles on the surface of the star leave the surface owing to the high speed of rotation of the star. [2]

(d) A stream of protons is accelerating towards the star. Suggest why this stream may be a source of X-ray radiation. [3]

N09/III/5

Solution

(a) (i) The gravitational field strength at a point in a gravitational field is the gravitational force per unit mass on a small point mass placed at that point.

(ii) Newton's law of gravitation states that the gravitational force of attraction F between two point masses M and m is directly proportional to the product of their masses, and inversely proportional to the square of their separation r.

From this definition, considering two point masses M and m separated a distance R apart, the gravitational force that M exerts on m is:

$$F = GMm / R^2$$

where G is a constant of proportionality known as the gravitational constant.

Since gravitational field strength g is the force per unit mass m, the gravitational field strength at a distance R from M is:

$$g = F/m = (GMm / R^2)/m = GM / R^2$$

(b) (i) Density = Mass / Volume.

$$\rho = M/V = (5.2 \times 10^{30})/((4/3)\, \pi\, (1.7 \times 10^{4})^{3})$$

$$= 2.5 \times 10^{17} \text{ kg m}^{-3}$$

(ii) The density is likely to be higher nearer the centre.

The closer it is to the centre, the greater the amount of matter above with gravity pressing inward and exerting a greater pressure, forcing neutrons closer towards one another.

Since gravitational force varies inversely with the square of separation, the closer spacing between neutrons near the centre will pull them even closer toward one another, making the core even denser.

Neutron stars are prevented from further gravitational collapse by Pauli exclusion principle, that no two neutrons (which are fermions) can occupy the same quantum state (H2 Physics – Semiconductor Band Theory).

(c) (i) Using the gravitational field strength equation,

$$g = GM / R^2$$

$$= (6.67 \times 10^{-11})(5.2 \times 10^{30}) / (1.7 \times 10^{4})^2$$

$$= 1.2 \times 10^{12} \text{ N kg}^{-1}$$

(ii) The centripetal acceleration:

$$a = R\omega^2 = R(2\pi / T)^2$$

$$= (1.7 \times 10^{4})(2\pi / 0.21)^2$$

$$= 1.5 \times 10^{7} \text{ m s}^{-2}$$

(iii) No. The particles will not leave the surface.

The gravitational field strength (1.2×10^{12} N kg^{-1}) is much greater than the centripetal acceleration (1.5×10^{7} m s^{-2}).

Therefore, the gravitational force acting on a particle on the surface is more than sufficient to provide the centripetal force required to keep it in circular motion about the centre of the star.

(d) The strong gravitational field strength (1.2×10^{12} N kg^{-1}) accelerates the protons to very high speeds as they stream towards the star.

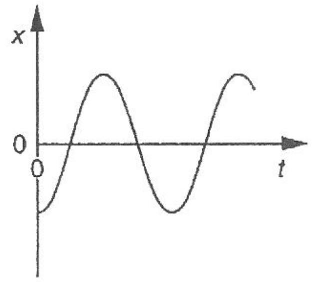
As the protons smash into the star's surface, they are decelerated and their kinetic energy is converted into electromagnetic waves as braking radiation.

The wavelength λ of the radiation emitted is inversely proportional to the kinetic energy K of the protons ($\lambda = hc/K$). Since the protons have very large kinetic energy, their wavelength is very short and in the X-ray region of the electromagnetic spectrum.

TOPIC 8 Oscillations

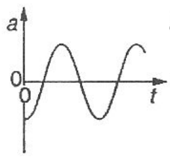
Simple Harmonic Motion

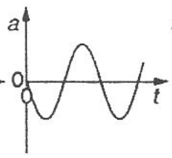
1 A body moves with simple harmonic motion about a point P. The graph shows the variation with time t of its displacement x from P.

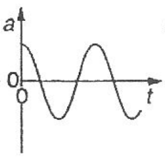


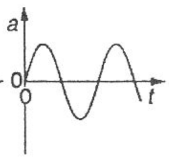
Which graph shows the variation with time t of its acceleration a?

A B C D









N03/I/14

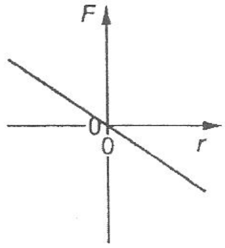
Solution Answer: **C**

The defining equation of simple harmonic motion is $a = -\omega^2 x$ where a is the acceleration of the particle and x its displacement from the equilibrium position.

It can be seen from the equation that the acceleration is directly proportional to, and in the opposite direction to, the displacement.

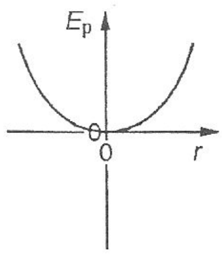
Since the displacement-time is a negative cosine graph, the acceleration-time is a cosine graph.

2 A particle is moving such that the force F on it changes with the distance r from a fixed point as shown.

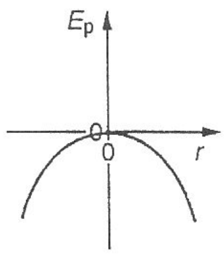


Which graph best shows the relationship between the potential energy E_p of the particle and the distance r?

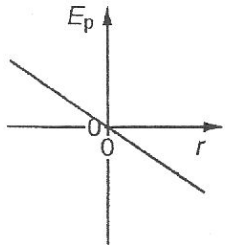
A



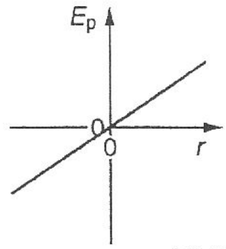
B



C



D



N04/I/16

Solution Answer: A

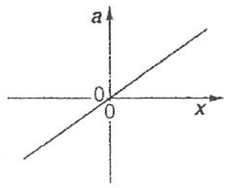
A particle acted upon by an elastic restoring force which is directly proportional to displacement executes simple harmonic motion.

Its potential energy $E_p = \int F\,dr = \int kr\,dr = \frac{1}{2}kr^2 = \frac{1}{2}m\omega^2r^2$.

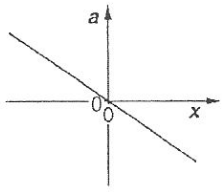
This is the equation of an upright parabola.

3 Which graph correctly represents the variation of acceleration a with displacement x for a body moving in simple harmonic motion?

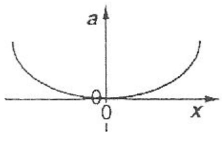
A



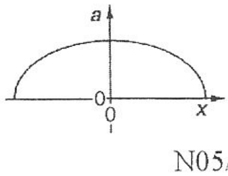
B



C



D



N05/I/15

Solution Answer: **B**

Simple harmonic motion (s.h.m.) is defined as oscillatory motion in which the acceleration a of the body is directly proportional to its displacement x from the equilibrium position, and directed opposite to its displacement, towards the equilibrium position.

The defining equation of s.h.m. is $a = -\omega^2 x$ where ω is the angular frequency of the body.

The graph is a straight line passing through the origin, with a negative gradient of $-\omega^2$.

4 A simple harmonic oscillator has a time period of 10 seconds.

Which equation relates its acceleration a and displacement x?

A $a = -10x$

B $a = -(20\pi)x$

C $a = -(20\pi)^2 x$

D $a = -(2\pi/10)^2 x$

N06/I/13

Solution Answer: **D**

The defining equation of simple harmonic motion is that the acceleration $a = -\omega^2 x$ where the angular frequency $\omega = 2\pi/T$, and T is the period of oscillation.

Therefore, $a = -(2\pi/T)^2 x = -(2\pi/10)^2 x$

5 A body performs simple harmonic motion with a period of 0.063 s. The maximum speed of the body is 3.0 m s^{-1}. What are the values of the amplitude x_0 and the angular frequency ω?

	x_0/m	ω/rad s^{-1}
A	0.030	100
B	0.19	16
C	5.3	16
D	33	100

N07/I/16

Solution Answer: **A**

Angular velocity $\omega = 2\pi/T = 2\pi/(0.063 \text{ s}) = 100$ rad s^{-1}.

Maximum velocity $v_0 = \omega x_0$ where x_0 is the amplitude of oscillation.

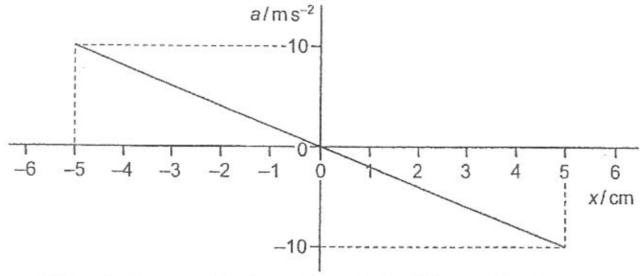
Hence $x_0 = v_0/\omega = (3.0 \text{ m s}^{-1})/(100 \text{ rad s}^{-1}) = 0.030$ m.

6 The defining equation for a particle moving in simple harmonic motion is

$$a = -\omega^2 x$$

where a is the acceleration of the particle, x is the displacement and ω is the angular frequency.

The graph shows how a varies with x for a particle moving in simple harmonic motion.



What is the amplitude and period of the motion?

	amplitude / cm	period / s
A	5.0	0.44
B	5.0	14
C	10	0.44
D	10	14

N08/I/15

Solution Answer: **A**

Amplitude refers to the maximum displacement in either direction, which can be read directly from the graph as 5.0 cm.

The maximum acceleration a_0 occurs when the particle is at its maximum displacement x_0 position,

$$a_0 = -\omega^2 x_0$$

$$a_0 = -(2\pi/T)^2 x_0$$

$$-10 \text{ m s}^{-2} = -(2\pi/T)^2 (0.050 \text{ m})$$

$$T = 0.44 \text{ s}$$

7 A body moves with simple harmonic motion and makes n complete oscillations in one second.

What is its angular frequency?

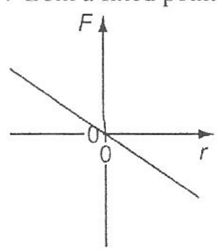
A n rad s^{-1} B $\frac{1}{2}$ rad s^{-1} C $2\pi n$ rad s^{-1} D $\frac{2\pi}{n}$ rad s^{-1}

N09/I/18

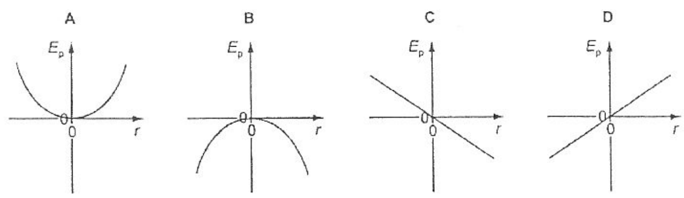
Solution Answer: **C**

Angular frequency $\omega = 2\pi f$ where f is the frequency (in this case n).

8 An particle is moving such that the force F on it changes with the distance r from a fixed point as shown.



Which graph shows the relationship between the potential energy E_p of the particle and the distance r?



N09/I/19

Solution Answer: **A**

A body on which the force F is directly proportional to its displacement r from a fixed point moves with simple harmonic motion.

The potential energy of a body moving in simple harmonic motion is given by the equation $E_p = ½m\omega^2r^2$ and has the shape of an upright parabola.

9 A mass of 8.0 g oscillates in simple harmonic motion with an amplitude of 5.0 mm at a frequency of 40 Hz.
What is the total energy of this simple harmonic oscillator?

A 0.16 mJ **B** 6.3 mJ **C** 13 mJ **D** 640 mJ

N10/I/17

Solution Answer: **B**

The total energy of a simple harmonic oscillator of mass m oscillating with an amplitude of x_0 at an angular frequency ω (or frequency f) is given by the equation

$$E = ½m\omega^2x_0^2 = ½m(2\pi f)^2x_0^2$$
$$= ½(0.008\text{ kg})(2\pi\ 40\text{ Hz})^2(0.005\text{ m})^2$$
$$= 6.3 \times 10^{-3}\text{ J}$$

Short Questions

10 A student sets up the apparatus illustrated in Fig. 4.1 in order to investigate oscillations.

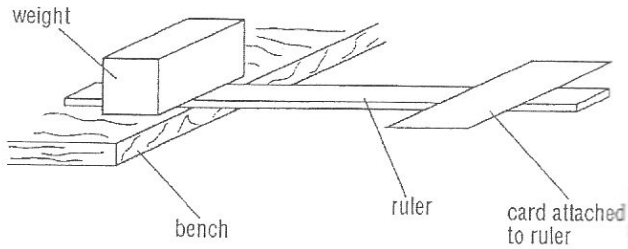


Fig. 4.1

The variation with time t of the displacement y of the end of the ruler is shown in Fig. 4.2.

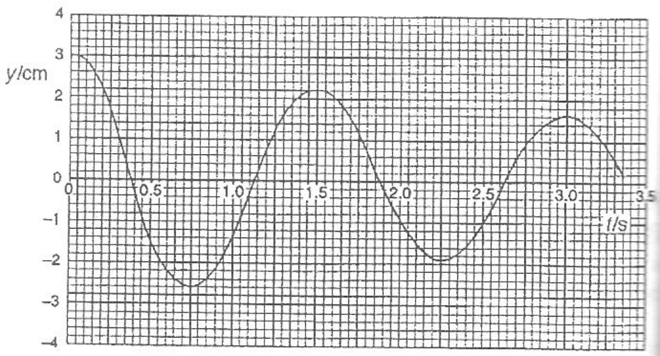


Fig. 4.2

(a) The student claims that the curve of Fig. 4.2 may be represented by the equation

$$y = y_0 \sin \omega t.$$

Identify two features of Fig. 4.2 which show that the student's claim is incorrect. [2]

(b) Calculate, using data from Fig. 4.2, values for

(i) the angular frequency of the oscillations,

(ii) the acceleration of the end of the ruler at time $t = 1.5$ s. [4]

(c) The card on the ruler is replaced with one having a larger surface area and the same mass. The experiment is then repeated.

On Fig. 4.2, draw another graph to show the effect of this change on the variation with t of the displacement of the ruler. [2] N02/II/4

Solution

(a) Firstly, a sine graph starts at the origin so that when time t = 0, the displacement y = 0. This is not the case here.

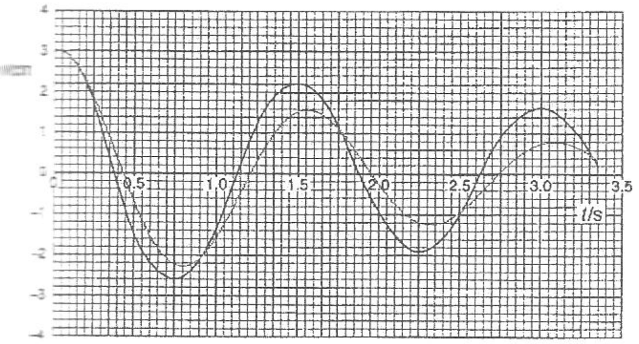
Secondly, a sine graph has a constant amplitude but the graph here has a decreasing amplitude.

(b) (i) Angular frequency

$\omega = 2\pi/T$
$= 2\pi/(1.5\text{ s})$
$= 4.189$
$= 4.2\text{ rad s}^{-1}$.

(ii) Acceleration
$a = -\omega^2 y$
$= -(4.189 \text{ rad s}^{-1})^2(0.022 \text{ m})$
$= -0.3860$
$= -0.39 \text{ m s}^{-2}$.

(c) A larger surface area results in a greater degree of damping.
Damping results in a more rapid reduction in the amplitude of oscillation, and a slight increase in natural period.
This is illustrated in the figure below.



11 *(a)* Define *simple harmonic motion.* [2]

(b) A horizontal metre rule is clamped at one end. The free end oscillates vertically as shown in Fig. 4.1.

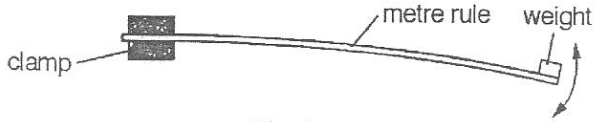


Fig. 4.1

Fig. 4.2 shows the variation with time t of the velocity v of a point at the free end of the rule.

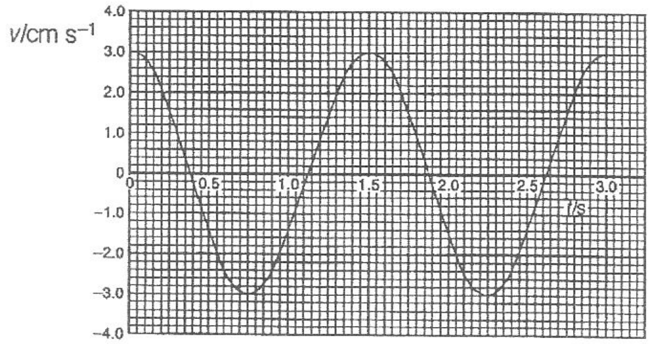


Fig. 4.2

(i) On Fig. 4.2, shade an area that represents the amplitude of the oscillations of the free end of the rule. [1]

(ii) Determine, for these oscillations,
1. the frequency, 2. the amplitude. [4]

(iii)

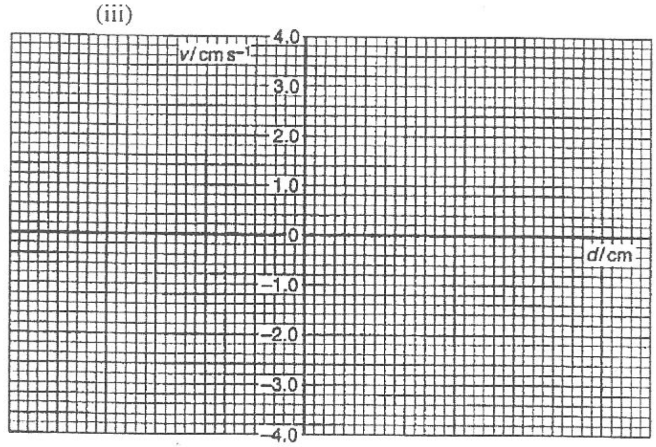


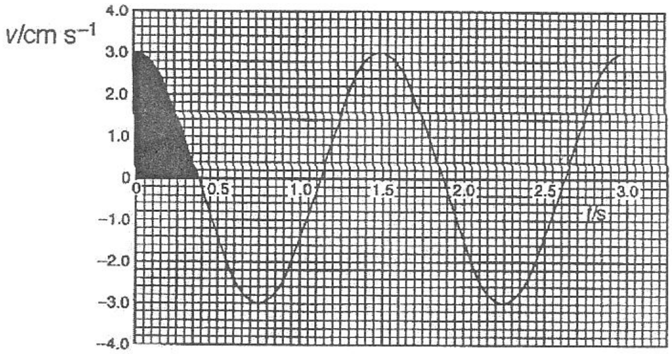
Fig. 4.3

On the axes of Fig. 4.3, sketch a graph to show the variation with displacement d of the velocity v of the end of the rule. Mark a scale on the d-axis.
[2] N03/II/4

Solution

(a) Simple harmonic motion is oscillatory motion in which the acceleration of the object is directly proportional to its displacement from the equilibrium position.
The acceleration is opposite in direction to its displacement, and towards the equilibrium position.

(b) (i) Displacement is equal to the area under the velocity time graph from $t = 0$ (when velocity is maximum) to $t = T/4$ (when velocity is zero).
The shaded area is shown in the figure below.

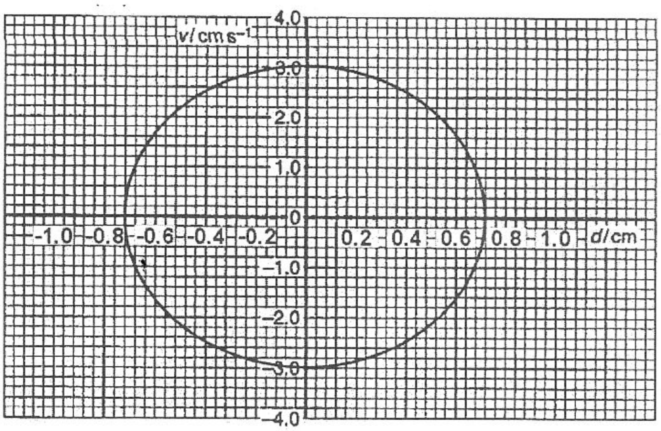


(ii) 1. Frequency
$f = 1/T$
$= 1/(1.5\text{ s})$
$= 0.6667$
$= 0.67\text{ Hz}$.

2. Amplitude
$x_0 = v_0/\omega$
$= v_0/(2\pi/T)$
$= (3.0\text{ cm s}^{-1})/(2\pi/1.5\text{ s})$
$= 0.7162$
$= 0.72\text{ cm}$.

(iii) The velocity-displacement equation
$v_0 = \pm\omega\sqrt{(x_0^2 - x^2)}$ is the equation of an ellipse.

This is illustrated in the figure below.



12 A metal tube of uniform cross-sectional area $1.3 \times 10^{-3}\text{ m}^2$ is sealed at one end. It floats upright in water, as shown in Fig. 1.2.

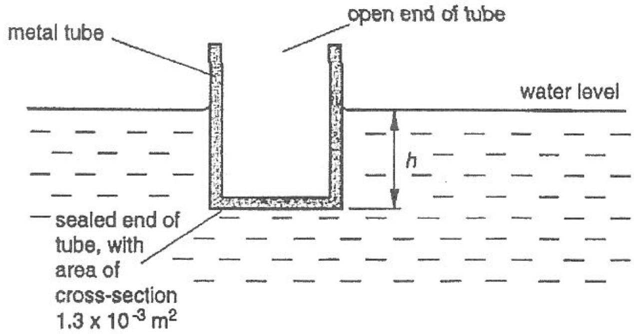


Fig. 1.2

When the tube is stationary, the base is 5.5 cm below the surface of the water.

The pressure p due to water of depth h is given by the expression

$$p = (9.8 \times 10^3)\,h,$$

where p is in pascals and h is in metres.

(i) 1. Calculate the force acting on the base of the tube due to the pressure of the water. [2]

2. State the direction of the force calculated in 1. [1]

3. State the weight of the tube. [1]

(ii) The tube is pushed vertically a short distance into the water and then released.

Fig. 1.3 shows the variation with time t of the depth h of the base of the tube below the surface.

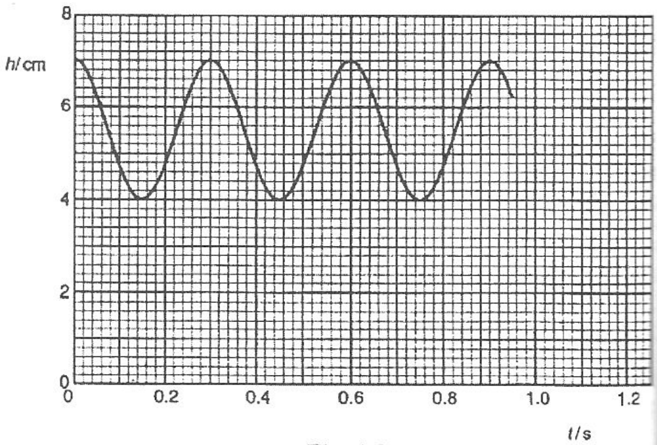


Fig. 1.3

1. Use Fig. 1.3 to determine the amplitude of the oscillations of the tube. [1]

2. Determine the frequency of the oscillations. [2]

3. Using your answers to 1. and 2., determine the maximum speed of the tube. [2]

N04/II/1 (part)

Solution

Using the equation, the pressure acting at the base of the tube is

$$p = (9.8 \times 10^3)\, h = (9.8 \times 10^3)\,(0.055 \text{ m}) = 539 \text{ Pa}$$

$$F = pA = (539 \text{ Pa})(1.3 \times 10^{-3} \text{ m}^2)$$

$$= 0.7007 = 0.70 \text{ N}$$

Upwards.

0.70 N (Principle of Floatation).

Amplitude x_0 = (7.0 cm – 4.0 cm) / 2 = 1.5 cm.

Frequency $f = 1/T$ (where T is the period of the oscillation, 0.30 s).
$f = 1/0.30$
$= 3.333$ Hz
$= 3.3$ Hz.

Maximum speed in simple harmonic motion $v_0 = \omega x_0$ where ω is the angular frequency and x_0 is the amplitude.
$v_0 = \omega x_0$
$= (2\pi f)x_0$
$= (2\pi)(3.333\text{Hz})(0.015 \text{ m})$
$= 0.3141$
$= 0.31 \text{ m s}^{-1}$

13

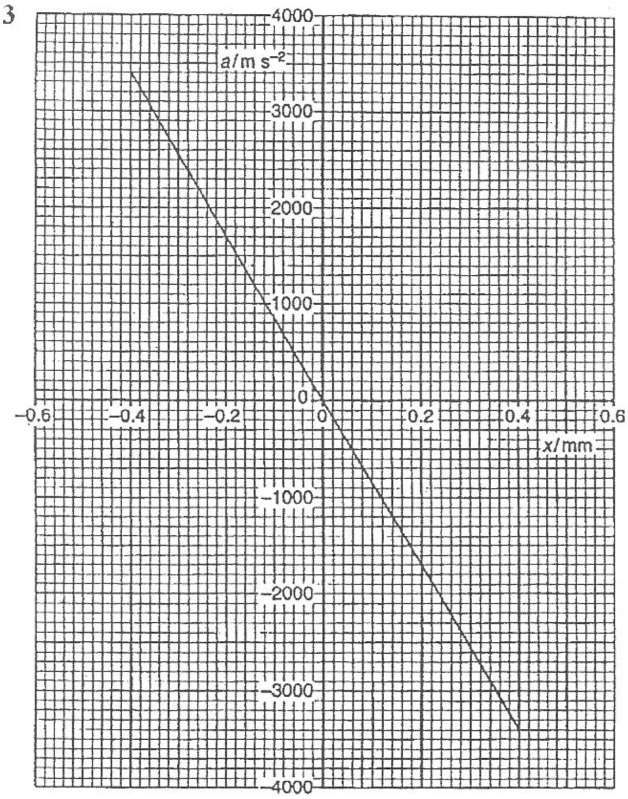


Fig. 3.1

Fig. 3.1 shows the variation with displacement x of the acceleration a of a particle P attached to the cone of a loudspeaker.

(a) Use Fig. 3.1 to

(i) explain why the motion of particle P is simple harmonic, [2]

(ii) show that the frequency of oscillation of particle P is 460 Hz. [2]

(b) (i) The magnitude of the gradient of the line in Fig. 3.1 is G. Show that, for a particle of mass m oscillating with amplitude A, its maximum kinetic energy E_{MAX} is given by [3]

$$E_{MAX} = \tfrac{1}{2} mGA^2.$$

(ii) Determine E_{MAX} for particle P of mass 2.5×10^{-3} kg. [2]

N06/II/3

Solution

(a) (i) Simple harmonic motion is oscillatory motion in which the acceleration a of a particle is directly proportional to its displacement x from its equilibrium position, and directed towards the equilibrium position, i.e. opposite to the direction of its displacement.

From the graph, it can be seen that the acceleration a of particle P is directly proportional to its displacement x (straight-line graph through origin); and a is directed towards the equilibrium position and opposite to the direction to x (negative gradient).

Thus the conditions for simple harmonic motion are satisfied.

(ii) The gradient of the graph is
$(-3400 - 3400 \text{ m s}^{-2})/(0.00040 - (-0.00040) \text{ m})$
$= -6800 \text{ m s}^{-2} / 0.00080 \text{ m}$
$= -8.5 \times 10^6 \text{ s}^{-2}$

Comparing this with the defining equation for simple harmonic motion $a = -\omega^2 x$, it can be seen that the gradient is equal in magnitude to ω^2.

$$\omega^2 = (2\pi f)^2 = 8.5 \times 10^6 \text{ s}^{-2}$$
$$2\pi f = 2916 \text{ s}^{-2}$$
$$f = 464 = 460 \text{ Hz}$$

(b) (i) As shown in (a)(ii), the gradient of the graph G has a magnitude equal to the square of the angular frequency, i.e. ω^2.

For a particle undergoing simple harmonic motion, its velocity $v = \pm\omega\sqrt{(A^2 - x^2)}$ and the maximum speed of the particle $v_{MAX} = \omega A$, which occurs when it is at its equilibrium position $x = 0$.

The maximum kinetic energy of the particle is:

$$E_{MAX} = \tfrac{1}{2}mv_{MAX}^2 = \tfrac{1}{2}m\omega^2 A^2 = \tfrac{1}{2}mGA^2.$$

(ii) From the graph, the amplitude $A = 0.40$ mm $= 0.40 \times 10^{-3}$ m.

$E_{MAX} = \tfrac{1}{2}mGA^2$
$= \tfrac{1}{2}(2.5 \times 10^{-3} \text{ kg})(8.5 \times 10^6 \text{ s}^{-2})(0.40 \times 10^{-3} \text{ m})^2$
$= 1.7 \times 10^{-3}$ J

14 A flat horizontal plate is made to vibrate in a vertical plane as shown in Fig. 2.1.

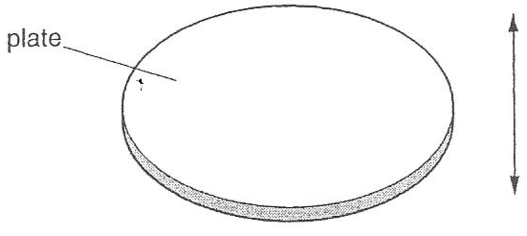


Fig. 2.1

The variation with displacement x of the acceleration a of the plate is shown in Fig. 2.2.

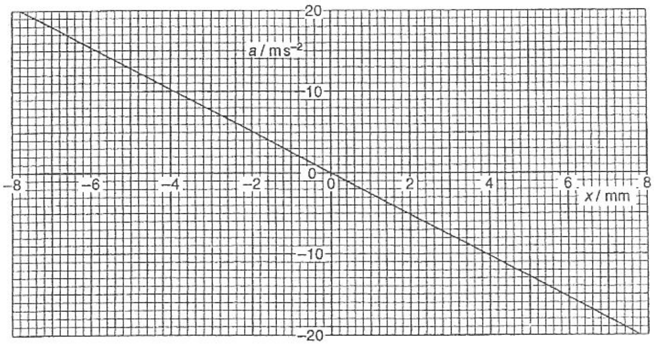


Fig. 2.2

(a) Explain how it may be deduced that the oscillations of the plate are simple harmonic.

....
....
....
.... [3]

(b) Some sand is sprinkled on to the plate. The amplitude of vibration of the plate is gradually increased from zero. At one particular amplitude, the sand is seen to lose contact with the plate.

(i) State the acceleration of the plate at which the sand first loses contact with the plate. Explain your reasoning.

....
....
....
.... [3]

(ii) Use Fig. 2.2 to determine the amplitude of vibration of the plate at which the sand first loses contact.

amplitude = ... mm [1]

N08/II/2

Solution

(a) It can be seen from the:

i. straight line graph passing through the origin that the acceleration a of the plate is directly proportional to the displacement x from its equilibrium position; and
ii. negative gradient that its acceleration a is opposite in direction to its displacement x, and towards the equilibrium position.

These two features defines simple harmonic motion.

(b) (i) -9.81 m s^{-2} – the acceleration of free fall.

When the sand is at its amplitude position, it is momentarily at rest and experiences a downward acceleration equal to that of free fall.

If the plate also has the same downward acceleration of free fall, then there is no relative acceleration and no relative velocity between the sand and the plate.

The contact force between the sand and the plate will disappear, and the sand loses contact with the plate.

(ii) The amplitude at which the acceleration of the plate is -9.81 m s^{-2} is 3.8 mm, as shown in Fig. 2.3.

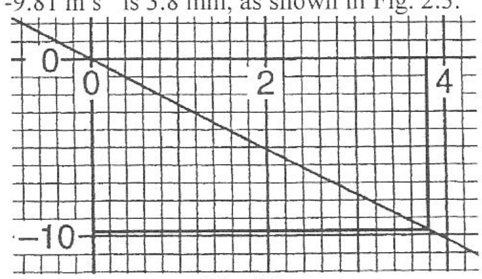


Fig. 2.3

Long Questions

(a) Distinguish between *frequency* and *angular frequency* for a body undergoing simple harmonic motion.

....

....

.... [2]

(b) A spring that has an unstretched length of 0.650 m is attached to a fixed point. A mass of 0.400 kg is attached to the spring and gently lowered until equilibrium is reached. The spring has then stretched elastically by a distance of 0.200 m.

Calculate, for the stretching of the spring,

(i) the loss in gravitational potential energy of the mass,
loss = ... J [1]

(ii) the elastic potential energy gained by the spring.
gain = ... J [2]

(c) Explain why the two answers to (b) are different.

....

....

.... [2]

(d) The load on the spring is now set into simple harmonic motion of amplitude 0.200 m.

Calculate

(i) the resultant force on the load at the lowest point of its movement,
resultant force = ... N [2]

(ii) the angular frequency of the oscillation,
angular frequency = ... rad s^{-1} [2]

(iii) the maximum speed of the mass.
maximum speed = ... m s^{-1} [1]

(e) Fig. 6.1 is a table of the energies of the simple harmonic motion. Complete the table.

	gravitational potential energy / J	elastic potential energy / J	kinetic energy / J	total energy / J
lowest point	0			
equilibrium position				
highest point				

Fig. 6.1

[5]

(f) On the axes of Fig. 6.2 below, sketch four graphs to show the shape of the variation with position of the four energies. Label each graph.

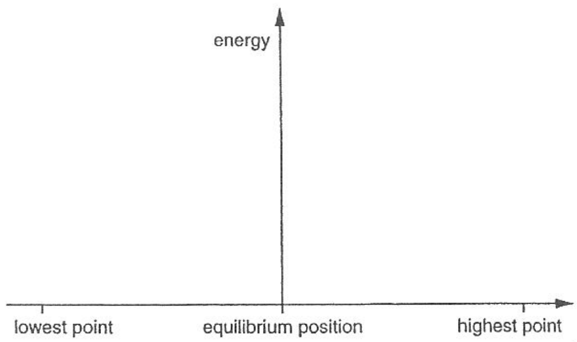


Fig. 6.2 [3]

N08/III/6

Solution

(a) *Frequency* is the number of complete oscillations per unit time, measured in hertz (Hz).

Angular frequency is the product of frequency and the angle subtended by the circumference of a circle (2π radians). It is a measure of the rate of change of angular displacement of the motion with time, measured in radians per second (rad s^{-1}).

(b) (i) Loss in gravitational potential energy

mgh = (0.400 kg)(9.81 N kg^{-1})(0.200 m) = 0.785 J

(ii) The spring constant

$k = F/e$ = (0.400 N x 9.81 N kg^{-1})/(0.200 m) = 19.62 N m^{-1}

The elastic potential energy stored

$U = ½ke^2$ = ½(19.62 N m^{-1})(0.200 m)^2 = 0.392 J

(c) The difference is work done against the external force needed to support the mass while lowering it gently. This force is the difference between the mass's weight and the tension in the spring. The work done is dissipated as heat.

(d) (i) At the lowest point, the total extension of the spring is 0.400 m.

Tension $T = ke$ = (19.62 N m^{-1})(0.400 m) = 7.848 N

Weight $W = mg$ = (0.400 kg)(9.81 N kg^{-1}) = 3.924 N

The resultant force
$F = T - mg$ = 7.848 N − 3.924 N = 3.92 N

(ii) Angular frequency of mass-spring system
$\omega = \sqrt{(k/m)} = \sqrt{(19.62 \text{ N m}^{-1} / 0.400 \text{ kg})}$ = 7.00 rad s^{-1}

(iii) Maximum speed
$v_0 = \omega x_0$ = (7.00 rad s^{-1})(0.200 m) = 1.40 m s^{-1}

(e) The completed table is at Fig. 6.3.

	gravitational potential energy / J	elastic potential energy / J	kinetic energy / J	total energy / J
lowest point	0	**1.57**	0	**1.57**
equilibrium position	**0.785**	**0.392**	**0.392**	**1.57**
highest point	**1.57**	0	0	**1.57**

Fig. 6.3

(f) The graph is shown at Fig. 6.4.

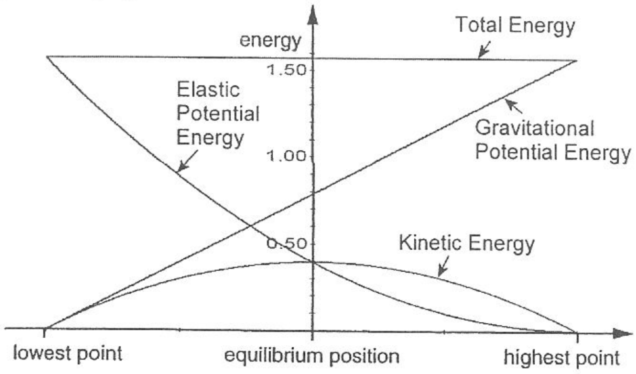


Fig. 6.4

16 (a) Define simple harmonic motion (s.h.m.).

....

....

.... [2]

(b) The cone of a loudspeaker is undergoing simple harmonic motion with a frequency of 620 Hz. The amplitude of the oscillations is 0.21 mm.

(i) Calculate, for the oscillations of the cone,

1. the angular frequency,
angular frequency = ... rad s^{-1} [1]

2. the maximum acceleration.
acceleration = ... m s^{-2} [2]

(ii) Use your answers in (i) to plot, on the axes of Fig. 1.1, a graph to show the variation with displacement x of the acceleration a of the cone.

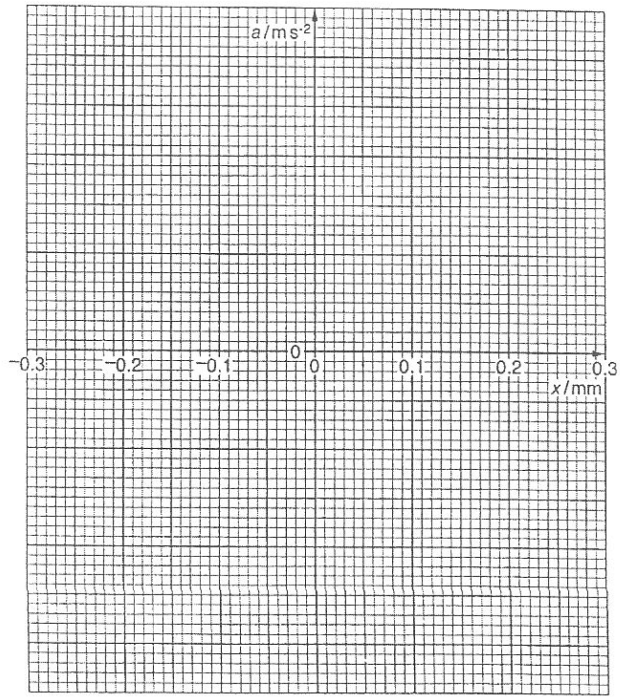


Fig. 1.1

(c) In normal use, the loudspeaker in (b) produces a range of frequencies of sound.
Suggest why it is important that the natural frequency of vibration of the cone of the loudspeaker is **not** within this range of frequencies.

....

....

....

....

.... [3]

N09/III/1

Solution

(a) Simple harmonic motion (s.h.m.) is a type of oscillatory motion of a body in which:

i. its acceleration a is directly proportional to its displacement x from the equilibrium position; and
ii. its acceleration a is always directed towards the equilibrium position, in a direction opposite to that of its displacement x.

(b) (i) 1. The angular frequency $\omega = 2\pi f$ where f is its frequency of oscillation.

$\omega = 2\pi f = 2\pi(620\text{ Hz}) = 3896\text{ rad s}^{-1}$.

angular frequency = <u>3.9 x 10^3 rad s^{-1}</u>

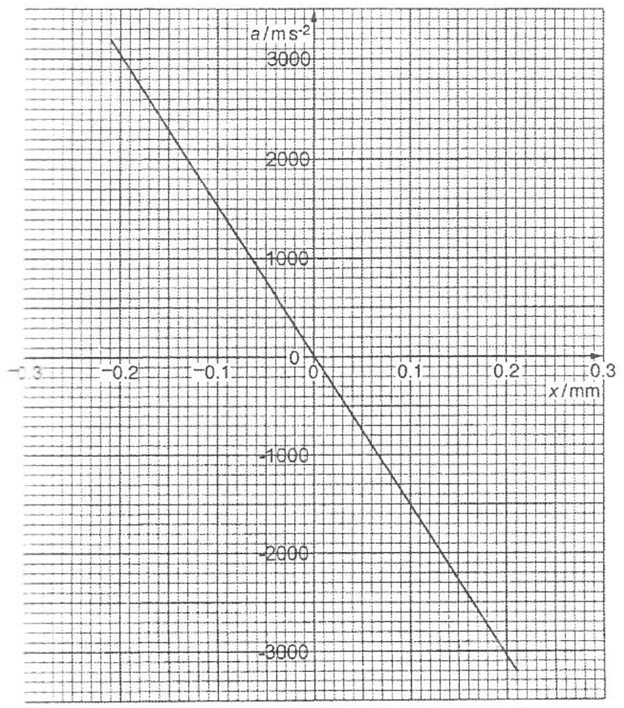
2. The acceleration a is related to angular frequency ω and displacement x by the equation: $a = -\omega^2 x$.

Hence the maximum acceleration $a_0 = -\omega^2 x_0$, where x_0 is the amplitude of oscillation.

$a_0 = -\omega^2 x_0 = -(3896)^2(0.21 \times 10^{-3}\text{ m}) = -3187\text{ m s}^{-2}$

acceleration = <u>3.2 x 10^3 m s^{-2}</u>

(ii) The graph is shown below.



(c) If the frequency of the sound produced matches the natural frequency of the cone of the loudspeaker, resonance will occur, causing the cone to vibrate at maximum amplitude. As a result, the frequency response of the loudspeaker will no longer be uniform, as it will vibrate more at certain frequencies and less at others. The sound output will be distorted.

Speakers that reproduce sound without unwanted frequency emphasis or resonance have high-fidelity (hi-fi). Those that resonate and distort the sound have "low-fidelity (lo-fi)".

17 (a) Define *force*. [2]

(b) A light helical spring is suspended vertically from a fixed position, as shown in Fig. 6.1.

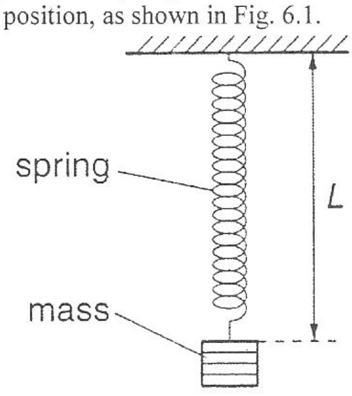


Fig. 6.1

Different masses are suspended from the spring. The weight W of the mass and the length L of the spring are noted.

The variation with weight W of the length L is shown in Fig. 6.2.

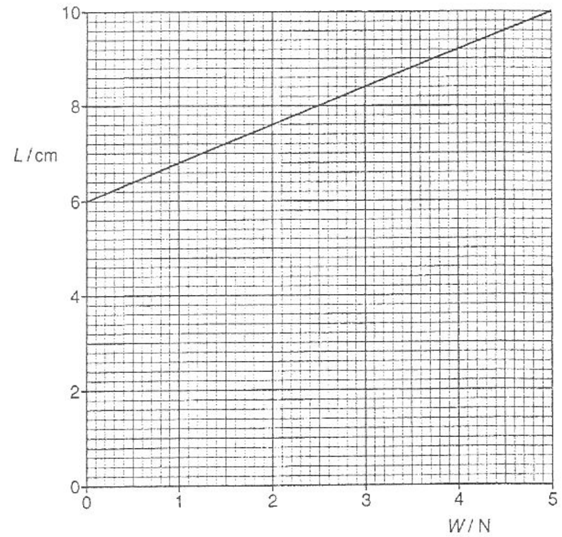


Fig. 6.2

(i) On Fig. 6.2, show clearly the area of the graph that represents the energy stored in the spring when the weight on the spring is increased from zero to 5.0 N. [1]

(ii) For a spring undergoing an elastic change, the force per unit extension of the spring is known as the force constant k.

Show that the energy E stored in the spring for an extension x of the spring is given by the expression

$$E = \tfrac{1}{2}kx^2.$$

[2]

(c) A mass of weight 4.0 N is suspended from the spring in (b).

When the mass is stationary, it is then pulled downwards a distance of 0.80 cm and held stationary.

(i) Determine the total length of the spring.

length = ... cm [1]

(ii) For the increase in extension of 0.80 cm, determine the magnitude of the change in

1. the gravitational potential energy of the mass,

change = ... J [2]

2. the elastic potential energy of the spring.

change = ... J [3]

(iii) Use your answers in (ii) to show that the work done to cause the additional extension of 0.80 cm is 4.0×10^{-3} J. [1]

(d) The mass in (c) is now released. The mass performs simple harmonic motion.

(i) State the total energy of oscillation of the mass.

energy = ... J [1]

(ii) Calculate, for the mass,

1. its maximum speed,

speed = ... m s^{-1} [2]

2. the frequency of oscillation.

frequency = ... Hz [3]

(e) The spring in (d) is assumed to be light. In practice, the spring will have some mass.

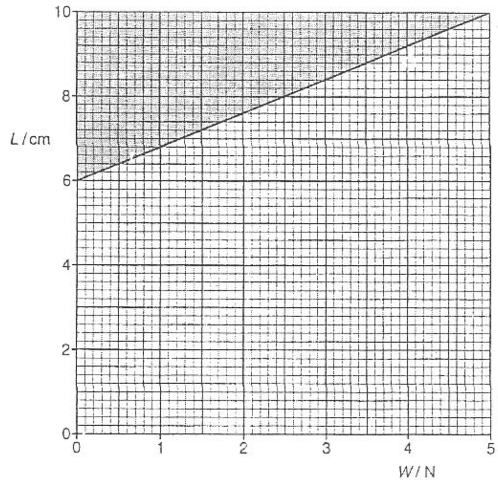
Assuming that the spring constant k is unchanged, suggest and explain the effect on the frequency of oscillation of having a spring with mass. [2]

N10/III/6

Solution

(a) The resultant force acting on a body is the rate of change of momentum of the body, and acts in the direction of the change in momentum. The unit for force is the newton, which is the force required to accelerate a mass of 1 kg at a rate of 1 m s^{-2} in the direction of the force.

(b) (i) The energy stored is represented by the shaded area:



(ii) The energy E stored in a spring is the work done to stretch it, which is the sum of the spring's force (F) multiplied by incremental displacements (δx). This can be found by integration with limits from 0 to x,

$$E = \int F\,dx = \int kx\,dx = \tfrac{1}{2}kx^2$$

Alternatively, this is also the triangular area under the force-extension graph.

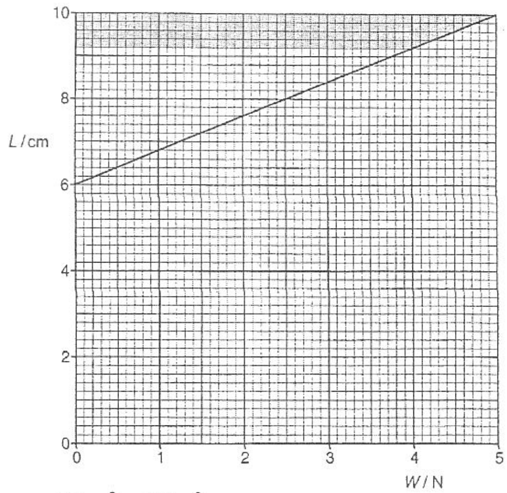
$$E = \tfrac{1}{2}Fx = \tfrac{1}{2}(kx)x = \tfrac{1}{2}kx^2$$

The spring constant for this spring
$k = (5.0 \text{ N})/(0.04 \text{ m}) = 125 \text{ N m}^{-1}$.

(c) (i) From the graph, the length of the spring is 9.2 cm when the weight is 4.0 N. Adding to this another 0.8 cm, the total length becomes 10 cm.
length = 10 cm

(ii) 1. The change in gravitational potential energy $= (mg)(h) = (4.0)(0.8 \times 10^{-2}) = 0.032$ J.

2. The change in elastic potential energy is the shaded area shown:



$= \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$
$= \tfrac{1}{2}(125)(0.04)^2 - \tfrac{1}{2}(125)(0.032)^2$
$= 0.036$ J

(iii) Work done to cause the additional 0.80 cm of extension = gain in elastic potential energy – loss in gravitational potential energy = 0.036 – 0.032 = 4.0×10^{-3} J.

(d) (i) 4.0×10^{-3} J.

(ii) 1. Maximum speed occurs when the mass possesses maximum kinetic energy while moving pass its equilibrium position, and its kinetic energy is equal to its total energy:

$$\tfrac{1}{2}mv^2 = 4.0 \times 10^{-3} \text{ J}$$
$$\tfrac{1}{2}(4.0/9.81)v^2 = 4.0 \times 10^{-3} \text{ J}$$
$$\tfrac{1}{2}(0.4077)v^2 = 4.0 \times 10^{-3} \text{ J}$$
$$v = 0.14 \text{ m s}^{-1}$$

2. For spring mass systems,

$$f = (1/2\pi)\sqrt{(k/m)}$$
$$= (1/2\pi)\sqrt{(125/0.4077)}$$
$$= 2.8 \text{ Hz}$$

(e) If the spring has mass, its frequency of oscillation will decrease.

If the spring has mass m_{spring}, the inertia of the system will increase since the mass of the system becomes effectively $m + fm_{spring}$ where f is some fraction depending on the shape of the spring.

Since the spring's force constant (k) remains unchanged, the acceleration of the system will decrease according to Newton's second law of motion.

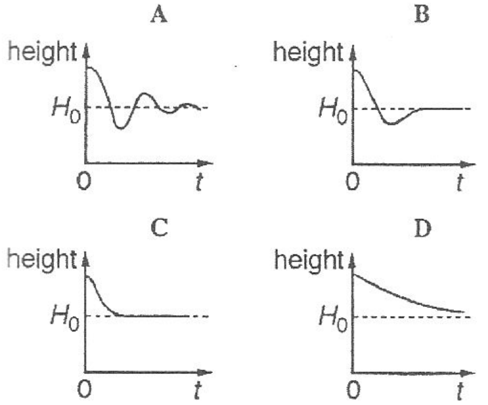
Using the defining equation for simple harmonic motion ($a = -\omega^2 x$), a smaller acceleration will result in a lower angular frequency and frequency of oscillation.

Oscillations & Resonance

18 It is important that a car suspension system should be critically damped.

The equilibrium height above the ground of the bodywork of such a car is H_0. The body of the car is raised to a greater height H and released at time $t = 0$. Assume that the car tyres remain in contact with the ground throughout and there is critical damping.

Which graph shows how the height of the car body above the ground varies with time?

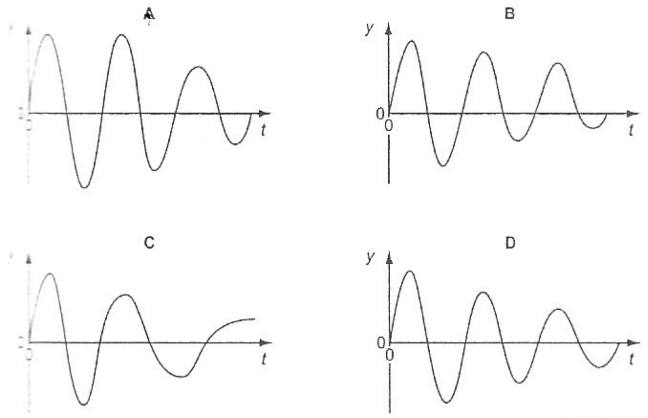


N02/I/13

Solution Answer: **C**

Critical damping occurs when the system returns to its equilibrium in the shortest possible time without overshooting the equilibrium position (Answer C).
Answer A is light damping.
Answer B is slightly under critical damping.
Answer D is heavy damping.

19 Which diagram shows the variation with time t of the displacement y of an object that is undergoing lightly-damped oscillations?



N08/I/16

Solution Answer: **D**

With light damping, there is a progressive decrease in the amplitude of the oscillation.

Answer A is wrong because the first two peaks are of the same height.

Answer B is wrong because the amplitude is not continuously decreasing (towards the end).

Answer C is wrong because the graph is no longer sinusoidal towards the end.

The damped graph is still sinusoidal, as seen from the complete equation for damped simple harmonic motion (mass-spring system):

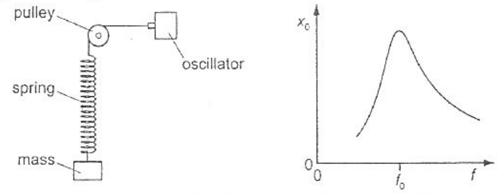
$$y = y_0 \, e^{\frac{-bt}{2m}} \sin(\omega t)$$

where

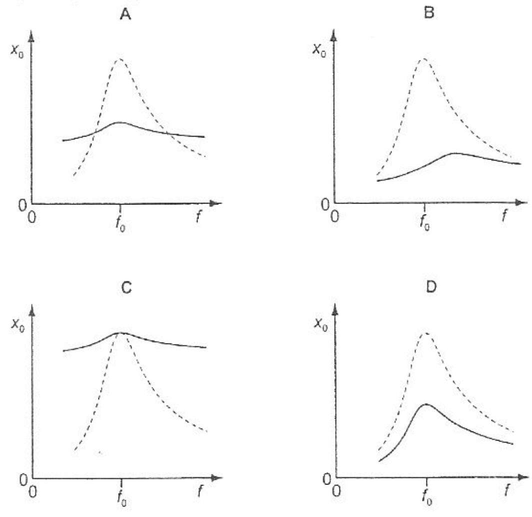
$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

where m is the mass, k the spring constant and b the damping constant.

20 A mass, suspended from a helical spring, is made to oscillate.
The graph shows the variation with frequency f of the amplitude x_0 of vibration of the mass.



A sheet of cardboard of negligible mass is now fixed to the mass on the spring to cause light damping of the oscillations.
Which graph shows how x_0 will vary with f over the same frequency range?



N10/I/16

Solution Answer: **D**

Damping causes a decrease in the amplitude x_0 at all frequencies. The resonant frequency f_0 is slightly reduced.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

where k is the spring constant, m the mass and b the damping constant.

Short Questions

21 Fig. 4.1 shows the variation with frequency f of the amplitude x_0 of the forced oscillations of a machine.

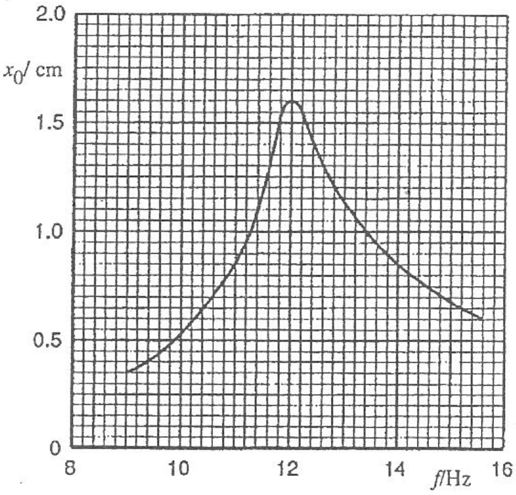


Fig. 4.1

(a) State

(i) what is meant by a forced oscillation, [1]

(ii) the name of the effect illustrated in Fig. 4.1. [1]

(b) At any value of frequency, the oscillations of the machine are simple harmonic.

(i) Calculate, for the machine vibrating at maximum amplitude, the maximum magnitudes of

1. the linear speed

maximum speed = ... ms^{-1} [3]

2. the linear acceleration.

maximum acceleration = ... ms^{-2} [2]

(ii) Determine the time interval between a maximum linear speed and the subsequent maximum linear acceleration. time = ... s [2]

(c) In order to reduce the maximum amplitude of the oscillations, the mass of the machine is increased. Suggest what effect this increased mass will have on the shape of Fig. 4.1. You may draw on Fig. 4.1 if you wish. [3]

N05/II/4

Solution

(a) (i) *Forced* oscillation refers to a situation in which a system is made to oscillate through the application of an external vibrating force of a frequency not necessarily equal to the natural frequency of the system.

(ii) Resonance.

(b) (i) **1.** Maximum velocity

$v_0 = x_0\omega$
$= x_0(2\pi f)$
$= (0.016 \text{ m})(2\pi \times 12 \text{ Hz})$
$= 1.206$
$= 1.2 \text{ m s}^{-1}$

2. Maximum magnitude of acceleration

$a_0 = \omega^2 x_0$
$= (2\pi f)^2 x_0$
$= (2\pi \times 12 \text{ Hz})^2 (0.016 \text{ m})$
$= 90.96$
$= 91 \text{ m s}^{-2}$

(ii) The acceleration-time graph leads the velocity-time graph by phase angle of $\pi/2$ rads, equivalent to a time interval of $T/4$.
The period $T = 1/f = 1/12$ s
Hence the time interval is 1/48 s = 0.02083 = 0.021 s.

(c) Generally a system's natural frequency f_0 varies inversely with mass, e.g. for a mass spring system with spring constant k and mass m, $f_0 = (1/2\pi)\sqrt{(k/m)}$.

By increasing the mass m, the system's natural frequency f_0 decreases and the peak of the graph shifts to a lower frequency. As a result, the machine's vibration (12 Hz) is no longer near the system's natural frequency. The system is no longer in resonance hence the amplitude of vibration at 12 Hz is reduced (refer to graph below).

Note:

The new frequency response graph has a peak which is higher and sharper than the original one due to a higher quality factor (Q factor) in the new system.

$$Q = \sqrt{(mk)} / R$$

where m is the mass of the oscillating object, k the spring constant and R the mechanical resistance (degree of damping). The higher the Q factor, the higher and sharper the resonant peak.

Increasing the mass of the system leads to a higher and sharper resonant peak. Conversely, reducing the mass leads to a lower and less sharp resonant peak. In fact if the mass is too low, resonance cannot be detected since the presence of inertia (in the form of a mass) is a prerequisite for simple harmonic motion of a mass-spring system.

The resultant graph is illustrated in Fig. 4.2 below as the solid curve (the original graph is shown dotted).

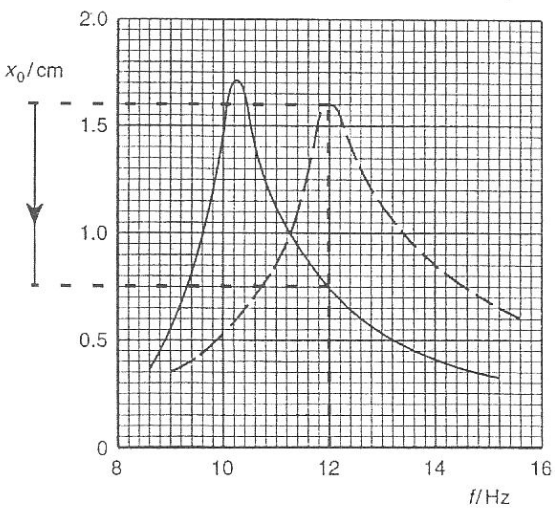


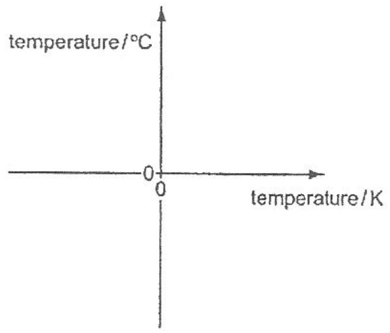
Fig. 4.2

TOPIC 9 Thermal Physics

Temperature Scales

1 A student draws a linear graph on the axes shown in order to convert temperatures in kelvin to temperatures in degrees Celsius.

What is the intercept on the vertical axis and the gradient of the line?



	intercept	gradient
A	0	1/273
B	−273	1
C	+273	1
D	0	273

N05/I/13

Solution Answer: **B**

The relationship between the Celsius scale °C and the Kelvin scale K is: °C = K − 273.15.

Plotting °C on the y-axis and K on the x-axis and comparing it with the equation of a straight line $y = mx + c$ where m is the gradient of the graph and c the y-intercept, it can be seen that the gradient is 1 and the y-intercept is -273.15.

Short Questions

2 *(a)* State two reasons why the temperature of a body is **not** a measure of the quantity of thermal energy in the body. [2]

(b) The variation with thermodynamic temperature T of the resistance R of a thermistor is shown in Fig. 4.1 .

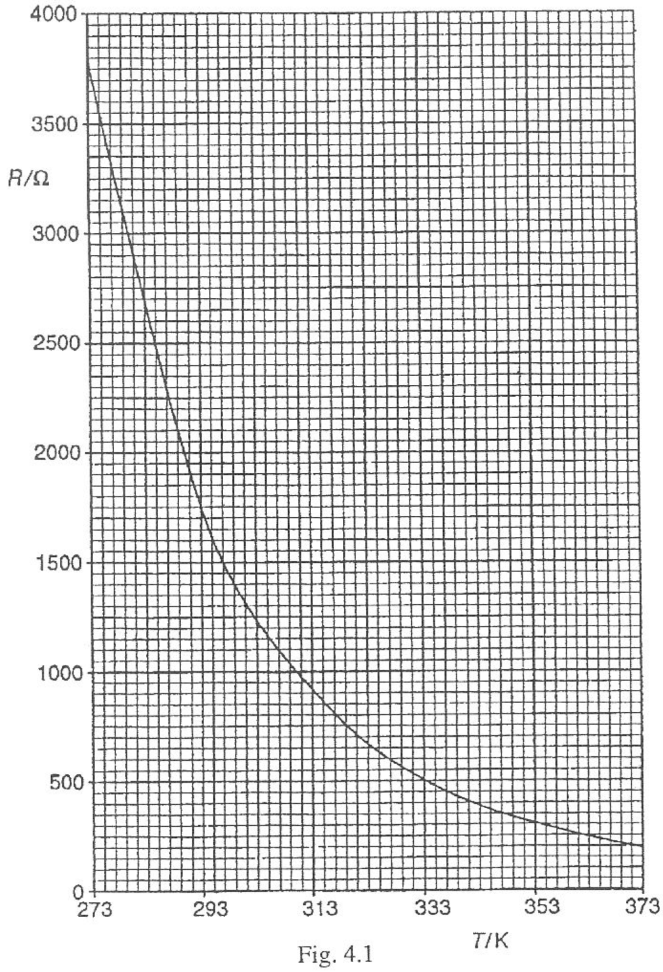


Fig. 4.1

(i) Suggest, with a reason, why the thermistor, used as a thermometer, is more appropriate for measuring temperatures in the range 273 K to 293 K than in the range 353 K to 373 K. [2]

(ii) Use Fig. 4.1 to determine the temperature corresponding to a thermistor resistance of 1500 Ω when measured on the thermodynamic scale of temperature.

temperature = ... K [1] *(c) The thermistor is connected into the circuit of Fig. 4.2.

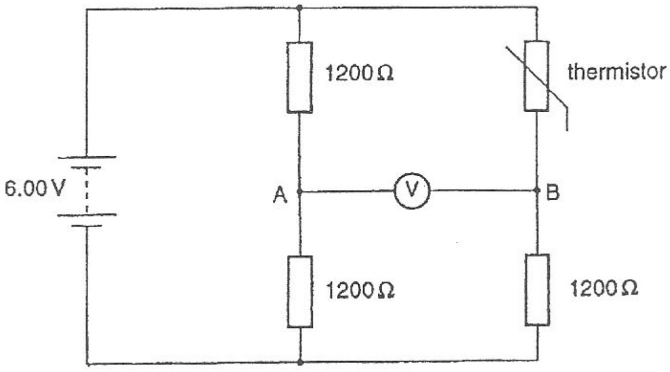


Fig. 4.2

The voltmeter connected between A and B has infinite resistance. The battery has e.m.f. 6.00 V and negligible internal resistance. The resistors each have a resistance of 1 200 Ω.

(i) Determine the thermodynamic temperature at which the voltmeter reads zero. Explain your working.

temperature = ... K [3]

(ii) The temperature of the thermistor is now changed and the voltmeter reads 1.20 V.

1. Suggest why the thermistor could be at one of two different thermodynamic temperatures.[2]

2. Calculate the lower of these two thermodynamic temperatures.

temperature = ... K [3]

N06/II/4

Solution

(a) 1. Temperature is a measure of the *average* kinetic energy of the molecules whereas the thermal energy in a body is its *total* energy.

2. Temperature only measures the *kinetic* energy of the molecules whereas thermal energy includes both their *kinetic* and *potential* energy.

(b) (i) This is because the graph is steeper between 273 K and 293 K as compared to between 353 K and 373 K.

This means that the rate of change of resistance with respect to temperature is greater between 273 K and 293 K as compared to between 353 K and 373 K.

Therefore, the thermometer is more sensitive to temperature changes between 273 K and 293 K as compared to between 353 K and 373 K.

(ii) Reading from the graph, the temperature is 297 K.

(c) (i) In order for the voltmeter to read zero, there must be no potential difference between points A and B, i.e. $V_{AB} = 0$. This means that the potential at A must be equal to the potential at B, i.e. $V_A = V_B$.

Using the potential divider principle, the potential difference across the lower resistor on the side of A is $[1200\ \Omega / (1200 + 1200)\ \Omega] \times 6.00\ V = 3.00\ V$.

For $V_A = V_B$, the potential difference across the lower resistor on the side of B must also be 3.00 V.
$[1200\ \Omega / (R_T + 1200)\ \Omega] \times 6.00\ V = 3.00\ V$.
$R_T = 1200\ \Omega$ (where R_T is the thermistor's resistance).

Reading from the graph, when $R_T = 1200\ \Omega$, temperature $T = 304$ K.

(ii) 1. A potential difference of 1.20 V could mean either $V_A > V_B$ or $V_A < V_B$.

$V_A > V_B$ means that $V_{AB} = 1.20$ V.
The thermistor has a resistance higher than 1200 Ω and a temperature lower than 304 K.

$V_A < V_B$ means that $V_{AB} = -1.20$ V.
The thermistor has a resistance lower than 1200 Ω and a temperature higher than 304 K.

2. At the lower temperature, the thermistor has a resistance higher than 1200 Ω, hence $V_A > V_B$ which means that $V_{AB} = 1.20$ V.

Since V_A is held permanently at 3.00 V, this means that $V_B = 1.80$ V.

Applying the potential divider principle,
$[1200\ \Omega / (R_T + 1200)\ \Omega] \times 6.00\ V = 1.80\ V$.
$R_T = 2800\ \Omega$ (where R_T is the thermistor's resistance).

From the graph, when $R_T = 2800\ \Omega$, the temperature is 282 K.

Specific Heat Capacity and Latent Heat

3 What is meant by the term *specific heat capacity* of a substance?

A The thermal energy required to change the state of 1 kilogram of the substance.

B The thermal energy required to change the state of the substance without change of temperature.

C The thermal energy required to raise the temperature of 1 kilogram of the substance by 1 kelvin.

D The thermal energy required to raise the temperature of the substance by 1 kelvin.

N03/I/12

Solution Answer: **C**

The *specific heat capacity* of a substance is the thermal energy required to raise the temperature of 1 kilogram of the substance by 1 kelvin.
The unit is J kg^{-1} K^{-1}.

4 In an experiment to determine the specific heat capacity of a liquid by an electrical method, a student obtained the following results.

mass of liquid heated	1.5 kg
initial liquid temperature	300 K
final liquid temperature	357 K
electrical power of heater	1.0 kW
time of heating	180 s

What is the specific heat capacity of the liquid?

A 2.1 J kg^{-1} K^{-1}

B 18 J kg^{-1} K^{-1}

C 1800 J kg^{-1} K^{-1}

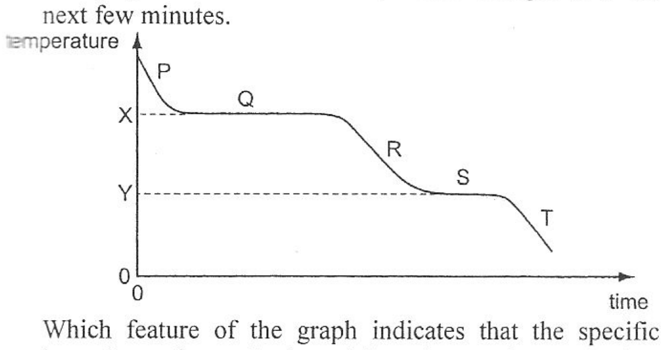
D 2100 J kg^{-1} K^{-1}

N04/I/15

Solution Answer: **D**

$Q = Pt = mc\Delta\theta$
$c = (Pt)/(m\Delta\theta)$
= (1000 W x 180 s)/(1.5 kg x 57 K) = 2105 = 2100 J kg^{-1} K^{-1}.

5 A vapour in a container is at a high temperature and loses heat to its surroundings.
The graph shows how its temperature changes over the next few minutes.



Which feature of the graph indicates that the specific latent heat of vaporisation of the substance is greater than its specific latent heat of fusion?

A The gradient of the graph at P is greater than the gradient at R.

B The gradient of the graph at T is greater than the gradient at R.

C The length of the line Q is greater than the length of the line S.

D The value of X is greater than the value of Y.

N08/I/17

Solution Answer: **C**

Latent heat = rate of heat loss during change of state x time for change of state to take place.

The rate of heat loss to surrounding during condensation (line Q) should have been higher than that during freezing (line S) due to higher temperature during condensation, yet the time taken for the substance to condense (line Q) is greater than that during freezing (line S). This shows that the specific latent heat of vaporisation is significantly greater than the specific latent heat of fusion.

Answers A and B are wrong because the gradient of P and T being greater than the gradient at R means that the specific heat capacities of the substance in the vapour and solid states, respectively, are lower than that in the liquid state.

Answer D is wrong because X being greater than Y means that the substance's boiling point is higher than its melting point.

6 An electric kettle contains 500 g of water at 15 °C. The heating element of the kettle is rated at 2.2 kW and the specific heat capacity of water is 4.2 x 10^3 J kg^{-1} K^{-1}.

What is the minimum time it takes to raise the temperature of the water to 100 °C?

A 22 s   B 81 s   C 95 s   D 8.1 x 10^4 s

N08/I/19

Solution Answer: **B**

Heat absorbed $Q = mc\Delta\theta$

Where m is the mass of the water, c its specific heat capacity and $\Delta\theta$ the increase in temperature.

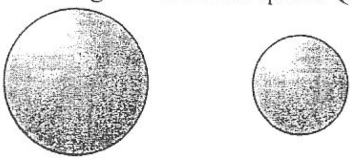
$$Pt = mc\Delta\theta$$

$$t = mc\Delta\theta / P$$

= (0.500 kg)(4.2 x 10^3 J kg^{-1} K^{-1})(100 – 15)K / (2200 W)

= 81 s

7 Two spheres P and Q are both made of steel. Sphere P has a radius that is larger than that of sphere Q.



Initially, sphere P is at temperature T_P and sphere Q is at temperature T_Q, where $T_P > T_Q$.
There spheres are brought into contact and their final temperature is T. No thermal energy is transferred from the spheres to the surroundings.
Which expression gives the relation between T_P, T_Q and T?

A $(T_P - T) = (T - T_Q)$

B $(T_P - T) > (T - T_Q)$

C $(T_P - T) < (T - T_Q)$

D $(T_P - T) = (T + T_Q)$

N10/I/19

Solution Answer: **C**

Since there is no thermal energy lost to the surroundings, the thermal energy lost by P must be equal to the thermal energy gained by Q, so that the final equilibrium temperature T is between T_P and T_Q. Since they are made of the same material, their specific heat capacity c is the same.

$$Q_{\text{lost by P}} = Q_{\text{gained by Q}}$$
$$m_P c(T_P - T) = m_Q c(T - T_Q)$$
$$(T_P - T)/(T - T_Q) = m_Q/m_P$$

Short Questions

8 A student sets up the apparatus illustrated in Fig. 2.1 in order to determine a value for the specific latent heat of fusion of ice.

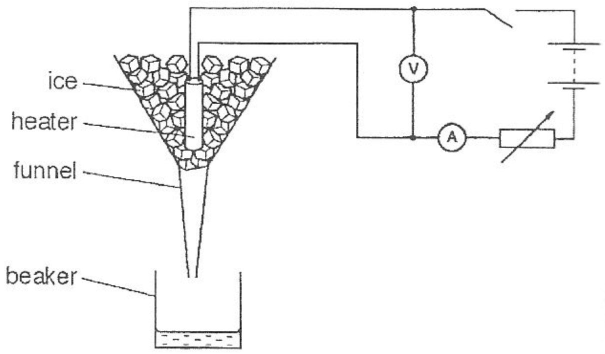


Fig. 2.1

A heater is placed in the funnel, surrounded by pure melting ice. The student measures the mass of melted ice in the beaker at regular time intervals before and after switching on the heater. The variation with time t of the mass m of melted ice in the beaker is shown in Fig 2.2.

During the heating process, the current is adjusted so that the readings on the ammeter and voltmeter are constant.

(a) By reference to Fig. 2.2,

(i) suggest a time at which the heater is switched on, [1]

(ii) determine the mass of ice melted in 1.0 minute

1. with the heater switched off,

2. with the heater switched on. [2]

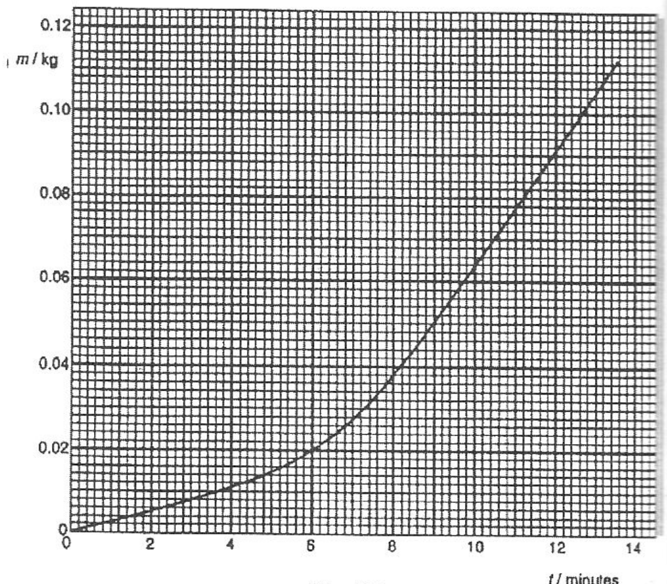


Fig. 2.2

(b) The readings of the ammeter and the voltmeter are 5.2 A and 11.5 V respectively. Use your answers in *(a)* to calculate a value for the specific latent heat of fusion of ice. [3]

(c) State and explain the effect on your calculated value for the specific latent heat of fusion if ice taken directly from a freezer were used to replace the ice in the funnel. [2]

N04/II/2

Solution

(a) (i) 4 minutes.

(ii) 1. 0.011 kg / 4 minutes
= 2.750×10^{-3}
= 2.8×10^{-3} kg

2. (0.105 – 0.05) kg / (13 – 9) minutes
= 1.375×10^{-2}
= 1.4×10^{-2} kg

(b) Amount of ice melted in one minute due to heater alone = 1.375×10^{-2} kg – 2.750×10^{-3} kg = 0.011 kg.

Heater's power $P = IV = 5.2$ A x 11.5 V = 59.8 W.

Amount of heat absorbed by ice in one minute
= 59.8 W x 60 s = 3588 J

Specific latent heat of fusion of ice
l_f = 3588 J / 0.011 kg
= 3.262×10^5
= 3.3×10^5 J kg^{-1}

(c) The calculated value of the specific latent heat of fusion will be an over-overestimate of the true value.
Ice taken from the freezer is below the melting point of water (0 °C).
Part of the heater's energy is used to raise the ice's temperature to its melting point, resulting in more energy used than is necessary.

Long Questions

9 (*a*) In a space, such as a swimming pool enclosure, water at 30°C and water vapour, also at 30°C, coexist.

(i) Compare the pattern of movement and the speed of molecules in water and water vapour at the same temperature. [4]

(ii) State what is meant by the *internal energy* of a system. [2]

(iii) Using your answer to (ii), compare the internal energy per unit mass of water and water vapour at the same temperature. [3]

(iv) Explain, in terms of internal energy, why the specific latent heat of vaporisation of a substance is greater than its specific latent heat of fusion. [2]

(*b*) A canal connects a reservoir to a town. The canal supplies the town with fresh water. During a summer month, the town is supplied with water at an average rate of 2.7 m^3 s^{-1}. The canal is 51 km long and 9.2 m wide. In the daytime, the average power from the Sun that is absorbed per unit area by the surface of the water is 900 Wm^{-2}. The temperature of the water does not change.

(i) Explain why more water needs to be supplied from the reservoir than is used by the town. [1]

(ii) The density of water is 1000 kg m^{-3} and the specific latent heat of vaporisation of water is 2.26×10^6 J kg^{-1}. Calculate the rate at which water needs to be supplied from the reservoir. [4]

(iii) Suggest **two** ways in which the loss of water from the canal could be reduced. Indicate the problems that could arise if your suggestions were to be implemented. [4]

N06/III/2

Solution

(a) (i) Water in both liquid and vapour states is a fluid, in which particles have freedom of motion and move about in continuous random motion.

However, water molecules in the liquid state are much closer spaced compared to those in the vapour state. Hence liquid molecules have much more restricted motion whereas vapour molecules move over larger distances.

Nevertheless, the root-mean-square speed of water molecules in both liquid and vapour states are the same. This is because root-mean-square speed is proportional to the square root of thermodynamic temperature. Fluids at the same temperature have the same average kinetic energy and same root-mean-square speed, regardless of whether they are in the liquid or vapour state.

(ii) The *internal energy* of a system is the sum of the kinetic energy of its molecules due to their random motion, and potential energy due to intermolecular forces of attraction between molecules.

(iii) The internal energy per unit mass of water is lower than that of water vapour.

This is <u>not</u> due to kinetic energy differences, since the average kinetic energy of liquid molecules at 30 °C is the same as that for vapour molecules at 30 °C (average kinetic energy is directly proportional to thermodynamic temperature).

The difference is due to potential energy differences as a result of intermolecular forces of attraction between molecules. Liquid molecules are spaced close to one another hence their potential energy is lower; vapour molecules are spaced far apart hence their potential energy is greater.

(iv) When water changes into vapour state at a constant temperature (100 °C), there is no change in the molecules' average kinetic energy but a large amount of work has to be done to separate the molecules to a large distance apart (approximately 2.09×10^6 J kg^{-1}) and a smaller amount to push back the atmosphere (approximately 1.70×10^5 J kg^{-1}). This work done is converted into the potential energy of the water vapour. Hence the specific latent heat of vaporisation is very large (2.26×10^6 J kg^{-1}).

When water changes from solid to liquid state at a constant temperature (0 °C), there is also no change in the molecules' average kinetic energy. Furthermore, the difference in volume between water in solid and liquid states is negligible hence there is negligible work done against intermolecular forces of attraction and by the atmosphere. A relatively smaller amount of energy (3.35×10^5 J kg^{-1}) is required to modify the intermolecular bonds to allow relative motion between molecules.

(b) (i) Some of the water is vapourised by the solar energy and escape into the atmosphere.

(ii) The surface area of the canal
$A = 51{,}000 \text{ m} \times 9.2 \text{ m} = 4.692 \times 10^5 \text{ m}^2$.

The power P incident on the water surface is related to its intensity I by the equation:

$$P = IA$$
$$= (900 \text{ W m}^{-2})(4.692 \times 10^5 \text{ m}^2)$$
$$= 4.223 \times 10^8 \text{ W}$$

The mass rate at which water is vapourised is:

$4.223 \times 10^8 \text{ J s}^{-1} / 2.26 \times 10^6 \text{ J kg}^{-1} = 186.9 \text{ kg s}^{-1}$

The volume rate at which water is vapourised is:
$186.9 \text{ kg s}^{-1} / 1000 \text{ kg m}^{-3} = 0.1869 \text{ m}^3 \text{ s}^{-1}$

Therefore, the rate at which water needs to be supplied from the reservoir
$= 2.7 + 0.1869 \text{ m}^3 \text{ s}^{-1}$
$= 2.887 = 2.9 \text{ m}^3 \text{ s}^{-1}$

(iii) One way is to cover up the canal and make it underground to shield it from the sunlight.

The cost of implementation will be prohibitive considering that it is 51 km long and not economically justifiable.

Another way is to narrow the canal to reduce the surface area of the water exposed to the Sun.

However, this will reduce the rate of flow of water through the canal and the rate of supply of water to the town.

10 (a) State what is meant by *latent heat of vaporisation.* [2]

(b) A student carries out an experiment to determine the specific latent heat of vaporisation of water.
Water is boiled in a beaker by means of an electric heater, as shown in Fig. 1.1.

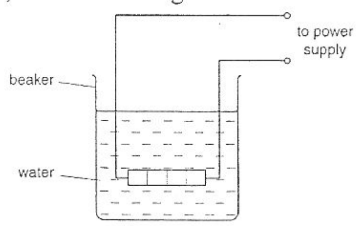


Fig. 1.1

The power P supplied to the heater is measured. When the water is boiling at a constant rate, the mass m of water evaporated in 5.0 minutes is determined. Data for the power P and the mass m for two different values of P are shown in Fig. 1.2.

P / W	m / g
140	14.1
95	8.2

Fig. 1.2

(i) Suggest why, in order to obtain a reliable result for the specific latent heat, the mass m is determined for two different values of P. [1]

(ii) Calculate the value for the specific latent heat L of vaporisation.

L = ... J g^{-1} [3]

N10/III/1

Solution

(a) The latent heat of vapourisation of a substance is the amount of thermal energy required to convert the substance from liquid to vapour phase (or vice versa), without a change in temperature.

(b) (i) This is to allow the power loss (P_{loss}) to be eliminated from the calculation of the specific latent heat of vaporisation. Since the temperature in both instances is the same, the power loss is also the same.

(ii) Heat absorbed by the boiling water $Q = Pt = ml_v$.
Using the first set of values,
$(140 - P_{loss})(300) = (14.1 \times 10^{-3})l_v$ ... (1)
Using the second set of values,
$(95 - P_{loss})(300) = (8.2 \times 10^{-3})l_v$ ... (2)
Subtracting (2) from (1),
$13500 \text{ J} = (5.9 \times 10^{-3} \text{ kg})l_v$
$l_v = 2.3 \times 10^6 \text{ J kg}^{-1} = 2300 \text{ J g}^{-1}$.

First Law of Thermodynamics

11 The first law of thermodynamics may be stated in terms of

ΔU, the **increase** in internal energy of the system,
Q, the energy transferred **to** the system by heating,
W, the work done **on** the system.

What is the formula connecting these quantities?

A $\Delta U = Q + W$
B $\Delta U = Q - W$
C $\Delta U = -Q - W$
D $\Delta U = W - Q$ N02/I/12

Solution Answer: **A**
For the terms defined in the manner above, the first law of thermodynamics can be stated as: the increase in internal energy of the system is equal to the sum of the heat absorbed by the system and the work done on the system.

12 The air in an aircraft when travelling has

8 MJ of kinetic energy as a result of the motion of the aircraft,

30 MJ of kinetic energy as a result of the random movement of the air molecules,

75 MJ of potential energy as a result of the altitude of the aircraft,

–3 MJ of potential energy as a result of intermolecular attraction between the air molecules.

What is the internal energy of the air in the aircraft?

A 27 MJ C 35 MJ
B 33 MJ D 110 MJ N03/I/13

Solution Answer: A
The kinetic and potential energy terms in internal energy refers only to the kinetic energy due to random motion of the molecules (30 MJ) and potential energy due to intermolecular forces (-3 MJ), and not the kinetic and potential energy of the entire bulk of gas.

13 The contents of a refrigerator are at a constant temperature, and the surroundings of the refrigerator are at a higher temperature. Because of this, thermal energy flows into the refrigerator from outside, and is removed at the same rate by the cooling mechanism.

The first law of thermodynamics may be applied to the contents of the refrigerator. This law is represented by $\Delta U = Q + W$, where ΔU is the increase of internal energy of the contents of the refrigerator, Q is the net heating of the contents, and W is the mechanical work done on the contents. For the refrigerator contents, which of the quantities ΔU, Q and W is/are zero?

A ΔU only
B Q only
C W only
D each of ΔU, Q and W N04/I/14

Solution Answer: **D**
$\Delta U = 0$ because there is no change in temperature (ΔK.E. = 0) and no change in volume of the contents (ΔP.E. = 0).
Q is zero because there is no net rate of heat transfer.
W is zero because the refrigerator contents' volume remains constant.

14 A system absorbs 80 J through heating while doing 100 J of external work.

What is the change in the internal energy of the system?

A –100 J
B –20 J
C +80 J
D +180 J N05/I/14

Solution Answer: **B**
Apply the first law of thermodynamics, the increase in internal energy $\Delta U = Q + W$ where Q is the heat transferred to the system and W the work done on the system.

In this case the work is done *by* the system hence the work done *on* the system W is negative.
$\Delta U = Q + W = +80 - 100 \text{ J} = -20 \text{ J}$.

15 Two bodies are in thermal equilibrium.

Which condition **must** apply?

A Their heat capacities are equal.
B Their internal energies are equal.
C They are at the same temperature.
D They emit and absorb electromagnetic radiation at the same rate. N06/I/11

Solution Answer: **C**
When two bodies are in thermal equilibrium, there is no heat transfer between them. This can only mean that they are at the same temperature.

Two bodies can have the same temperature even if they have different heat capacities, internal energies and radiation.

16 An ideal gas is compressed at constant temperature.
Which line of the table is correct?

	work done	heating of gas
A	work is done **by** gas	heat energy goes **into** gas
B	work is done **by** gas	heat energy goes **out of** gas
C	work is done **on** gas	heat energy goes **into** gas
D	work is done **on** gas	heat energy goes **out of** gas

N06/I/12

Solution
Answer: **D**

The internal energy U of an ideal gas is its kinetic energy alone and is directly proportional to its thermodynamic temperature T ($U = 3nRT/2$) where n is the amount of gas molecules in moles and R the molar gas constant.

Therefore, if temperature remains constant, the internal energy remains constant hence the increase in internal energy $\Delta U = 0$.

The first law of thermodynamics, $\Delta U = Q + W$ where Q is the amount of heat energy transferred to the gas and W the work done on the gas.

Since $\Delta U = Q + W = 0$, it can be inferred that $Q = -W$.

W is positive since the gas is being compressed and there is positive work done on the gas.

Therefore, Q must be negative implying that heat energy goes out of the gas.

17 An ideal gas undergoes an expansion in volume from 1.3×10^{-4} m^3 to 3.6×10^{-4} m^3 at a constant pressure of 1.3×10^5 Pa. During this expansion, 24 J of heat is supplied to the gas.

What is the overall change in the internal energy of the gas?

A decrease of 54 J
B decrease of 6 J
C increase of 6 J
D increase of 54 J

N07/I/20

Solution
Answer: **B**

Work done by the gas

$$W = p\Delta V = (1.3 \times 10^5 \text{ Pa})(2.3 \times 10^{-4} \text{ m}^3) = 29.9 \text{ J}$$

Hence work done on the gas is -29.9 J.

Apply the first law of thermodynamics,

$$\Delta U = Q + W = +24 \text{ J} - 29.9 \text{ J} = -5.9 \text{ J}$$

18 The first law of thermodynamics may be expressed as shown.

$$\Delta U = q + w$$

where ΔU is the change in internal energy,
q is the heating of the system,
w is the work done on the system.

A fixed mass of ideal gas at high pressure is contained in a balloon. The balloon suddenly bursts, causing the gas to expand and cool.

In this situation, which row describes the values of ΔU, q and w.

	ΔU	q	w
A	negative	negative	positive
B	negative	zero	negative
C	positive	zero	negative
D	positive	negative	positive

N09/I/21

Solution
Answer: **B**

In a sudden expansion, there is no time for heat exchange between the gas and its surrounding, hence such processes are adiabatic ($q = 0$).
Since the gas expands, work is done by the gas, hence the work done on the gas, w, is negative.
Applying the first law of thermodynamics,

$$\Delta U = 0 + w$$

ΔU must be negative if w is negative.

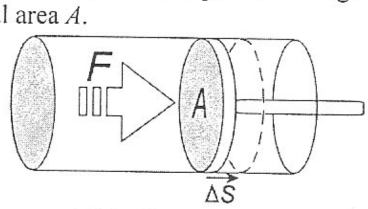
In an ideal gas, there being no potential energy due to the absence of intermolecular forces of attraction, the internal energy of the gas comprises only the kinetic energy of its molecules and is directly proportional to its thermodynamic temperature. Hence a decrease in internal energy results in a cooling of the gas.

19 The work done W by an expanding gas is calculated using $W = p\Delta V$.
What must remain constant for this equation to be used?
A the pressure of the expanding gas
B the pressure of the surroundings
C the temperature of the expanding gas
D the temperature of the surroundings

N10/I/10

Solution
Answer: A

The work done (W) by a constant force is $W = F\Delta s$ where F is the constant force and Δs the displacement in the direction of the force. Consider a gas exerting a force F against a piston of cross-sectional area A.



Since pressure $p = F/A$ where A is the area of the surface perpendicular to the displacement, substituting $F = pA$ into the work done equation, $W = pA\Delta s = p\Delta V$ where p is the pressure of the expanding gas which thus must be constant. If pressure is not constant, the equation becomes $W = \int p\, dV$.
The pressure is that of the gas and not the surrounding. Even if the surrounding is a vacuum but the piston is rusty and exerts a constant frictional force against the moving piston, work done by the gas is still $p\Delta V$ where p is the pressure of the expanding gas.

20 Which statement about the first law of thermodynamics is correct?

A The heating of a system equals the increase of its thermal energy plus the work done on the system.

B The increase in the internal energy of a system equals the heating of the system plus the work done by the system.

C The increase in the internal energy of a system equals the heating of the system plus the work done on the system.

D The work done on a system equals the increase of its thermal energy plus the heating of the system.

N10/I/18

Solution Answer: **C**

The first law of thermodynamics can be expressed in equation form as: $\Delta U = Q + W$ where ΔU is the increase in the internal energy of a system, Q the heating of the system and W the work done on the system.

Short Questions

21 (a) State

(i) the meaning of the term *internal energy*,

....

.... [2]

(ii) the first law of thermodynamics.

....

.... [2]

(b) An ideal gas undergoes a cycle of changes A → B → C → A, as shown in Fig. 4.1.

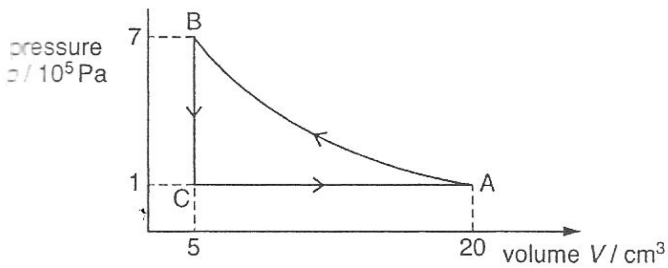


Fig. 4.1

(i) Calculate the work done by the gas during the change C → A.

work done by the gas = ... J [2]

(ii) Fig. 4.2 is table of energy changes during one cycle. Complete Fig. 4.2.

section of cycle	heating supplied to gas / J	Work done on gas / J	Increase in Internal energy of gas / J
A → B	zero	4.2	
B → C	-8.5		
C → A			

Fig. 4.2 [4]

N08/III/4

Solution

(a) (i) The *internal energy* of a system is the sum of the kinetic energy due to the random motion of its molecules, and the potential energy due to intermolecular forces of attraction.

(ii) The first law of thermodynamics states that the increase in internal energy of a system is the sum of the thermal energy added to the system by heating and the work done on the system.

(b) (i) Work done

$W = p\Delta V = (1 \times 10^5 \text{ Pa})(20 - 5) \times 10^{-6} \text{ m}^2 = 1.5 \text{ J}$

(ii) The completed table is shown in Fig. 4.3. Using the first law of thermodynamics $Q + W = \Delta U$.

section of cycle	heating supplied to gas / J (Q)	Work done on gas / J (W)	Increase in Internal energy of gas / J (ΔU)
A → B	zero	4.2	**4.2**
B → C	-8.5	**zero**	**-8.5**
C → A	**5.8**	**-1.5**	**4.3**
Net	*-2.7*	*2.7*	*zero*

The last row 'Net' is included to enable the completion of the table.

The net increase in internal energy in a cyclic process is zero, as internal energy is a function of state, i.e. internal energy only depends on the state of the system, and not how the state is arrived at.

Long Questions

22 (a) Convert the temperature of the boiling point of nitrogen, 77.30 K, to a temperature on the Celsius scale.

temperature = ... °C [2]

(b) State what is meant by saying that a temperature is on an *absolute* scale.

....

.... [1]

(c) Explain what is meant by

(i) the *internal energy* of a gas,

....

....

.... [2]

(ii) the *ideal* gas.

....

.... [1]

(d) (i) A car tyre has a fixed internal volume of 0.0120 m^3. On a day when the temperature is 25 °C the pressure in the tyre has to be increased from 2.62×10^5 Pa to 3.23×10^5 Pa. Assuming the air is an ideal gas, calculate the amount of air which has to be supplied at constant temperature.

amount of air = ... mol [3]

(ii) A portable supply of air used to inflate tyres has a volume of 0.0108 m^3 and is filled with air at a pressure of 8.72×10^5 Pa. Show that, at 25 °C, there is more than enough air in it to supply four tyres, as in (i), without the pressure falling below 3.23×10^5 Pa. [3]

(e) (i) Show that the internal energy of a molecule of air at a temperature of 25 °C is 6.17×10^{-21} J. Assume that the air behaves as an ideal gas. [2]

(ii) Hence calculate the internal energy of one mole of the air at a temperature of 25 °C.

internal energy = ... J [1]

(iii) Calculate the increase in the internal energy of the air in the tyre in (d)(i) as a result of increasing the pressure.

increase = ... J [2]

(f) (i) Give a word equation stating the first law of thermodynamics.

....

.... [2]

(ii) Apply this law to the pressure increase in (d)(i) in order to calculate the amount of work which has to be done to increase the pressure.

work done = ... J [1]

N07/III/6

Solution

(a) Temperature in Celsius = 77.30 – 273.15 = -195.85 °C.

(b) This means that the temperature on such a scale is not dependent on the thermometric property of any particular substance and has absolute zero as its minimum temperature.

Its fixed point, absolute zero, is not dependent on any particular substance but rather is the temperature at which all substances have a minimum internal energy.

(c) (i) The *internal energy* of a gas is the sum of the kinetic energy due to the microscopic random motion of its molecules, and potential energy due to intermolecular forces of attraction.

(ii) An *ideal gas* is a hypothetical gas that obeys the equation of state of an ideal gas perfectly at all temperatures and pressures.

A real gas approaches ideal behaviour at very low pressures and high temperatures.

(d) (i) Apply the equation of state of an ideal gas,

$$pV = nRT$$
$$n = pV/RT$$
$$\Delta n = \Delta pV/RT$$
$$= (61000 \text{ Pa} \times 0.0120 \text{ m}^3)/(8.31 \text{ J K}^{-1} \text{ mol}^{-1} \times 298 \text{ K})$$
$$= 0.296 \text{ mol.}$$

(ii) To supply four tyres, the amount of gas in moles transferred from the supply to the tyres is

$$\Delta n = 4 \times 0.296 \text{ mol} = 1.18 \text{ mol}$$

The fall in the supply's pressure is

$$\Delta p = \Delta nRT/V$$
$$= (1.18 \text{ mol} \times 8.31 \text{ J K}^{-1} \text{ mol}^{-1} \times 298 \text{ K})/(0.0108 \text{ m}^3)$$
$$= 2.71 \times 10^5 \text{ Pa}$$

The supply's pressure will fall to $8.72 \times 10^5 - 2.71 \times 10^5 = 6.01 \times 10^5$ Pa.

This is still above 3.23×10^5 Pa.

(e) (i) For an ideal gas, the internal energy is its kinetic energy alone (since potential energy is zero).

$$\tfrac{1}{2}m\langle c^2\rangle = (3/2)kT$$
$$= (3/2)(1.38 \times 10^{-23} \text{ J K}^{-1})(298 \text{ K})$$
$$= 6.17 \times 10^{-21} \text{ J.}$$

(ii) One mole ≡ 6.02×10^{23} molecules.
Internal energy,
$U = 6.02 \times 10^{23} \times 6.17 \times 10^{-21} \text{ J} = 3.71 \times 10^3 \text{ J}$

(iii) Increase in internal energy,
$\Delta U = 0.296 \text{ mol} \times 3.71 \times 10^3 \text{ J mol}^{-1} = 1.10 \times 10^3 \text{ J}$

(f) (i) The increase in the internal energy of a system is equal to the sum of the thermal energy added to the system by heating and the work done on the system.

(ii) Apply the equation of the first law of thermodynamics. Since there is no heat supplied to or removed from the system, $Q = 0$.

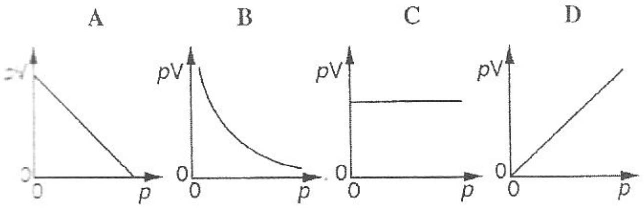
$$\Delta U = Q + W$$
$$1.10 \times 10^3 \text{ J} = W$$

Work done on the system is 1.10×10^3 J.

The Ideal Gas Equation

23 In an experiment to investigate the relationship between the volume V of a fixed mass of an ideal gas and its pressure p, a graph of pV against p is plotted.

Which graph shows the correct relationship at constant temperature?



N03/I/10

Solution Answer: **C**

From the equation of state for an ideal gas: $pV = nRT$ it can be seen that for a fixed mass of gas at constant temperature, the product pV is constant regardless of pressure.

24 The equation of state for an ideal gas is $pV = nRT$.

Which of the following correctly describes n and T?

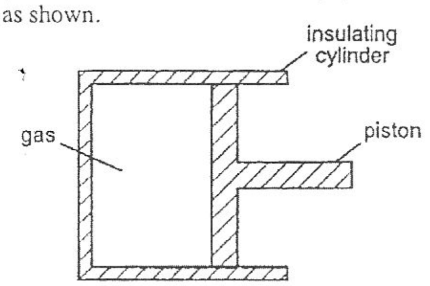
	n	T
A	amount of gas in moles	kelvin temperature
B	amount of gas in moles	temperature change
C	number of molecules of gas	kelvin temperature
D	number of molecules of gas	temperature change

N04/I/12

Solution Answer: **A**

In the equation of state for an ideal gas, 'n' is the amount of gas in moles and T the kelvin temperature.

25 An ideal gas is contained in an insulating cylinder fitted with a piston, as shown.



The piston is suddenly moved inwards so that the volume of the gas is reduced.

What happens to the temperature of the gas?

A It increases because the gas molecules bounce off the piston at higher speeds.

B It increases because the gas molecules collide with each other more often.

C It stays constant because *pressure* × *volume* is constant for an ideal gas.

D It stays constant because the cylinder is an insulator.

N05/I/12

Solution Answer: **A**

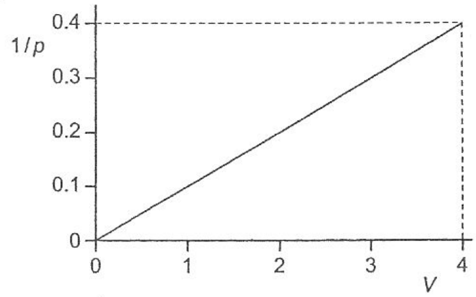
When the piston is moved inward *suddenly*, there is no time for heat energy to enter or leave the cylinder (which is insulated anyway). Hence the change is an adiabatic compression ($Q = 0$).

Apply the first law of thermodynamics, the increase in internal energy $\Delta U = Q + W$ where Q is the heat transferred to the system (in this case zero) and W the work done on the system. Since $Q = 0$, $\Delta U = W$. When a gas is compressed, the work done on the system is positive hence there is an increase in its internal energy U.

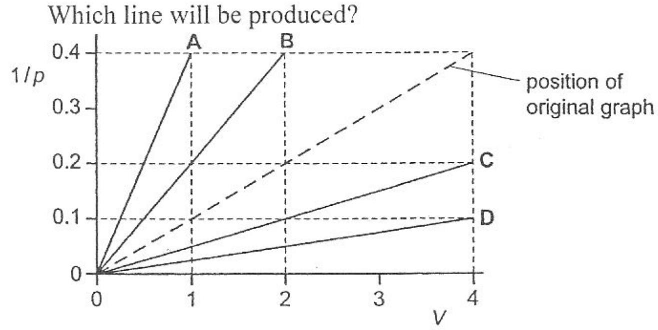
For an ideal gas, the internal energy is directly proportional to thermodynamic temperature ($U = (3/2)nRT$) hence the temperature increases.

What happened at the molecular level is that the molecules bounce off the in-moving piston with a speed higher than the speed of collision. The molecules gain kinetic energy from the in-moving piston.

26 A fixed amount of an ideal gas has pressure p and volume V.
The graph shows the variation of $1/p$ with V at a constant temperature.



The amount of gas and the thermodynamic temperature are both doubled.
Which line will be produced?



N08/I/18

Solution Answer: **D**

The ideal gas equation of state $pV = nRT$ where p is the pressure of the gas, V its volume, n the amount of gas in moles, R the molar gas constant and T the thermodynamic temperature.
Rearranging this into a linear equation of $1/p$ vs V,

$$1/p = (1/nRT)\ V$$

The gradient is $1/nRT$.
Hence if the amount of gas (n) and thermodynamic temperature (T) are both doubled, the gradient will become one-quarter of its initial value.

27 What is the number of hydrogen atoms, in terms of the Avogadro constant N_A, in one mole of water (H_2O)?

A $(1/3)N_A$ B $(2/3)N_A$ C N_A D $2N_A$

N09/I/20

Solution Answer: **D**

The number of water molecules in one mole of water is the Avogadro constant N_A.

Since there are 2 hydrogen atoms per molecule of water, the number of hydrogen atoms is $2N_A$.

Short Questions

28 (a) The variation with volume V of the pressure p of a fixed mass of gas at constant temperature is shown in Fig. 1.1.

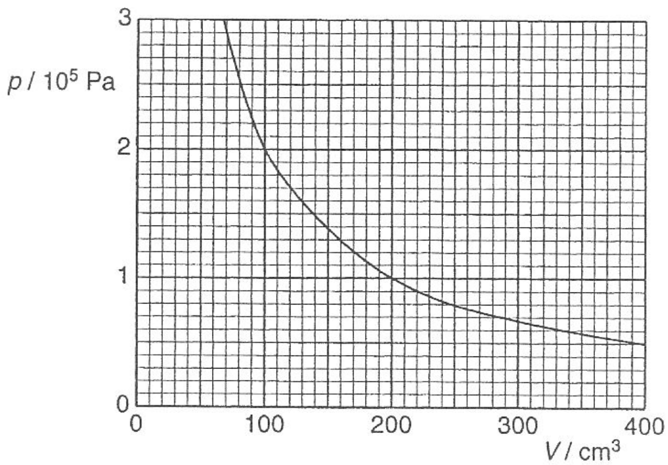


Fig. 1.1

(i) Using values from Fig. 1.1, plot a second graph using the axes drawn in Fig. 1.2, to show that p is inversely proportional to V.

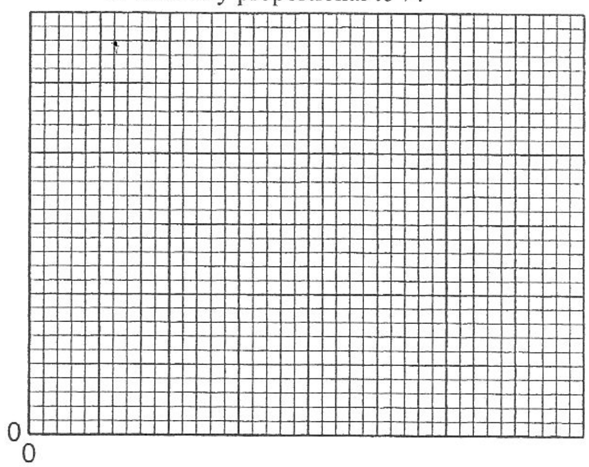


Fig. 1.2 [2]

(ii) State how inverse proportion is demonstrated by your graph.

....

.... [1]

N08/III/1(a)

Solution

(a) (i) A graph of $1/p$ plotted against V is shown in Fig. 1.6.

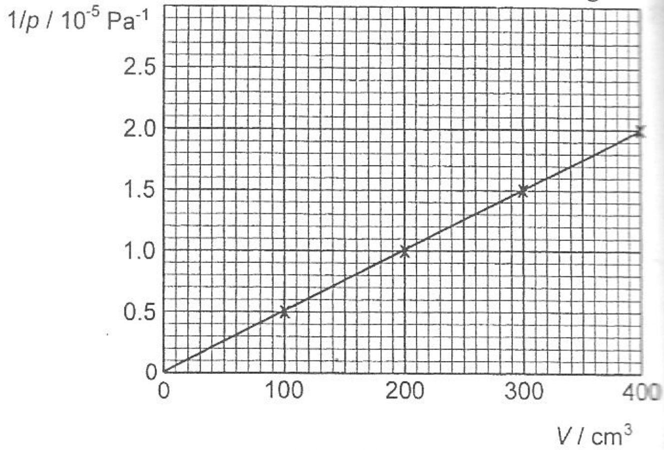


Fig. 1.6

V / cm^3	p / 10^5 Pa	$1/p$ / 10^{-5} Pa^{-1}
100	2.00	0.5
200	1.00	1.0
300	0.67	1.5
400	0.50	2.0

(ii) The graph of $1/p$ against V is a straight line through the origin. This shows that $1/p$ is directly proportional to V and pV = constant, hence p = constant/V.

29 (a) (i) Explain the concept of *absolute zero* on the thermodynamic temperature scale. [1]

(ii) State how the temperature of an ideal gas is related to the energy of the molecules of the gas. [1]

(b) An oven has a volume of 0.064 m^3. The pressure and temperature of the air in the oven are 1.0 x 10^5 Pa and 27 °C respectively. The mass of one mole of air is 0.030 kg. The air behaves as an ideal gas.

(i) Calculate the mass of air in the oven.

mass = ... kg [3]

(ii) The oven is heated to a temperature of 180 °C. The oven door is opened.

Calculate the mass of air that must escape from the oven for the pressure in the oven to return to 1.0 x 10^5 Pa.

mass = ... kg [2]

N10/II/5

Solution

(a) (i) Absolute zero is the temperature at which all substances have a minimum internal energy.

(ii) The temperature of an ideal gas is directly proportional to the average kinetic energy of the molecules in the gas.

(b) (i) Applying the equation of state of an ideal gas, $pV = nRT$, the amount of air

$n = pV/RT = (1.0 \times 10^5)(0.064)/(8.31)(27 + 273)$

= 2.567 moles

Since one mole of air has a mass of 0.030 kg, the mass of 2.567 moles = 0.030 x 2.567 = 0.077 kg.

(ii) Applying the equation of state of an ideal gas, $pV = nRT$, the amount of air remaining
$n = pV/RT = (1.0 \times 10^5)(0.064)/(8.31)(180 + 273)$
$= 1.700$ moles
Mass of air remaining = 0.030 x 1.700 = 0.051 kg.
Mass of air that escaped = 0.077 – 0.051 = 0.026 kg.

Kinetic Theory

Short Questions

30 (a) (i) State which **two** quantities are conserved in any collision.

(ii) State what is meant by an *elastic collision*. [3]

(b) An ideal gas is contained in a cylinder by means of a piston, as shown in Fig. 3.1.

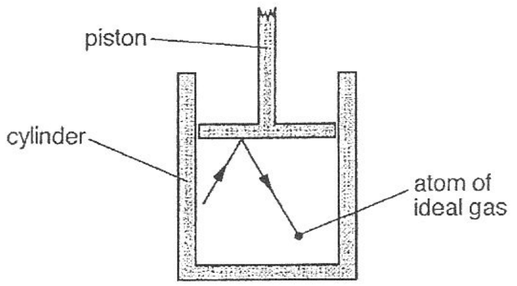


Fig. 3.1

The cylinder and piston are made of a thermal insulator. An atom of the gas collides with the piston, as illustrated.

State, with a reason, whether the momentum of the atom is conserved in the collision. [2]

(c) The piston in *(b)* is lowered so that the volume of the gas is reduced.

(i) State what change, if any, occurs in the speed of a gas atom during an elastic collision with the moving piston. [1]

(ii) Use the kinetic theory of gases to explain why the temperature of the gas rises. [2]

(iii) Write down a word equation to represent the conservation of energy for this change in volume of the gas. [2]

N03/II/3

Solution

(a) (i) Linear momentum and total energy.

(ii) An *elastic collision* is a collision in which the total kinetic energy of the system is conserved.

(b) The momentum of the atom is not conserved.
Momentum is a vector quantity hence there is a change in momentum when there is change in the direction of the atom's velocity.
The law of conservation of linear momentum states that momentum remains constant provided here is no net external force acting on the system.
If the atom is considered as the system, the system experiences a net external force during the collision (the reaction force by the piston on the atom) causing the atom's momentum to change.
However, if the piston, cylinder and atoms are considered as one system, then the momentum of the whole system is conserved since the forces are internal.

(c) (i) The speed of the gas atom increases.

(ii) The internal energy U of an ideal gas is the sum total of the kinetic energy of all the atoms in the gas, which is directly proportional to the thermodynamic temperature T ($U = ½M\langle c^2\rangle = (3/2)nRT$) where n is the amount of gas in moles and R the molar gas constant.
When the downward-moving piston collides with the gas atoms, it imparts energy on the atoms and causes their speed and kinetic energy to increase.
As a result, the mean-square speed, kinetic energy and temperature of the gas increase.

(iii) Work done W on the gas by the piston equals the increase in internal energy ΔU of the gas (heat transfer Q is zero due to thermal insulation).

Long Questions

31 A container of gas holds 3.6×10^{25} molecules of an ideal gas each with a mass of 4.6×10^{-26} kg. The root-mean-square speed of the molecules is 270 m s^{-1} and the container is on an aircraft travelling at 240 m s^{-1}. Calculate

(i) the kinetic energy as a result of the random motion of all the molecules in the gas,

(ii) the kinetic energy the gas has as a result of being on the aircraft,

(iii) the internal energy of the gas. [5]

N02/III/3 (part)

Solution

(i) Kinetic energy due to random motion of gas molecules
$½M\langle c^2\rangle = ½(3.6 \times 10^{25} \times 4.6 \times 10^{-26}\text{ kg})(270\text{ m s}^{-1})^2$
$= 6.036 \times 10^4$
$= 6.0 \times 10^4$ J.

(ii) Kinetic energy due to the bulk motion of the entire gas
$½Mv^2 = ½(3.6 \times 10^{25} \times 4.6 \times 10^{-26}\text{ kg})(240\text{ m s}^{-1})^2$
$= 4.769 \times 10^4$
$= 4.8 \times 10^4$ J.

(iii) The internal energy of an ideal gas is the total kinetic energy due to the random motion of its gas molecules, and not the kinetic energy due to the bulk motion of the entire gas.
Hence the internal energy is 6.0×10^4 J.

32 *(c)* The four main constituents of air are nitrogen, oxygen, argon and carbon dioxide. The gases, assumed to be ideal, are all in thermal equilibrium. The masses of the molecules are listed in Fig. 3.1.

	mass of molecule/u
nitrogen	28
oxygen	32
argon	40
carbon dioxide	44

Fig. 3.1

(i) State what is meant by *thermal equilibrium*. [1]

(ii) Show that the mean-square speed of the molecules in air is inversely proportional to the mass of each molecule. [3]

(iii) State which gas has the molecules with the smallest root-mean-square speed. [1]

(iv) The root-mean-square speed of oxygen molecules is 480 m s^{-1}. Calculate the root-mean-square speed of nitrogen molecules. [3]

(b) Estimate the root-mean-square speed of a particle of dust, of mass 1.2×10^{-12} kg, in the air.

State any assumption you make. [4]

N03/III/3 (part)

Solution

(c) (i) Two different substances are at *thermal equilibrium* when there is no heat transfer between them.
This happens when they are at the same temperature.

(ii) The average kinetic energy of each molecule $\frac{1}{2}m\langle c^2\rangle = (3/2)kT$ hence mean-square-speed $\langle c^2\rangle = 3kT/m$.
Since the four gases are at thermal equilibrium, they have the same temperature T.
The Boltzmann constant k is also a constant.
Hence $\langle c^2\rangle \propto 1/m$.

(iii) Carbon dioxide (which has the largest mass).

(iv) Root-mean-square speed $\sqrt{\langle c^2\rangle} \propto 1/\sqrt{m}$.
Therefore, root-mean-square speed of nitrogen
$\sqrt{\langle c^2\rangle} = \sqrt{(32/28)} \times 480\ \text{m s}^{-1} = 513\ \text{m s}^{-1}$.

(b) Assuming a room temperature of 300 K,

$\sqrt{\langle c^2\rangle} = \sqrt{(3kT/m)}$
$= \sqrt{(3 \times 1.38 \times 10^{-23}\ \text{J K}^{-1} \times 300\ \text{K}/1.2 \times 10^{-12}\ \text{kg})}$
$= 1.017 \times 10^{-4}$
$= 1.0 \times 10^{-4}\ \text{m s}^{-1}$.

An assumption is that the dust particle behaves like an ideal gas molecule.

33 *(d)* (i) By putting each quantity into base units, show that the equation

$$pV = nRT$$

is homogeneous. [4]

(ii) Explain why a homogeneous equation may not necessarily be correct. [1]

(e) The pressure of an ideal gas, as derived from the kinetic theory, is given by the equation

$$p = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle.$$

(i) State the meaning of each of the symbols N, m and $\langle c^2\rangle$. [2]

(ii) Use this equation, together with the one in (d)(i), to calculate the root mean square speed of the oxygen molecules in air at a temperature of 27 °C. The mass of one mole of oxygen molecules is 32 g. [3]

(iii) Assuming the gas to behave ideally, calculate the internal energy of a mole of oxygen at this temperature. [2]

N04/III/1(part)

Solution

(d) (i) The unit for pressure p is the pascal (Pa)
One pascal Pa = N m^{-2} = (kg m s^{-2}) m^{-2} = kg m^{-1} s^{-2}.
The base unit for volume V is m^3.
The base units for the expression pV on the left-hand-side of the equation is (kg m^{-1} s^{-2})(m^3) = kg m^2 s^{-2}.

The base unit for n is the mole (mol).
The base units for R is J K^{-1} mol^{-1} = [N m] K^{-1} mol^{-1} = (kg m s^{-2}) m K^{-1} mol^{-1} = kg m^2 s^{-2} K^{-1} mol^{-1}.
The base unit for T is the kelvin (K).
The base units for the expression nRT on the right-hand-side of the equation is
(mol)(kg m^2 s^{-2} K^{-1} mol^{-1})(K) = kg m^2 s^{-2}.

Since the base units for the expression on the left-hand-side of the equation are the same as those on the right-hand-side of the equation, the equation is homogeneous.

(ii) An expression may contain dimensionless constants which do not appear in the dimensional analysis.

(e) (i) N is the total number of gas molecules in the gas.
m is the mass of one gas molecule.
$\langle c^2\rangle$ is mean-square speed of the gas molecules.

(ii) Use the equation in (e), $pV = \frac{1}{3}Nm\langle c^2\rangle$.

Equating this with the equation in (d)(i),
$\frac{1}{3}Nm\langle c^2\rangle = nRT$.

The number of moles of gas molecule n is equal to N/N_A where N_A is the Avogadro constant.
$\frac{1}{3}Nm\langle c^2\rangle = (N/N_A)RT$

Eliminating N from both sides of the equation and multiplying N_A across the equation,

$$\tfrac{1}{3} N_A m\langle c^2\rangle = RT$$
$$\tfrac{1}{3} M\langle c^2\rangle = RT$$
$$\sqrt{\langle c^2\rangle} = \sqrt{(3RT/M)}$$

where M is the molar mass
Substituting $R = 8.31$ J K^{-1} mol^{-1}, $T = 300$ K and $M = 0.032$ kg,

$$\sqrt{\langle c^2\rangle} = 483.4 = 480 \text{ m s}^{-1}$$

(iii) Internal energy of one mole of ideal gas\
$U = \tfrac{1}{2}M\langle c^2\rangle = \tfrac{1}{2}(0.032 \text{ kg})(483.4 \text{ ms}^{-1})^2$
$= 3738 = 3700$ J.

34 *(a)* The ideal gas equation is $pV = nRT$. Explain why non-SI units may be used for p and V but the temperature T cannot have the unit °C. [2]

(b) Write down the exact temperature on the Kelvin scale of zero degrees Celsius. [1]

(c) In an attempt to beat the world altitude record for a balloon, a helium balloon containing 15 000 m^3 of helium at a temperature of 288 K was launched from sea level, where the pressure of the gas was 101 kPa. The balloon, carrying a payload, rose to an altitude of 32.0 km before reaching equilibrium. Data concerning atmospheric conditions are given in the table.

	sea level altitude = 0	equilibrium altitude = 32.0 km
pressure of helium	101 kPa	0.890 kPa
temperature	288 K	228 K
density of air	1.23 kg m^{-3}	0.0134 kg m^{-3}

Calculate

(i) the volume of helium at 32.0 km, [2]

(ii) the weight of air displaced by the balloon at equilibrium altitude, [2]

(iii) the total weight of balloon, helium and payload, [1]

(iv) the resultant force on the balloon at sea level, [2]

(v) the acceleration of the balloon at take-off. [2]

(d) (i) Using the equations $pV = \frac{1}{3}Nm < c^2 >$ and $pV = NkT$, derive an expression for the relationship between the average translational kinetic energy of a helium atom and the temperature. [1]

(ii) Hence find the average translational kinetic energy of one of the helium atoms in the balloon in *(c)*, when the balloon is at an altitude of 32.0 km. [1]

(iii) Calculate the amount, in mol, of helium in the balloon. [2]

(iv) Assuming that the gas behaves as an ideal gas, calculate the kinetic energy of all the helium at equilibrium altitude. [2]

(e) Suggest why the change in the potential energy of the gas in the balloon as it rises does **not** change its internal energy. [2]

N05/III/2

Solution

(a) Non-SI units for p and V are directly proportional to their SI counterparts, thus the non-SI product of pV will still be directly proportional to nRT.

However, the Celsius scale, even though linearly-related, is not directly proportional to the Kelvin scale. A graph of pV against T in °C will not start at the origin, hence pV is not directly proportional to T in °C.

(b) 273.15 K.

(c) (i) Assuming no leakage of helium gas so that the amount of gas in moles (n) remains constant,

$$p_1V_1/T_1 = p_2V_2/T_2$$

$$(101{,}000 \text{ Pa})(15{,}000 \text{ m}^3) / (288 \text{ K}) = (890 \text{ Pa})\ V_2 / (228 \text{ K})$$

$$V_2 = 1.3476 \times 10^6 = 1.35 \times 10^6 \text{ m}^3$$

(ii) Mass of air displaced
$m = V_2$ x density
$= 1.3476 \times 10^6$ m^3 x 0.0134 kg m^{-3}
$= 1.8058 \times 10^4$ kg

Weight of air displaced
$W = mg$
$= 1.8058 \times 10^4$ kg x 9.81 N kg^{-1}
$= 1.7715 \times 10^5 = 1.77 \times 10^5$ N

(iii) Magnitude of weight of air displaced
= magnitude of upthrust.

At equilibrium altitude, resultant force is zero hence the total weight = 1.77×10^5 N

(iv) At sea level, mass of air displaced
$m = V_1$ x density
$= 15{,}000$ m^3 x 1.23 kg m^{-3}
$= 1.845 \times 10^4$ kg

Weight of air displaced
$W = mg$
$= 1.845 \times 10^4$ kg x 9.81 N kg^{-1}
$= 1.8099 \times 10^5$ N

Magnitude of weight of air displaced
= magnitude of upthrust = 1.8099×10^5 N

Resultant force = upthrust – total weight
$= 1.8099 \times 10^5$ N – 1.7715×10^5 N = 3840 N

(v) Acceleration = Resultant Force / Mass
= 3840 N / 1.8058×10^4 kg
$= 0.21265 = 0.213$ m s^{-2}

(d) (i) Equating,

$$\tfrac{1}{3}Nm\langle c^2\rangle = NkT$$

Dividing by N on both sides of the equation,

$$\tfrac{1}{3}m\langle c^2\rangle = kT$$

Multiplying by 3/2 to both sides of the equation,

$$E_k = \tfrac{1}{2}m\langle c^2\rangle = (3/2)kT.$$

(ii) At 32.0 km, the temperature is 228 K.

$$E_k = (3/2)kT$$
$$= (3/2)(1.38 \times 10^{-23}\ \text{J K}^{-1})(228\ \text{K})$$
$$= 4.7196 \times 10^{-21} = 4.72 \times 10^{-21}\ \text{J}.$$

(iii) Using the equation of state for ideal gas, $pV = nRT$ at sea level,

$$(101 \times 10^3\ \text{Pa})(15{,}000\ \text{m}^3) = n(8.31\ \text{J K}^{-1}\ \text{mol}^{-1})(288\ \text{K})$$
$$n = 6.3302 \times 10^5 = 6.33 \times 10^5\ \text{mol}.$$

(iv) The total kinetic energy at equilibrium altitude, where the temperature T = 228 K, is given by:

$$(3/2)NkT = (3/2)nRT$$
$$= (3/2)(6.3302 \times 10^5\ \text{mol})(8.31\ \text{J K}^{-1}\ \text{mol}^{-1})(228\ \text{K})$$
$$= 1.80 \times 10^9\ \text{J}.$$

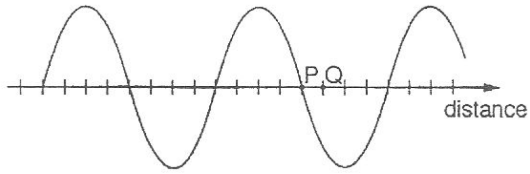
(e) The internal energy of a system is the sum of its kinetic energy due to the random motion of its molecules and potential energy due to intermolecular forces of attraction.

When the balloon rises, there is an increase in the bulk gravitational potential energy of the entire system, but not in the potential energy due to internal interaction between molecules.

Bulk changes in the potential energy of the entire system have no effect on internal energy.

TOPIC 10 Wave Motion

1 The diagram shows a transverse wave at a particular instant. The wave is travelling to the right. The frequency of the wave is 12.5 Hz.



At the instant shown the displacement is zero at the point P.

What is the shortest time to elapse before the displacement is zero at point Q?

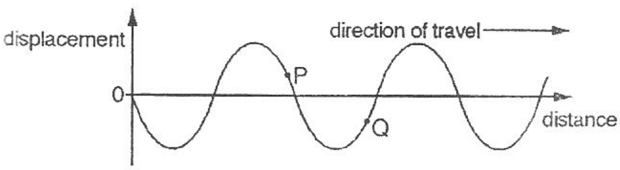
A 0.01 s
B 0.03 s
C 0.08 s
D 0.10 s

N02/I/15

Solution Answer: A

Period $T = 1/f = 1/(12.5 \text{ Hz}) = 0.08$ s.
A wave travels a distance of one wavelength in a time interval of one period.
To travel 1/8 of a wavelength, the time needed is 1/8 of a period = (1/8) x 0.08 s = 0.01 s.

2 A transverse progressive wave travels along a rope. The graph shows the variation of displacement with distance along the rope, at a certain time. The wave is travelling to the right.



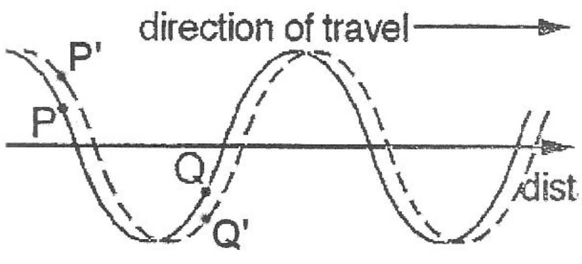
In which direction are P and Q moving?

	movement of P	movement of Q
A	downwards	downwards
B	downwards	upwards
C	upwards	downwards
D	upwards	upwards

N03/I/15

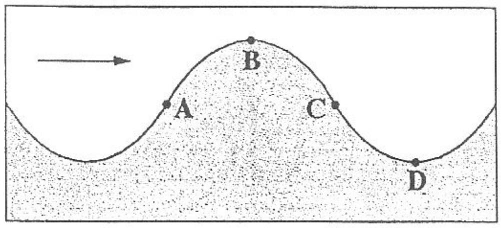
Solution Answer: C

Draw the graph an instant later (the dotted curve shown below) and observe the positions of particles P and Q, indicated as P' and Q' respectively.
It can be seen that P' is above P, while Q' below Q.



3 The diagram shows a vertical cross-section through a water wave moving from left to right.

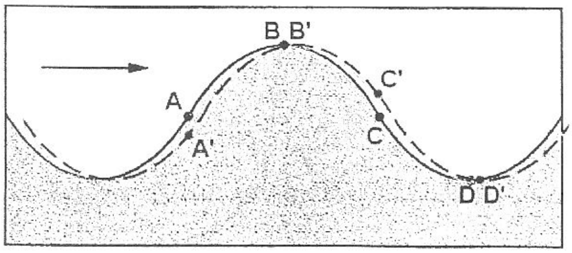
At which point is the water moving upwards with maximum speed?



N04/I/17

Solution Answer: C

Draw the graph an instant later (the dotted curve shown below) and observe the positions of the particles, indicated as A', B', C' and D'.
It can be seen that particle C' is above C while particle A' is below A.
Particles B' and D' are at the same positions as B and D respectively, which means they are instantaneously at rest.



4 The intensity of a wave depends on the amplitude. The intensity is also proportional to the square of the frequency.

A wave has frequency 3.0 Hz, amplitude 1.5 cm and intensity I.

What is the intensity of a similar wave of frequency 6.0 Hz and amplitude 0.5 cm?

A $\frac{4}{9}I$ B $\frac{4}{3}I$ C $\frac{9}{4}I$ D $36I$

N05/I/16

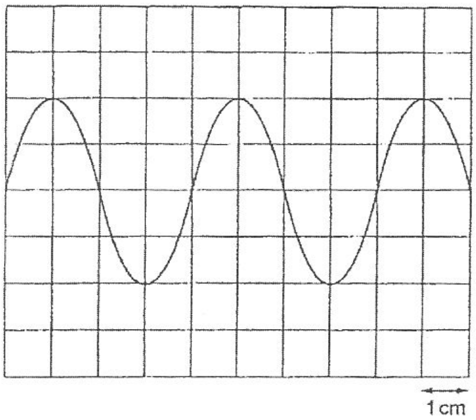
Solution Answer: A

The intensity of a wave is proportional to the square of the amplitude A and also the square of the frequency f.

The relationship can be expressed as $I = kA^2f^2$ where k is a constant of proportionality.

If A is replaced with $A/3$ while f replaced with $2f$, the new intensity $I' = k(A/3)^2(2f)^2 = (4/9)\, kA^2f^2 = (4/9)\, I$.

5 The diagram shows the trace produced by a sound wave on a cathode-ray oscilloscope. The time base is calibrated at 2.00 ms cm^{-1}.



What is the frequency of the sound wave?

A 62.5 Hz
B 125 Hz
C 250 Hz
D 500 Hz

N05/I/17

Solution Answer: **B**

The period of the wave $T = (4\text{ cm})(2.00\text{ ms cm}^{-1}) = 8.00$ ms.
Frequency $f = 1/T = 1/(8.00 \times 10^{-3}\text{ s}) = 125$ Hz.

6 A sound wave is emitted from a point source. The intensity of the sound wave is inversely proportional to the square of the distance from the source. At a distance *r* from the source, the amplitude of the wave is 8*X*

What is the amplitude at a distance 2r from the source?

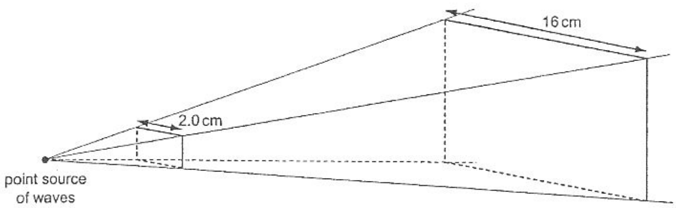
A 8X C 2X
B 4X D X

N06/I/15

Solution Answer: **B**

Intensity $I \propto 1/r^2$.
Intensity $I \propto A^2$ where *A* is the amplitude of the wave.
Hence $A \propto 1/r$.
At a distance 2*r*, the amplitude becomes $8X(1/2) = 4X$.

7 Waves from a point source pass through an area that is 2.0 cm wide, as shown.



Within this area, the intensity of the waves is *I* and their amplitude is *A*. The waves reach a second area of width 16 cm.

What will be the intensity and amplitude of the waves when they reach the second area?

	intensity	amplitude
A	$\frac{I}{8}$	$\frac{A}{4}$
B	$\frac{I}{64}$	$\frac{A}{4}$
C	$\frac{I}{64}$	$\frac{A}{8}$
D	$\frac{I}{256}$	$\frac{A}{16}$

N08/I/20

Solution Answer: C

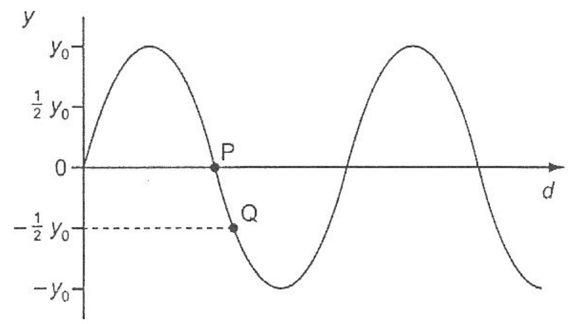
Intensity = Power / Area.

The second area's linear dimension is 8 times the first's, hence the second area is 8^2 or 64 times the first area. Since the same amount of power passes through the two areas, the intensity of the waves when they reach the second area is *I*/64.

The intensity *I* of a wave is proportional to the square of its amplitude *A*. Therefore, amplitude *A* is proportional to the square-root of intensity *I*.

Hence the amplitude of the waves when they reach the second area is $A/\sqrt{64}$ or $A/8$.

8 The diagram shows the variation with distance *d* along a sinusoidal wave of displacement *y* of particles in the wave. The amplitude of the wave is y_0.



What is the phase angle between the two particles P and Q in the wave?

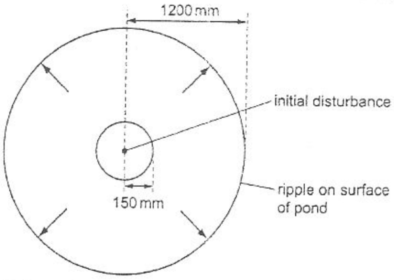
A 30° **B** 45° **C** 90° **D** 180°

N09/I/23

Solution Answer: A

In a sinusoidally-varying motion, the phase difference between a particle at equilibrium and a particle mid-way between equilibrium and amplitude is 30°, based on the trigonometric quantity sin(30°) = 0.5.
In this example, particle P is at a phase angle of 180° from the origin and Q at a phase angle of 210°. The trigonometric quantity sin(180°) = 0 and sin(210°) = -0.5.

9 Ripples on the surface of a pond spread out in circles from the point of an initial disturbance.
Assume that the energy of the wave is spread over the entire circumference of the ripple.
For one such ripple, the amplitude of the ripple at a distance of 150 mm from the disturbance is 2.0 mm.



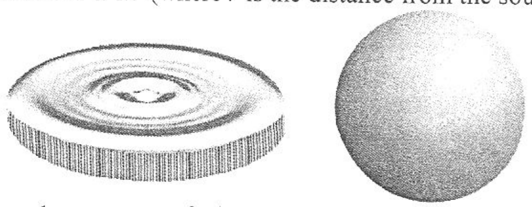
What will be the amplitude of the ripple at a distance of 1200 mm from the disturbance? (Assume that no energy is lost in the propagation of the ripple).

A 0.031 mm **B** 0.13 mm **C** 0.25 mm **D** 0.71 mm

N10/I/20

Solution Answer: **D**

A surface wave's power is confined to the water surface and distributed around the circumference (diagram below left). This is different from an isotropic wave where the power is distributed around a spherical surface ($4\pi r^2$) (diagram below right). Hence a surface wave's intensity $I \propto 1/r$ and amplitude $A \propto 1/\sqrt{r}$ whereas an isotropic wave's intensity $I \propto 1/r^2$ and amplitude $A \propto 1/r$ (where r is the distance from the source).



Since the energy of the wave is spread over the entire circumference of the ripple, the intensity of the wave $I \propto 1/(2\pi r)$ and is hence inversely proportional to the distance r from the source. At a distance of 8 times (1200 mm: 150 mm), the intensity will fall to 1/8. However, intensity I is proportional to the square of amplitude A hence $A \propto \sqrt{I}$. Therefore, the amplitude will become 2 mm/$\sqrt{8}$ = 0.71 mm.

Short Questions

10 Explain the meaning of each of the following terms as applied to waves. You may include labelled diagrams if you wish.

(c) coherence [2]

N08/III/3

Solution

(c) Coherence refers to the relationship between two wave sources which emit waves with a constant phase difference. Waves from two coherent sources have a constant phase difference as time goes on. If they are in phase, they remain constantly in phase; if they are out-of-phase, they remain constantly out-of-phase by the same angular difference as time goes on.

Coherence is a critical condition for the interference of waves to produce maxima and minima in space. Fig. 3.4 shows two coherent waves, arriving at a point, which are constantly out-of-phase by 90°.

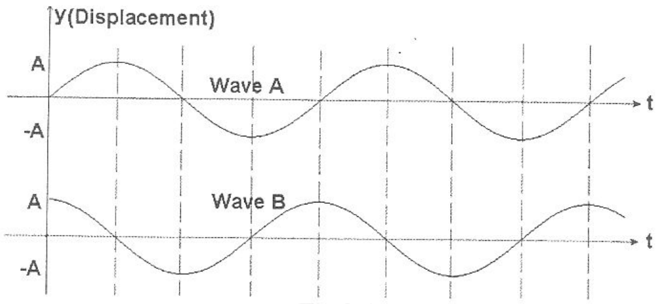


Fig.3.4

Long Questions

11 *(a)* Illustrate and explain the meaning of the terms *longitudinal* and *transverse* when applied to a wave. [4]

(b) A satellite passing the planet Neptune communicates with its controller on the Earth using a microwave transmitter with output power 22.0 W and wavelength 79 600 μm. Neptune is 4.35×10^{12} m from the Earth at the time when the communication takes place.

(i) State whether the microwaves are progressive or stationary.

(ii) State whether the microwaves are longitudinal or transverse.

(iii) Calculate the time taken for a signal to travel from the satellite to the Earth.

* (iv) Calculate the energy of a photon of the microwaves. [8]

(c) (i) Assuming that the power transmitted by the satellite in *(b)* is radiated uniformly in all directions, calculate the power received on the Earth by a dish aerial of effective area 260 m^2.

(ii) The actual power received at the dish aerial is 1.2×10^{-15} W.

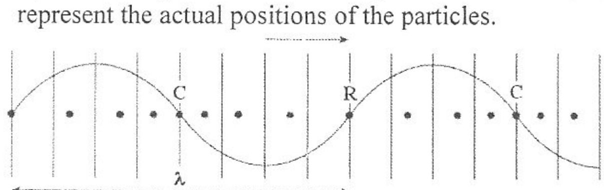
1. Suggest why the actual power received is greater than that calculated in *(c)*(i).

2. Calculate the actual rate at which photons of microwaves arrive at the dish aerial. [8]

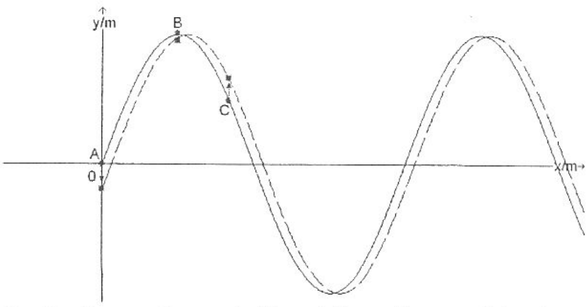
N02/III/6

Solution

(a) In a *longitudinal* wave, the direction of the disturbances is parallel to the direction of propagation of the wave, forming regions known as compressions (C) and rarefactions (R), as illustrated below.
In the figure below, the vertical lines represent the equilibrium positions of the particles and the dots represent the actual positions of the particles.



In a *transverse* wave, the direction of the disturbances is perpendicular to the direction of propagation of the wave, as illustrated below.



In the figure above, A, B and C are three points along the wave.
The solid curve represents the wave at one instant and the dotted curve represents the location of the wave a short instant later.

(b) (i) Progressive.
(The wave energy needs to be transported from the satellite to the controller).

(ii) Transverse.
(All electromagnetic waves are transverse waves).

(iii) Time = distance / speed
$= (4.35 \times 10^{12}\text{ m})/(3.00 \times 10^{8}\text{ m s}^{-1}) = 1.45 \times 10^{4}\text{ s}.$

(iv) $E = hc/\lambda$
$= (6.63 \times 10^{-34}\text{ J s})(3.00 \times 10^{8}\text{ m s}^{-1})/(79600 \times 10^{-6}\text{ m})$
$= 2.50 \times 10^{-24}\text{ J}.$

(c) (i) Intensity at a distance $r = 4.35 \times 10^{12}$ m away (where the Earth is located) is
$I = P/A = P/(4\pi r^2) = (22.0\text{ W})/(4\pi(4.35 \times 10^{12}\text{ m})^2)$
$= 9.2520 \times 10^{-26}\text{ W m}^{-2}.$

Power received
$P = IA = (9.2520 \times 10^{-26}\text{ W m}^{-2})(260\text{ m}^2)$
$= 2.4055 \times 10^{-23} = 2.41 \times 10^{-23}\text{ W}.$

(ii) 1. In practice, the antenna is designed to focus the power in the direction towards the Earth and not radiate the power uniformly in all directions.

2. Rate of photon arrival
n = rate of energy arrival / energy of each photon
$= (1.2 \times 10^{-15}\text{ J s}^{-1})/(2.50 \times 10^{-24}\text{ J photon}^{-1})$
$= 4.8 \times 10^{8}\text{ photons s}^{-1}.$

12 Fig. 4.1 shows a displacement-distance graph for two sound waves **A** and **B**, of the same frequency and amplitude. Wave **A** is travelling to the right and wave **B** is travelling to the left.

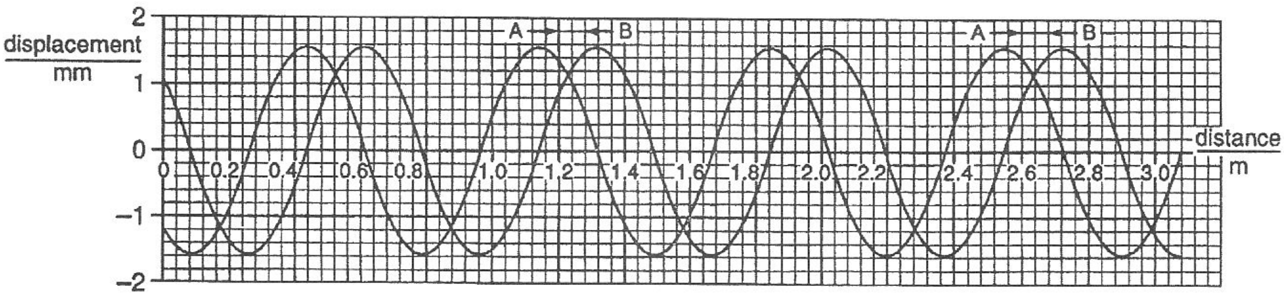


Fig. 4.1

(a) (i) From the graph, deduce the phase difference between the two waves at the instant shown. [1]

(ii) The period of each wave is T. Determine the maximum displacement of the resultant of the two waves

1. at the instant shown,

2. at the instant shown + $\frac{1}{8}T$,

3. at the instant shown + $\frac{3}{8}T$. [5]

(b) Describe features of the resultant wave set up by the two waves travelling in opposite directions. [3]

(c) Fig. 4.1 has the disadvantage that it makes the sound wave appear to be a transverse wave, when it is not.

(i) What type of wave is sound? [1]

(ii) Explain why sound waves cannot be polarised. [2]

(d) Describe how a cathode-ray oscilloscope (c.r.o.) may be used to determine the frequency of a sound wave. [5]

(e) The waves in Fig. 4.1 have a frequency of 484 Hz. Deduce their speed to **three** significant figures. [3]

N03/III/4

Solution

(a) (i) Phase difference $\Delta\Phi = (1/4)\, 2\pi = \pi/2$ rad.

(ii) 1. $1.55\sqrt{2}$ = 2.2 mm (at the intersection points of the two graphs).
Alternatively, add the individual displacements: 1.1 mm + 1.1 mm = 2.2 mm.

2. 2 x 1.55 mm = 3.1 mm
(twice the amplitude since the two waves will be in phase and undergo constructive interference).

3. 0 mm
(since the two waves will be π rad out of phase and undergo destructive interference).

(b) The resultant wave will be a stationary wave. It will have an amplitude twice that of each individual wave.
The waveform of this resultant wave will not advance in either direction and there is no transportation of energy either way.
The particles will vibrate about their equilibrium positions with simple harmonic motion, with the maximum amplitude of vibration at locations known as antinodes and zero amplitude of vibration at locations known as nodes.
All particles between two adjacent nodes vibrate in phase with one another, but are π rad out of phase with particles in the adjacent half-wavelength segment.
The wavelength of the wave is equivalent to twice the distance between successive nodes or between successive antinodes.

(c) (i) Sound wave is a longitudinal wave.

(ii) Only transverse waves can be polarised as polarisation involves the transmission of vibrations in a plane perpendicular to the direction of the wave's propagation.
Sound waves, being longitudinal waves, do not have vibrations in the plane perpendicular to the direction of propagation.
In longitudinal waves, the vibrations are parallel to the direction of propagation hence the entire wave will pass through the polariser.
They cannot be polarised.

(d) A microphone can be placed in the path of the sound wave, with its output connected to a cathode-ray oscilloscope (c.r.o.).
The microphone acts as a transducer which converts sound energy into electrical energy, such that the amplitude of microphone's voltage output is proportional to the amplitude of the sound wave.
Since the x-axis of the oscilloscope is the time base, the oscilloscope displays the voltage-time graph of the microphone output, and thus the displacement-time graph of the sound wave.
The time taken for one complete oscillation displayed in the c.r.o. is the period T.
Frequency $f = 1/T$.

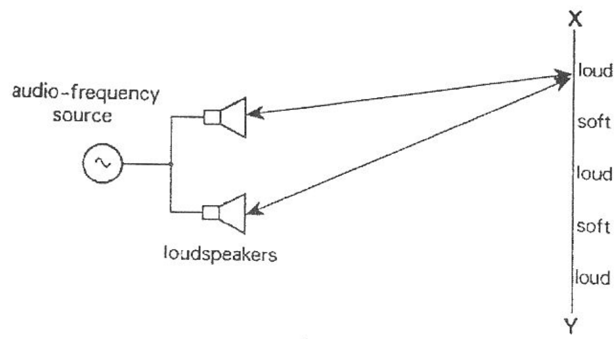
(e) The wavelength
λ = 2.82 m / 4 wavelengths
= 0.705 m.

Speed $v = f\lambda$
= (484 Hz)(0.705 m)
= 341.22
= 341 m s^{-1}.

13 The wavelength of sound in air is of the order of one million times greater than the wavelength of light in air. Describe how you could check this statement experimentally. [8]
N04/III/3 (part)

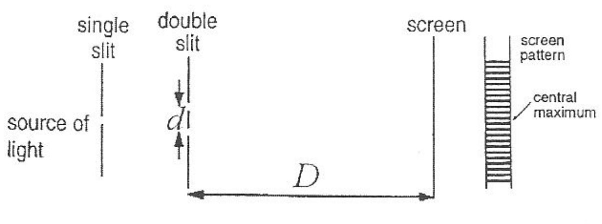
Solution
The wavelength of a wave can be determined using two-source interference experiments.

To determine the wavelength of sound, connect two loudspeakers to a single sound oscillator (to ensure coherence).
Face the two speakers towards a wall several metres away.
Move a microphone along the wall to detect variations in sound intensity.
There will be intensity maxima corresponding to constructive interference of the two sound waves, and minima corresponding to destructive interference, as illustrated in the figure below.



Select a maximum that is n^{th} order from the central axis maximum.
Measure the distance from each speaker to this maximum and subtract them to obtain the path difference.
The path difference divided by 'n' will be the wavelength of the sound wave.
(In the illustration above, n = 1).

To determine the wavelength of light, use Young's double-slit experiment.
A monochromatic light source is placed behind the double slits, with a third slit (single slit) between them to ensure coherence.
Bright and dark fringes will be observed on the screen, as illustrated in the figure below.



Since the angle of diffraction is small in this experiment, the Young's double-slit formula $y = \lambda D/d$ can be used to calculate the wavelength λ of the light (where y is the fringe separation, D the slit-to-screen distance and d the slit separation).

The experimental results will show that the wavelength of sound waves is in the order of metres; while that of light waves in the order of micrometres.

The ratio is approximately a million is to one.

14 (a) Explain what is meant by a *progressive transverse* wave.
progressive ……………………………………………
transverse …………………………………….. [3]

(b) State how a polarised transverse wave differs from an unpolarised transverse wave. [2]

(c) Light is polarised when it passes through a sheet of material known as polaroid.
The component of the displacement of the wave in the direction of polarisation produced by the polaroid is unaffected as the wave passes through the polaroid. The component normal to the direction of polarisation is completely absorbed by the polaroid.
Two sheets of polaroid P and Q are placed close to each other. Their directions of polarisation are parallel to one another, as shown in Fig. 7.1.

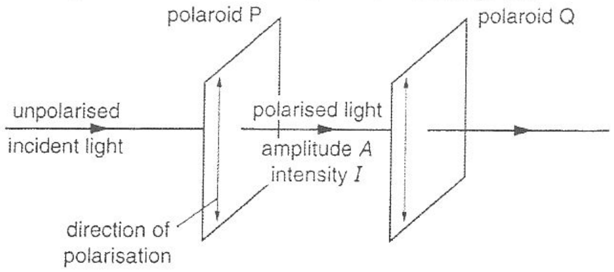


Fig. 7.1

A parallel beam of light passes through polaroid P. The beam, after passing through polaroid P, has amplitude A and intensity I.
The beam then passes through polaroid Q.
For the light transmitted through polaroid Q, state
(i) the amplitude (in terms of A),
amplitude = ………………
(ii) the intensity (in terms of I),
intensity = …………………
(iii) the relation between the answers to (i) and (ii).
………………………… [2]

(d) The polaroid Q in (c) is now rotated about the axis of the light beam, as shown in Fig. 7.2.

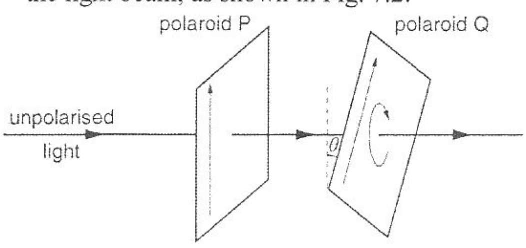


Fig. 7.2

The plane of polaroid Q remains parallel to the plane of polaroid P.
The angle between the direction of polarisation of polaroid P and polaroid Q is θ.
Complete Fig. 7.3 to show the amplitude, in terms of A, and the intensity, in terms of I, for angle θ equal to 180°, 90° and 60°.

angle θ	amplitude	intensity
180°	…………	…………
90°	…………	…………
60°	…………	…………

Fig. 7.3

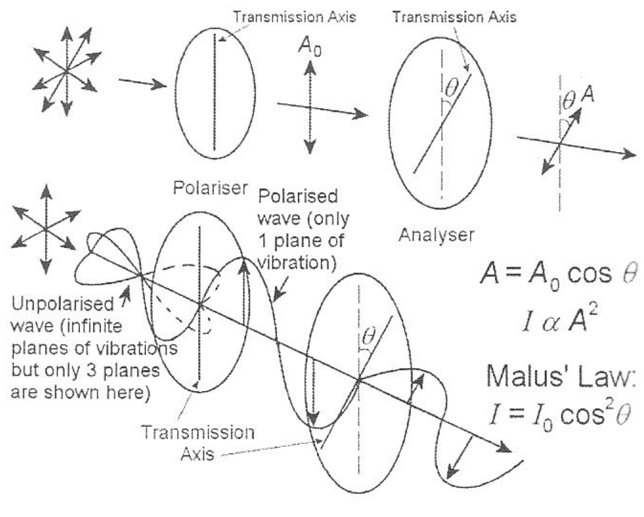
N10/III/7 (part)

Solution

(a) A progressive wave is a wave in which the waveform advances and there is a transfer of energy along the direction of propagation of the wave.

A transverse wave is a wave in which the vibrations are perpendicular to the direction of propagation of the wave.

(b) In a polarised wave, the vibrations are in only one plane; whereas in an unpolarised wave there are infinite planes of vibrations.



(c) (i) amplitude = A.

(ii) intensity = I.

(iii) $I = kA^2$, where k is a constant of proportionality.

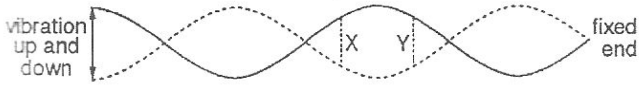
(d)

angle θ	amplitude	intensity
180°	A	I
90°	0	0
60°	0.5 A	0.25 I

TOPIC 11 Superposition

Stationary Waves

1 The diagram shows a long rope fixed at one end. The other end is moved up and down, setting up a stationary wave.



What is the phase difference between the oscillations at X and Y?

A 0

B $\frac{1}{4}\pi$ rad

C $\frac{1}{2}\pi$ rad

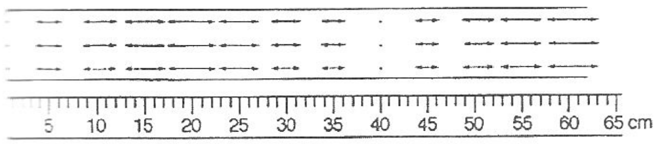
D $\frac{3}{4}\pi$ rad

N02/I/16

Solution Answer: **A**

All particles between two successive nodes oscillate in phase with one another.
Therefore, the phase difference between X and Y is zero.

2 A stationary sound wave is set up in a pipe using an oscillator of frequency 440 Hz. The extent of the vibration of the molecules in the pipe is illustrated. A centimetre scale is shown alongside the pipe.



What is the speed of sound in the pipe?

A 176 m s^{-1}

B 328 m s^{-1}

C 337 m s^{-1}

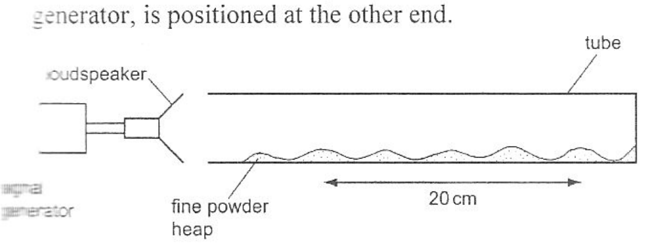
D 352 m s^{-1}

N04/I/18

Solution Answer: **D**

The inter-nodal distance is 0.40 m.
The wavelength of a stationary wave is twice the inter-nodal distance, i.e. $\lambda = 0.80$ m.
The speed of sound $v = f\lambda$ = (440 Hz)(0.80 m) = 352 m s^{-1}.

3 A long horizontal tube, containing a fine powder, is closed at one end. A loudspeaker, connected to a signal generator, is positioned at the other end.



At a particular frequency, a stationary wave is set up inside the tube and the powder forms heaps at the nodes. The speed of sound is 330 m s^{-1}.

What type of wave is the stationary wave and what is its frequency?

	type of wave	frequency / kHz
A	longitudinal	3.3
B	transverse	3.3
C	longitudinal	6.6
D	transverse	6.6

N07/I/21

Solution Answer: **A**

Sound wave is a longitudinal wave.

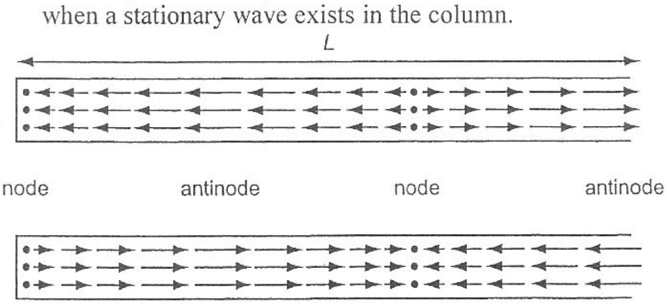
The distance between two successive nodes is half-a-wavelength.

$$\lambda/2 = 0.05 \text{ m}$$

$$\lambda = 0.10 \text{ m}$$

Frequency $f = v/\lambda$ = 330 m s^{-1} / 0.10 m = 3300 Hz.

4 The diagrams show particle movement in an air column when a stationary wave exists in the column.



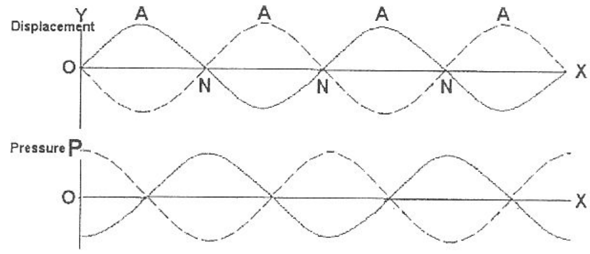
The first diagram shows the displacement of some particles at one instant and the second diagram shows the displacement of some particles half a cycle later.
What is the length L of the column in terms of wavelength λ, and at which position within the column does the pressure change by the largest amount?

	length L	maximum pressure change at
A	$\frac{3}{4}\lambda$	node
B	$\frac{3}{4}\lambda$	antinode
C	$\frac{3}{2}\lambda$	node
D	$\frac{3}{2}\lambda$	antinode

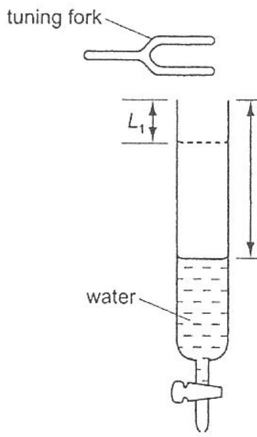
N08/I/22

Solution Answer: **A**

The distance between two adjacent nodes is half-a-wavelength. Therefore, $\lambda/2 = 2L/3$ and $L = 3\lambda/4$.
The pressure antinode (where pressure changes by the largest amount) is at the displacement node. This is because the displacement node experiences the greatest compression and rarefaction. On the other hand, the pressure node (where there is no pressure change) is at the displacement antinode.



5 A tuning fork is made to vibrate above a tube filled with water. The water is allowed to run out of the tube.



A loud sound is heard when the length of the air column is L_1 and again when the length is L_2. What is the wavelength of the sound in the tube?

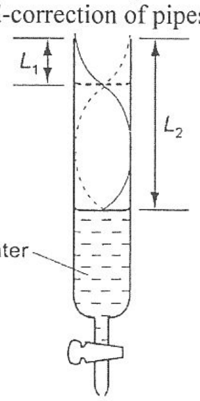
A $2L_1$ **B** $L_2 - L_1$ **C** $2(L_2 - L_1)$ **D** $2L_2$

N09/I/22

Solution Answer: **C**

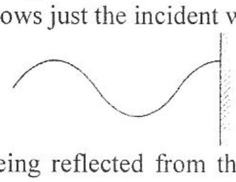
A loud sound is heard (resonance) when a stationary wave is formed within the tube.

A stationary wave is formed in a closed pipe with a node at the closed end and an antinode at the open end (or slightly above the open end due to end-correction of pipes).



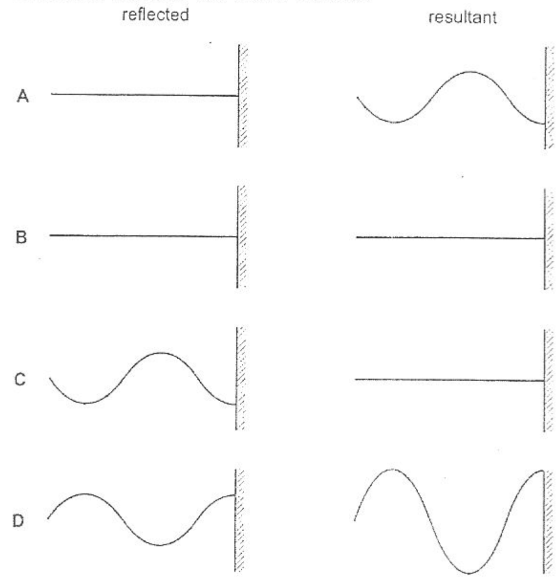
The distance between successive nodes ($L_2 - L_1$) is half-a-wavelength, hence the wavelength is $2(L_2 - L_1)$.

6 A continuous progressive wave is incident on a barrier. The diagram shows just the incident wave at one instant.



The wave is being reflected from the barrier, forming a stationary wave.

Which diagrams represent the reflected wave and the resultant wave at the same instant?



N10/I/21

Solution Answer: **C**

When a progressive wave is reflected from a barrier, it undergoes a π radians phase change at the point of reflection. The incident and reflected waves undergo destructive interference at the barrier to produce a node in the resultant wave at the point of reflection (eliminate answers A and D). The incident and reflected waves further undergo superposition at all other locations along the wave to produce the resultant wave (eliminate B) which at this instant in time all happen to have zero displacement (not zero amplitude) as shown in answer C.

Short Questions

7 A string is stretched between two points. When the string is plucked, a stationary wave is produced on the string, as illustrated in Fig. 4.1.

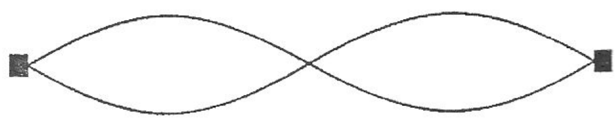


Fig. 4.1

The stationary wave has frequency f and wavelength λ.

(i) 1. State, in terms of λ, the minimum separation of two points that have zero amplitude of vibration. [1]

2. State the phase angle between the vibrations of points on the string situated at adjacent antinodes. [1]

(ii) The speed v of a progressive wave is given by the expression

$$v = f\lambda.$$

A stationary wave does not have a speed. By reference to the formation of a stationary wave, explain the significance of the product $f\lambda$ for a stationary wave. [3]

N04/II/4 (part)

Solution

1. 2λ.
2. 180°.

A stationary wave is formed by the superposition of two progressive waves of the same type with equal speed, frequency, wavelength and amplitude travelling in opposite directions.
The product $f\lambda$ in a stationary wave refers to the speed of the two progressive waves that superpose to form the stationary wave.
Due to the two waves having the same speed $f\lambda$ but different directions, their velocity vectors cancel out resulting in the stationary wave not travelling in either direction.

8 Explain the meaning of each of the following terms as applied to waves. You may include labelled diagrams if you wish.

(a) stationary wave [2]

N08/III/3

Solution

(a) A stationary wave is a wave which's waveform does not advance, and there is no transfer of energy in any direction. However, there are localised vibrations with amplitude ranging from maximum at points known as antinodes, to zero at points known as nodes. Points in adjacent half-wavelength segments vibrate 180° out-of-phase. The wavelength of the wave is twice the distance between two successive nodes or two successive antinodes. It is formed by the superposition of two progressive waves of the same type, frequency, wavelength and amplitude moving in opposite directions, as shown in Fig. 3.1.

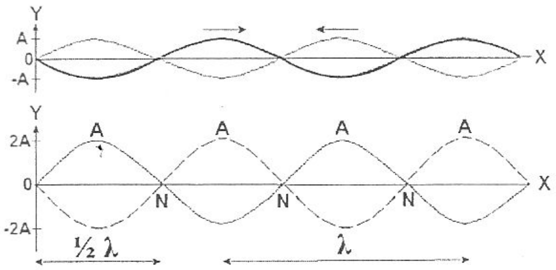
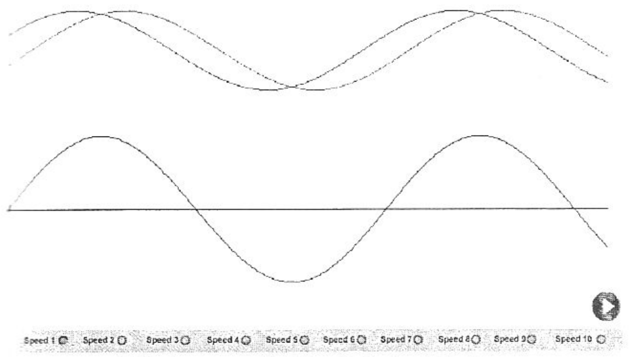


Fig. 3.1

The superposition of two progressive waves to produce a stationary wave is shown in the Flash animation below.



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http://www.physics.com.sg/H2/Stationary_Waves.swf

Long Questions

9 (a) Explain what is meant by the term *stationary wave*. [3]

(b) A long glass tube has a small loudspeaker, connected to a signal generator, placed at one end. A small micro-phone that can be moved through the tube is connected to a cathode-ray oscilloscope (CRO), as shown in Fig.3.1.

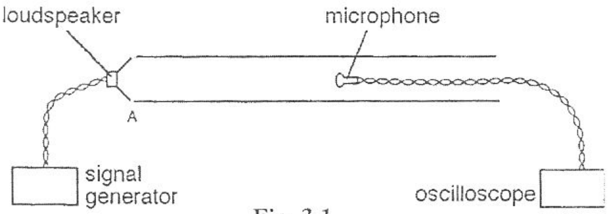


Fig. 3.1

For some frequencies of the signal generator, the microphone detects large-amplitude signals at certain positions and low-amplitude signals at other positions. The results obtained are given in the table of Fig.3.2.

frequency/ Hz	distance of microphone from the loudspeaker for maximum amplitude signal / mm						
270	102	724	1348				
405	82	499	913	1332			
540	62	375	685	1000	1310		
675	52	302	552	800	1050	1300	
810	47	253	462	670	880	1088	1296

Fig. 3.2

(i) Using the distances in the table, deduce the wave-length of the sound at each frequency. Show your working. [4]

(ii) Use your values from (i) to determine five values for the speed of sound in the tube. [3]

(iii) Calculate the average value for the speed of sound. [1]

(iv) Suggest a value for the uncertainty in the speed of sound you have found in (iii). Explain how you obtained this value. [2]

(c) Microphones are devices that detect variations of air pressure. By describing the movement of molecules in a stationary sound wave, explain where the air pressure varies most and where it varies least. [4]

(d) Suggest how stationary electromagnetic waves might be set up. [3]

N05/III/3

Solution

(a) A stationary wave is a wave produced by the superposition of two progressive waves of the same type, with the same speed, frequency, wavelength and amplitude, travelling in opposite directions.

A stationary wave's waveform does not advance, nor is there transportation of energy in either direction. However, there are localized vibrations with varying amplitudes ranging from maximum amplitude at locations known as antinodes, to zero amplitude at locations known as nodes.

All particles between two successive nodes vibrate in phase with one another, but are 180° out-of-phase with all particles in the adjacent half-wavelength segment.

The wavelength in a stationary is twice the distance between two successive nodes or two successive antinodes.

(b) (i) The wavelength of a stationary wave is twice the distance between two successive nodes or two successive antinodes.

Since the positions of the antinodes (maximum amplitude signals) are given, the antinodes are used for determining the wavelength.

At 270 Hz,
wavelength = 1348 – 102 = 1246 mm = 1.246 m.

At 405 Hz,
take average of 831 mm (913 – 82) and 833 mm (1332 – 499).
Wavelength = 832 mm = 0.832 m.

At 540 Hz,
take average of 623 mm (685 – 62), 625 mm (1000 – 375) and 625 mm (1310 – 685).
Wavelength = 624 mm = 0.624 m.

At 675 Hz,
take average of 500 mm (552 – 52), 498 mm (800 – 302), 498 mm (1050 – 552) and 500 mm (1300 – 800).
Wavelength = 499 mm = 0.499 m.

At 810 Hz,
take average of 415 mm (462 – 47), 417 mm (670 – 253), 418 mm (880 – 462), 418 mm (1088 – 670) and 416 mm (1296 – 880).
Wavelength = 417 mm = 0.417 m.

(ii) Speed $v = f\lambda$ where f is the frequency of the wave and λ its wavelength.

270 Hz: v = 270 Hz x 1.246 m = 336.42 = 336 m s^{-1}
405 Hz: v = 405 Hz x 0.832 m = 336.96 = 337 m s^{-1}
540 Hz: v = 540 Hz x 0.624 m = 336.96 = 337 m s^{-1}
675 Hz: v = 675 Hz x 0.499 m = 336.83 = 337 m s^{-1}
810 Hz: v = 810 Hz x 0.417 m = 337.77 = 338 m s^{-1}

(iii) The average value for the speed of sound
v = (336 + 337 + 337 + 337 + 338) / 5
= 1685 / 5 = 337 m s^{-1}

(iv) The uncertainty is 1 m s^{-1}.

The data for frequency are all given to 3 significant figures.
The antinodal distances, being the subtraction of two whole numbers, are all evaluated to the lowest order of 10^0 (i.e. a place value of 1), which translates into either 3 or 4 significant figures. Therefore, the value for speed is evaluated to 3 significant figures (take the lower of 3 and 4 s.f.s).

This implies an uncertainty of ± 1 m s^{-1}.

The sum of the five speed values is thus 1685 ± 5 (for additions, the absolute uncertainties are added).

Dividing this sum by five yields 337 ± 1 m s^{-1} (when dividing by a constant "five", the uncertainty is also divided by the same constant "five").

(c) In a sound wave, which is a longitudinal wave, air molecules vibrate in a direction parallel to the direction of propagation of the wave.

Although the stationary sound wave does not propagate, there are localized vibrations with amplitudes ranging from maximum at the antinodes to zero at the nodes.

The molecules at the antinodes have maximum freedom of motion and hence experience the lowest pressure variation. The molecules at the nodes, however, are not free to move.

Their pressure increases when molecules on both sides of the node move towards it (compression), and decreases when molecules on both sides move away from it (rarefaction).

Therefore, the displacement nodes are the locations of the pressure antinodes (maximum pressure variation) while the displacement antinodes are the pressure nodes (minimum pressure variation).

(d) To set up a stationary electromagnetic wave e.g. microwave, the loudspeaker should be replaced by a microwave antenna and the signal generator by a microwave source.

Instead of a tube with an open end, two microwave sources can be placed facing each other, one at each end of the tube. This ensures that the two interfering progressive waves are as similar in amplitude as possible (whereas reflected waves are weaker than the incident wave, resulting in incomplete cancellations at the nodes).

To detect the microwave nodes and antinodes, the microphone is replaced with an antenna connected to an amplifier attached to a high-frequency oscilloscope with a range of up to several GHz.

(e) (i) State the *principle of superposition*. [3]

(ii) A sound wave passes into a pipe that is open at both ends, as shown in Fig. 7.4.

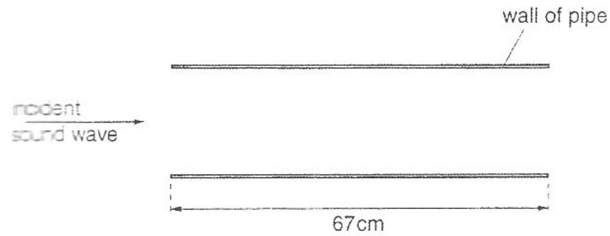


Fig. 7.4

The sound wave travels along the axis of the pipe.

Explain the formation of a stationary (standing) wave in the pipe. [2]

(f) The frequency of the sound wave in (e)(ii) is gradually increased from a low value.

A loud sound is heard in the pipe for the first time at a frequency of 250 Hz.

The length of the pipe is 67 cm.

(i) On Fig. 7.4, mark all the positions of

1. the displacement antinodes (use the letter A),
2. the displacement nodes (use the letter N).

[1]

(ii) Calculate a value for the speed of the sound in the pipe.

speed = ... m s^{-1} [2]

(g) An alternative, more reliable, method of measuring the speed of sound shows that the value in (f)(ii) is an underestimate.

This underestimate cannot be attributed to the uncertainty in the measurement of either the frequency or the length of the pipe.

State and explain what can be deduced about the positions of either the nodes or the antinodes of the stationary wave in the pipe. [2]

N10/III/7 (part)

Solution

(e) (i) The *principle of superposition* states that when two or more waves of the same kind exist simultaneously at a point in space, the displacement of the resultant wave at that point is the vector sum of displacements of the individual waves.

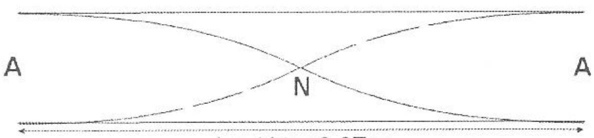
(ii) When the sound wave reaches the open end, it is reflected due to a change in boundary conditions.

Since the boundary is an open end, the reflected wave experiences no phase change at the point of reflection and interferes constructively with the incident wave to produce an antinode there with twice the amplitude of the incident wave.

The incident and reflected waves, travelling in opposite directions, also undergo superposition throughout the tube to produce regions of high and low amplitude along the pipe.

The highest amplitude positions occur at locations where the two waves always arrive in phase to undergo constructive interference, and are known as antinodes; while the zero amplitude positions occur at locations where the two waves always arrive π radians out of phase to undergo destructive interference, and are known as nodes.

f. (i) 1. The displacement antinodes are marked A.
2. The displacement nodes are marked N.



$L = ½\lambda = 0.67$ m

Open Pipe (First Harmonic)

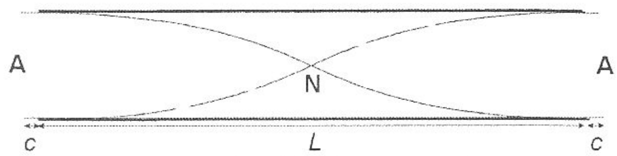
(ii) The distance between two adjacent antinodes is half-a-wavelength.

$½\lambda = 0.67$ m

$\lambda = 1.34$ m

Speed $v = f\lambda$ = (250 Hz)(1.34 m) = 335 m s^{-1}.

g. The underestimation is due to the end correction (c) of pipes, whereby the antinodes are not exactly at the open ends but located slightly outside. The nodes, however, remain at the same position, as shown.

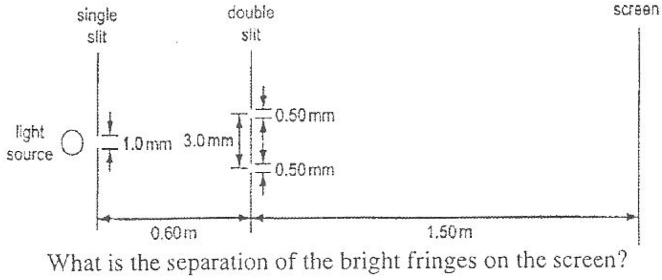


Open Pipe (First Harmonic)

Therefore, the wavelength is not $2L$ but actually $2(L + 2c)$ where c is the end correction (distance between the antinode and the open end of the pipe).

Interference

11 A student sets up an experiment to demonstrate double slit interference, using light of wavelength 6.0×10^{-7} m. The main features of the apparatus, and some of the dimensions, are illustrated.



What is the separation of the bright fringes on the screen?

A 0.12 mm
B 0.30 mm
C 0.90 mm
D 1.80 mm

N06/I/16

Solution Answer: **B**

Using Young's double-slit formula, the fringe separation:

$y = \lambda D/d = (6.0 \times 10^{-7}\text{ m})(1.50\text{ m})/(3.0 \times 10^{-3}\text{ m})$
$= 3.0 \times 10^{-4}\text{ m} = 0.30$ mm.

12 A double-slit light interference experiment is set up.

Under which conditions will the separation of bright fringes be greatest?

	distance between slits	distance from slits to screen	wavelength of source
A	small	large	short
B	small	large	long
C	large	small	short
D	large	small	long

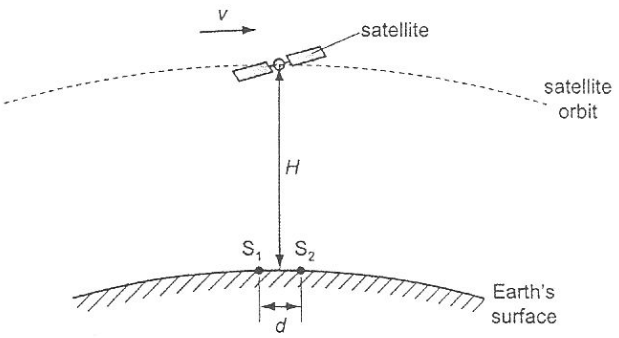
N07/I/22

Solution Answer: **B**

Fringe separation $y = \lambda D/d$ where λ is the wavelength of the source, D is the distance from the slits to the screen and d the distance between slits.

13 Two coherent sources S_1 and S_2 of radio waves are separated by a distance d.

A satellite is travelling overhead in the direction shown.



The satellite, travelling at speed v and at altitude H, detects f maxima of intensity of the radio waves per unit time.

What is the wavelength of the radio waves?

A $\dfrac{d}{fH}$ B $\dfrac{df}{H}$ C $\dfrac{df}{vH}$ D $\dfrac{dv}{fH}$

N09/I/24

Solution Answer: **D**

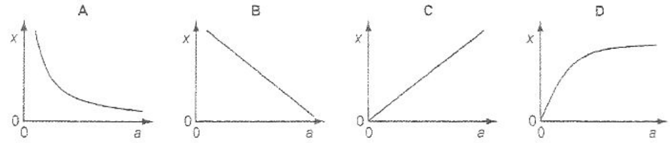
In one second, the satellite travels v metres and detects f maxima. Hence the separation between successive maxima (or fringe separation) is v/f.

Assuming the altitude H to be significantly larger than the source separation d, Young's double slit equation can be applied,

$$v/f = \lambda H/d$$
$$\lambda = dv/fH$$

Since this is a multiple choice question, the answer can also be selected based on dimensional analysis of the answers. The first answer (A) has the SI unit of second (s), the second answer (B) has the unit of s^{-1}, the third answer (C) has the unit of m^{-1}. Only the last answer (D) has the unit of m.

14 A double-slit interference experiment uses coherent monochromatic light.
Which graph shows how the distance x between fringes varies with slit separation a, when the distance from the double slits to the screen is kept constant?



N10/I/22

Solution Answer: A

Fringe separation $x = \lambda D/a$. Since wavelength λ and distance from the double slits to the screen D are kept constant, x is inversely proportional to a.

Short Questions

15 *(a)* Explain what is meant by *coherent* sources. [1]

(b) Two small coherent sound sources S_1 and S_2 are set up as shown in Fig. 5.1.

Fig. 5.1

A sound detector is moved along a line AB that is parallel to S_1S_2. N is the point on AB such that $S_1N = S_2N$.

The sound waves from S_1 and S_2 have frequency 2.80 kHz and speed 336 m s^{-1}.

(i) Show that the wavelength of the waves is 12.0 cm [1]

(ii) The detector, when placed at N, indicates a maximum intensity of sound. As it is moved from N to a point P, the intensity varies between high and low values. At P, the distance S_1P is 372 cm and S_2P is 402 cm.

Determine, with suitable explanation,

1. whether the intensity of sound at P is high or low,
2. the number of high intensity regions that are found between N and P. Do not include the maximum at N.

number = ... [6]

(iii) The intensity of sound produced at N by S_1 alone or by S_2 alone is I. Calculate the intensity at N, in terms of I, when both S_1 and S_2 are producing sound.

intensity = ... [3]

N02/II/5

Solution

(a) Two sources are said to be coherent if they produce waves with a constant phase difference.

(b) (i) Wavelength
$\lambda = v/f$
$= (336\ \text{m s}^{-1})/(2.80 \times 10^3\ \text{Hz})$
$= 0.120$ m
$= 12.0$ cm.

(ii) 1. Low.
The fact that N, which is equidistant from both sources, is an intensity maximum shows that the two sources oscillate in phase.
Since the two waves that are emitted in phase, a point on the screen will have an intensity maximum if the path difference $S_1P - S_2P = n\lambda$, and a minimum if $S_1P - S_2P = (n + ½)\lambda$, where n is an integer.
The path difference between S_1P and S_2P
$= S_1P - S_2P$
$= 402$ cm $- 372$ cm
$= 30$ cm $\equiv 2.5$ wavelengths.
Since the path difference $= (n + ½)\lambda$, where $n = 2$, the two waves arrive 180° out-of-phase and undergo destructive interference, producing a low intensity at P.

2. Two.
For waves that are emitted in phase, intensity maximum occurs at every location where the path difference between S_1P and S_2P is an integer multiple of λ.
Since P is the third minimum where the path difference $= 2.5\ \lambda$, there is one maximum where the path difference is λ and another one where it is 2λ.
There are altogether 2 maxima between N and P, excluding the central maximum at N where the path difference is zero.

(iii) The two waves arrive in phase at N and undergo constructive interference such that the resultant amplitude at N is $2A$, where A is the amplitude of each wave.
Since intensity is proportional to the square of amplitude, the intensity at N is $4I$, where I is the intensity of each wave.

16 Coherent light is incident normally on a double slit, as shown in Fig. 6.1.

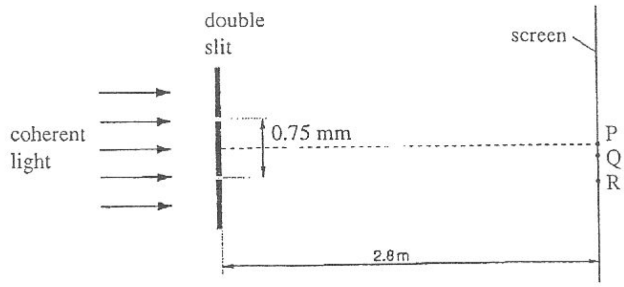


Fig. 6.1 (not to scale)

The separation of the slits in the double-slit arrangement is 0.75 mm.

A screen is placed parallel to, and at a distance of 2.8 m from, the double slit. P is a point on the screen that is equidistant from the two slits.

Fig. 6.2 shows the variation with distance from P of the intensity I of the light on the screen.

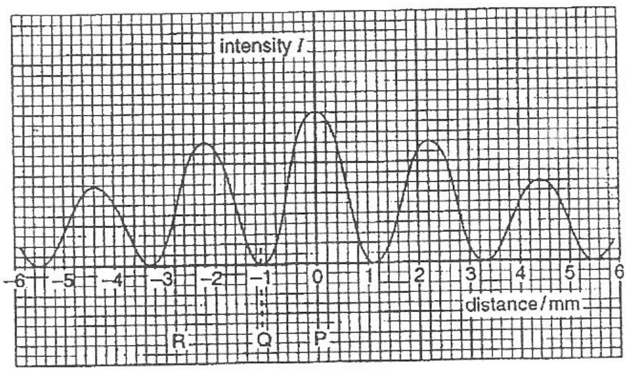


Fig. 6.2

(a) Calculate the wavelength, in nm, of the coherent light.

wavelength = ... nm [4]

(b) Points Q and R are points on the screen. Their positions are indicated on Fig. 6.2. Determine the phase angle between the waves from the double slit when the waves meet at

(i) point Q,
phase angle = ... rad [1]

(ii) point R.
phase angle = ... rad [2]

(c) Suggest why the maxima on Fig. 6.2 are not all of the same intensity. [2]

N06/II/6

Solution

(a) The fringe separation is 2.2 mm (Fig. 6.2).

The wavelength can be calculated using Young's double-slit formula: fringe separation $y = \lambda D/d$, where λ is the wavelength of the light, D the distance from the double-slits to the screen (2.8 m) and d the slit separation (0.75 x 10^{-3} m). Rearranging the equation,

$$\lambda = yd/D$$

$$\lambda = (2.2 \times 10^{-3}\ \text{m})(0.75 \times 10^{-3}\ \text{m})/(2.8\ \text{m})$$

$$\lambda = 5.89 \times 10^{-7}\ \text{m} = 589\ \text{nm}$$

(b) (i) Point Q has zero intensity hence the waves must have undergone destructive interference.

The phase difference is thus π rad.

(ii) The separation between a bright and dark fringe on the screen is 1.1 mm and this corresponds to a phase difference of π rad.

Point R is 0.60 mm from the first maximum. The phase angle = $(0.60 / 1.1) \times \pi = 0.5455\ \pi = 1.71$ rad.

A more accurate way is to use trigonometric relations. Let the phase angle be θ.

If one of the waves is assumed to be a cosine wave starting at the origin, the other wave is displaced to the right by angle θ.

The point of intersection between the two waves will be at phase angle $\theta/2$, and this point will be where the two waves superpose to produce the amplitude of the resultant wave.

Since the intensity of the resultant wave at R is 5/12 that of the first order maximum (Fig. 6.2), the amplitude of the resultant wave is $\sqrt{5/12}$ that of the first order maximum (which is 2 A, where A is the amplitude of each individual wave).

$A \cos(\theta/2) = \sqrt{5/12}\ A$ hence $\theta = 1.74$ rad.

(c) The light diffracted out from each slit produces an intensity pattern which is greatest at the central position and tapers off towards both sides.

Since the intensity of the light from each individual slit decreases with distance from the central axis, the higher order maxima will correspondingly be of lower intensity as they are constrained by the diffraction envelope (shown as the dotted line in Fig. 6.3 below).

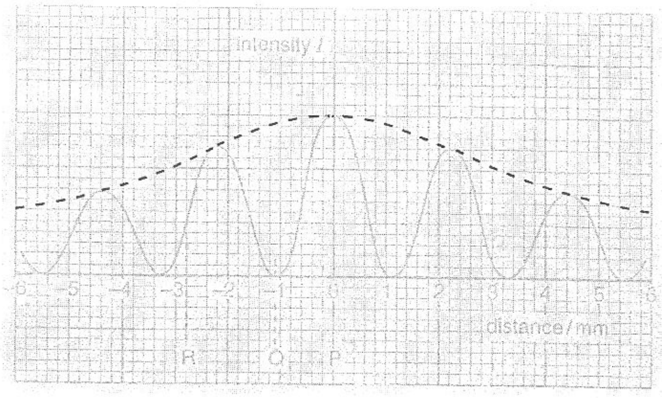


Fig. 6.3

17 (a) State what is meant by a *progressive wave*.

....

....

.... [2]

(b) Two sources S_1 and S_2 produce waves of the same frequency on the surface of some water.

(i) State three conditions that must be satisfied for waves from the two sources to produce an observable interference pattern on the water surface.

1. ....

....

2. ....

....

3. ....

.... [3]

(ii) Wavefronts produced by the two sources are illustrated in Fig. 3.1.

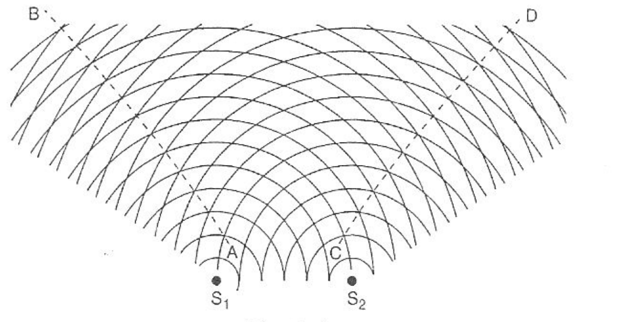


Fig. 3.1

Waves of maximum amplitude are observed along the directions AB and CD, amongst others.

1. Determine the number of directions between AB and CD along which maximum amplitude occurs.

number = ... [1]

2. On Fig. 3.1, draw a line to show one direction along which the waves have minimum amplitude. [1]

N07/II/3

Solution

(a) A *progressive wave* is a wave which's waveform advances and there is a transfer of energy along the direction of propagation of the wave.

(b) (i) 1. The two sources must be coherent, meaning they must have a constant phase relationship.

2. The two sources should produce waves with approximately the same amplitude.

3. The separation between the two sources should be greater than the wavelength of the wave.

4. The waves should meet.

There is another condition which applies to transverse waves in general:

The two sources should produce waves which are either unpolarised, or polarised in the same plane. However, it is difficult to imagine how surface water waves could be polarised in different planes.

(ii) 1. Seven (excluding AB and CD).

These are lines joining the intersections between the wavefronts. Refer to Fig. 3.2.

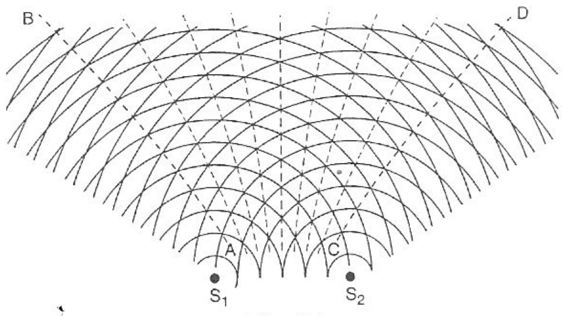


Fig. 3.2

2. The line is shown in Fig. 3.3.

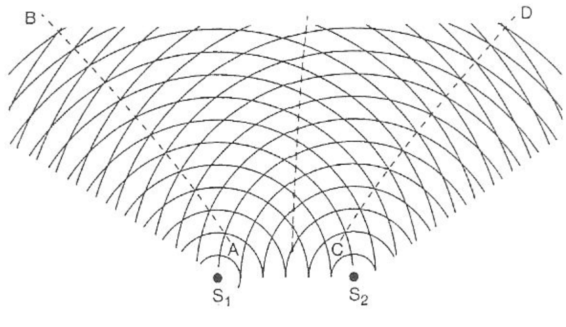


Fig. 3.3

Assuming the wavefronts to be crests, then the troughs lie mid-way between the crests. Destructive interference occurs where the crests of one wave meet the troughs of the other.

18 Explain the meaning of each of the following terms as applied to waves. You may include labelled diagrams if you wish.

(b) diffraction [2]

N08/III/3

Solution

(b) Diffraction is the spreading of waves through an aperture, or bending of waves around an obstacle. It is observable when the wavelength of the wave is of the same order of magnitude as the width of the aperture (Fig. 3.2). When the width of the aperture is significantly greater than the wavelength of the wave, the diffraction is less observable (Fig. 3.3).

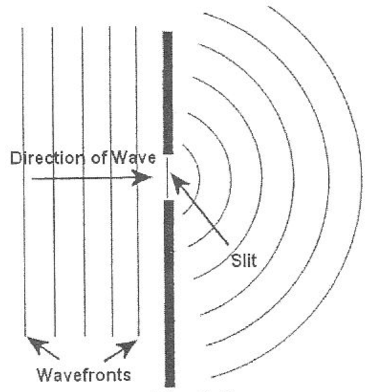


Fig. 3.2

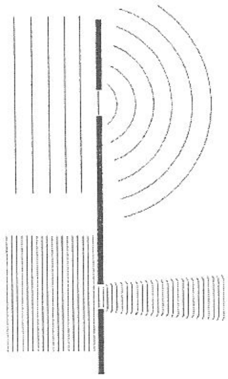


Fig. 3.3

19 (a) State the meaning of

(i) *diffraction,*

(ii) *phase difference,*

(iii) *coherence.* [3]

(b) Fig. 3.1 shows two microwave emitters M_1 and M_2. A microwave detector is moved along the line AB.

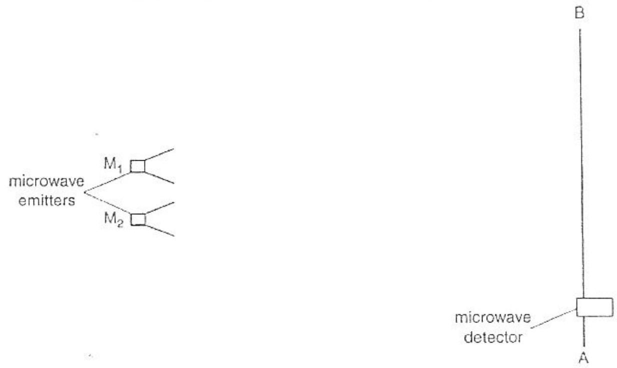


Fig. 3.1

(i) Explain how interference fringes are formed along the line AB. [3]

(ii) The following changes are made independently. Describe, in each case, the effect on the position and intensity of the fringes.

1. The intensity of the microwave emitted from both M_1 and M_2 is increased. [2]
2. The phase difference between the microwaves emitted from M_1 and M_2 is changed by π radians. [2]

N10/II/3

Solution

(a) (i) *Diffraction* is the spreading of waves through an aperture or around an obstacle. It is observable when the wavelength of the wave is of the same order of magnitude as the width of the aperture or size of the obstacle.

(ii) The *phase difference* between two waves meeting at a point (or two points along a wave) is an angular measure of the difference in their states of motion as a fraction of a complete cycle, expressed in degrees or radians, where one complete cycle is equivalent to 360° or 2π radians.

(iii) Two wave sources possess coherence if the waves emerging from them have a constant phase difference (or phase relationship).

(b) (i) The microwaves diffract from each emitter and spread out.

Where the waves meet, they undergo interference.

Assuming the emitters are coherent and also emit waves which are in phase, at locations along AB where their path difference is $n\lambda$ (where n is an integer and λ the wavelength), they arrive in phase and undergo constructive interference producing fringes with maximum intensity.

At locations where their path difference is $(n + \frac{1}{2})\lambda$, they arrive π radians out-of-phase and undergo destructive interference producing fringes with zero intensity.

(ii) 1. Increasing the intensity of the microwave does not affect the position of the fringes.

However, the maximum intensity fringes become more intense due to the constructive interference of two waves of larger amplitude; while the minimum intensity fringes remain unchanged due to the destructive interference of two waves of larger but equal amplitude.

The contrast increases.

2. Changing the phase difference by π radians does not affect the intensity of the fringes but causes the maximum and zero intensity fringes to exchange places.

Whereas previously a path difference of $n\lambda$ results in the two waves arriving in phase and undergoing constructive interference to produce high intensity fringes; while a path difference of $(n + \frac{1}{2})\lambda$ results in the two waves arriving π radians out-of-phase and undergoing destructive interference to produce zero intensity fringes, the reverse is true when the phase difference between the emitters is changed by π radians.

Long Questions

20 (i) Show how the principle of superposition of waves can be used to explain the formation of two source interference fringes. [3]

(ii) Two-source interference fringes using light can only be obtained if light from the two sources is coherent. Explain

1. the meaning of the term *coherent,* [1]
2. why, in practice, interference fringes can be seen only if light from a single source is split into two. [2]

(iii) Coherent, monochromatic light from two narrow slits a distance 0.38 mm apart causes an interference pattern on a screen 1.20 m from the slits, as illustrated in Fig. 3.1.

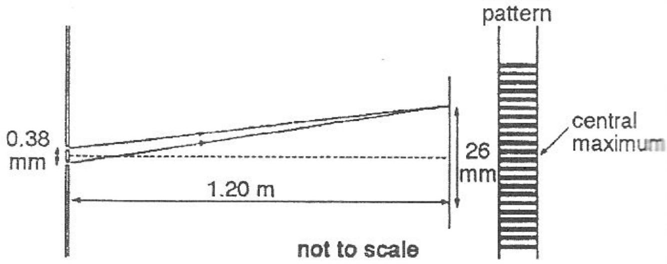


Fig. 3.1

The distance from the sixth bright fringe on one side of the pattern to the sixth bright fringe on the other side of the pattern is found to be 26 mm. Calculate the wave-length of the monochromatic light. [3]

(iv) State the experimental advantage gained by determining the fringe width in the way that was used in (iii). [1]

(v) Another way of obtaining fringes similar to those described in (iii) is illustrated in Fig. 3.2.

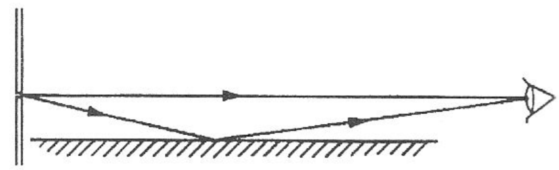


Fig. 3.2

A single slit is viewed both directly and by reflection from a mirror surface. Explain why this system produces a fringe pattern. [2]

N04/III/3 (part)

Solution

(i) When waves spread out from the two coherent sources, the spreading wavefronts interfere with each other.
At locations where the wave trains are in phase, their displacements add up vectorially, according to the principle of superposition, to produce a resultant wave with double the amplitude and four times the intensity. These are the regions of constructive interference, also known as maxima.
At locations where the wave trains are 180° out-of-phase with each other, their displacements cancel vectorially to produce a resultant wave with zero amplitude and intensity. These are the regions of destructive interference, also known as minima.
At other locations where the wave trains are neither in phase nor completely out-of-phase, the amplitude and intensity are between the minimum and maximum.
Energy is neither being created at the maxima or destroyed at the minima. It is merely re-distributed from the regions of lower intensity to those of higher intensity.

(ii) 1. Two sources are said to be *coherent* if they produce waves with a constant phase difference.

2. Most light sources are not coherent (one exception is the LASER).
The splitting of a single source ensures that the two interfering wave trains are from a single point source so that they are coherent.
Source coherence is a condition for interference fringes to be observed, as the bright and dark fringes can only be formed if there is a constant phase relationship between the two interfering wave trains.

(iii) The fringe separation y = 26 mm / 12 = 2.167 mm.
Use the Young's double slit formula, fringe separation $y = \lambda D/d$ where λ is the wavelength, D the distance between the screen and the slits and d the distance between the slits.
Wavelength
$\lambda = yd/D$
$= (2.167 \times 10^{-3}\text{ m})(0.38 \times 10^{-3}\text{ m})/(1.20\text{ m})$
$= 6.862 \times 10^{-7}$
$= 6.9 \times 10^{-7}$ m.

By taking the average of 12 fringe separations rather than just one fringe, the uncertainty in the measurement is reduced from ±1 mm to ±(1/12) mm.

The mirror image of the single slit in the mirror acts as a second light source. The reflected beam of light appears as though it originates from the image of the slit.
The source and the image (acting as another source) have the same frequency and constant phase difference (coherent) hence their waves are able to interfere.
At locations where the two waves are in phase, constructive interference occurs to produce intensity maxima.
At other locations where the two waves are 180° out-of-phase, destructive interference occurs to produce intensity minima.

Diffraction Grating

21 A narrow beam of monochromatic light falls at normal incidence on a diffraction grating. Third-order diffracted beams are formed at angles of 45° to the original direction.

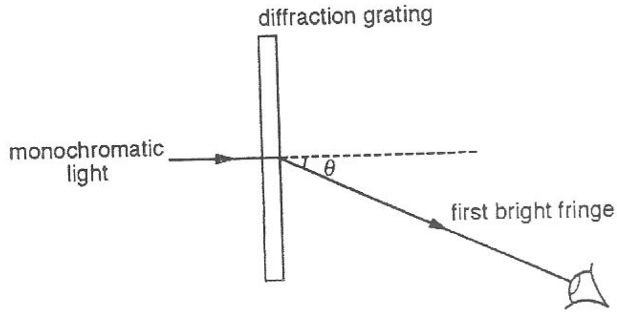
What is the highest order of diffracted beam produced by this grating?

A 3rd **C** 5th
B 4th **D** 6th

N02/I/17
Answer: **B**

Solution
Use the diffraction grating formula $d \sin \theta = n\lambda$.
For the third order diffracted beam, $n = 3$ hence $d \sin 45° = 3\lambda$.
The highest order n's angle θ cannot exceed 90°.
To find the maximum order, calculate n from $d \sin 90° = n\lambda$.
Dividing the second equation by the first, d and λ are eliminated, resulting in $n = 4.24$.
The highest order is 4 (always round down).

22 A source of monochromatic light is viewed through a diffraction grating.



Which of the following causes the angle θ to increase?

A decreasing the distance between adjacent slits on the grating
B decreasing the wavelength of the monochromatic light
C increasing the distance of the eye from the grating
D increasing the number of slits of the grating but keeping the slit spacing the same

N03/I/16
Answer: A

Solution
Use the diffraction grating formula $d \sin \theta = n\lambda$.
Rearranging the equation, $\sin \theta = n\lambda/d$.
To increase θ, the denominator d can be decreased.

23 When monochromatic light of wavelength 5.0×10^{-7} m is incident normally on a plane diffraction grating, the second-order diffraction lines are formed at angles of 30° to the normal to the grating. What is the number of lines per millimetre of the grating?

A 250
B 500
C 1000
D 2000

N04/I/19
Answer: B

Solution
Use the diffraction grating formula $d \sin \theta = n\lambda$.
Rearranging the equation,
$d = = n\lambda/\sin \theta = 2(5.0 \times 10^{-7}\text{ m})/(\sin 30°) = 2.0 \times 10^{-6}$ m.
Number of lines per metre = $1/d = 1/(2.0 \times 10^{-6}\text{ m})$ = 500,000.
The number of lines per mm = 500.

24 Monochromatic light is incident normally on a diffraction grating. Second order diffraction is observed at an angle of 38.9°.

At what angles will the first and third orders be observed?

	first order	third order
A	18.3°	57.2°
B	18.3°	70 4o
C	19.5°	58.5°
D	19.5°	77.8°

N05/I/19

Solution Answer: **B**

Use the diffraction grating formula $d \sin \theta = n\lambda$ where d is the slit separation, θ the observed angle of the intensity maxima, n the order and λ the wavelength.

For the second order,

$$d \sin 38.9° = 2\lambda$$
$$\lambda / d = 0.31398$$

For the first order,

$$d \sin \theta = \lambda$$
$$\sin \theta = \lambda / d = 0.31398$$
$$\theta = 18.3°$$

For the third order,

$$d \sin \theta = 3\lambda$$
$$\sin \theta = 3\lambda / d = 3(0.31398) = 0.94194$$
$$\theta = 70.4°$$

25 Light of frequency 6.0×10^{14} Hz passes through a diffraction grating with 4.0×10^3 lines per centimetre.

What is the angle between the two third-order diffraction maxima?

A 12° B 23° C 37° D 74°

N07/I/23

Solution Answer: **D**

Slit separation $d = 1/p = 1/(4.0 \times 10^5 \text{ m}^{-1}) = 2.5 \times 10^{-6}$ m.

Wavelength

$$\lambda = c/f = 3.00 \times 10^8 \text{ m s}^{-1} / 6.0 \times 10^{14} \text{ Hz} = 5.0 \times 10^{-7} \text{ m}.$$

Apply the diffraction grating formula,

$$d \sin \theta = 3\lambda$$

$$(2.5 \times 10^{-6} \text{ m}) \sin \theta = 3 (5.0 \times 10^{-7} \text{ m})$$

$$\theta = 37°$$

where θ is the angle between each third-order diffracted maximum and the central axis.

The angle between the two third-order diffracted maxima is thus 2 x 37° = 74°.

26 A beam of monochromatic light of wavelength 600 nm is incident normally on a diffraction grating that has 3.0×10^5 lines per metre.

What is the total number of images produced by light transmitted through this grating?

A 5 B 8 C 9 D 11

N08/I/23

Solution Answer: **D**

Slit separation $d = 1/p = 1/(3.0 \times 10^5 \text{ m}^{-1}) = 3.333 \times 10^{-6}$ m.

Apply the diffraction grating formula with maximum angle of diffraction being 90°,

$$d \sin \theta = n\lambda$$
$$(3.333 \times 10^{-6} \text{ m}) \sin 90° = n (6.0 \times 10^{-7} \text{ m})$$
$$n = 5.56 = 5$$

(always round down to the lower integer)

The total number of images is 11, comprising 5 on each side of the central maximum, plus the central maximum.

27 Monochromatic light of wavelength λ is incident normally on a diffraction grating with line separation d.
At which angle with the normal to the grating is a diffracted beam observed?

A $\sin^{-1} \frac{2\lambda}{d}$ B $\sin^{-1} \frac{\lambda}{2d}$ C $\sin^{-1} \frac{d}{\lambda}$ D $\sin^{-1} \frac{2d}{\lambda}$

N10/I/23

Solution Answer: **A**

Diffracted beams are observed at angles θ which satisfy the following condition: $d \sin \theta = n\lambda$ or $\theta = \sin^{-1} n\lambda / d$, where n is an integer.

Short Questions

28 *(a)* Explain what is meant by

(i) the *diffraction* of a wave,

(ii) the *principle of superposition.* [4]

(b) A narrow beam of coherent light of wavelength 589 nm is incident normally on a diffraction grating having 4.00×10^5 lines per metre.

(i) Determine the number of orders of diffracted light that are visible on each side of the zero order. [3]

(ii) A student suspects that there are two wavelengths of light in the incident beam, one at 589.0 nm and the other at 589.6 nm.

1. State the order of diffracted light at which the two wavelengths are most likely to be distinguished.

2. The minimum angular separation of the diffracted light for which two wavelengths may be distinguished is 0.10°. Make calculations to determine whether the student can observe the two wavelengths as separate images. [4]

N03/II/5

Solution

(a) (i) *Diffraction* is the spreading of waves through an aperture or around an obstacle.
It is observable when the wavelength of the wave is of the same order of magnitude as the dimensions of the aperture or obstacle.

(ii) The *principle of superposition* states that when two or more waves of the same kind exist simultaneously at a point in space, the displacement of the resultant wave at that point is the vector sum of displacements of the individual waves.

(b) (i) Slit separation $d = 1/(4.00 \times 10^5 \text{ m}^{-1}) = 2.5 \times 10^{-6}$ m.
The maximum possible angle of a diffracted beam is 90°.
Using the diffraction grating formula $d \sin \theta = n\lambda$ with $\theta = 90°$,

$$(2.5 \times 10^{-6} \text{ m}) \sin 90° = n(589 \times 10^{-9} \text{ m})$$

$$n = 4.24 = 4 \text{ (always round down)}$$

(ii) 1. 4
(The higher the order, the greater the separation between fringes).

2. Use the diffraction grating formula $d \sin \theta = n\lambda$, where the order $n = 4$.

For the wavelength 589.0 nm,

$$(2.5 \times 10^{-6} \text{ m}) \sin \theta = 4(589.0 \times 10^{-9} \text{ m})$$

$$\theta = 70.459°$$

For the wavelength 589.6 nm,

$$(2.5 \times 10^{-6} \text{ m}) \sin \theta = 4(589.6 \times 10^{-9} \text{ m})$$

$$\theta = 70.624°$$

The angular separation
$\Delta\theta = 70.624° - 70.459° = 0.165°$.

This is greater than the minimum required, hence the student can observe the two wavelengths as separate images.

Long Questions

29 *(a)* Give reasons for the following.

(i) Sound waves and water waves can go round corners but light waves seem to travel only in straight lines. [2]

(ii) Sound waves cannot be polarised but radio waves can be polarised. [1]

(iii) Fig. 4.1 represents a stationary sound wave in a pipe. This figure looks like a transverse wave although sound waves are longitudinal waves. [2]

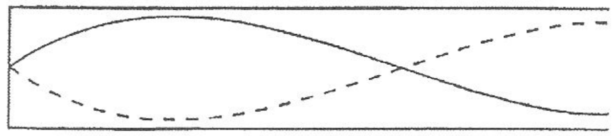


Fig. 4.1

(iv) A small radio receiver situated between a radio transmitter and a large reflector will not detect a signal in certain areas. [2]

(c) (i) State a wavelength of light in the visible spectrum. [1]

(ii) Calculate the angle of the first-order diffraction when light of the wavelength you have given in (i) is incident normally on to a diffraction grating having 200 lines per millimetre. [3]

N06/III/4 part

Solution

(a) (i) When waves arrive at an aperture or obstacle, they may undergo a process known as diffraction. Diffraction is the spreading of waves through an aperture, or in this case, the bending of waves around an obstacle or corner. However, diffraction is only observable if the wavelength of the wave is of the same order of magnitude as the dimension of the obstacle.

The wavelengths of sound and water waves are of the order of magnitude of metres, similar to that of the corner around which they are diffracting, hence diffraction is observable. Light waves, with wavelengths of the order of magnitude of micrometres, do not undergo observable diffraction around corners. However, they may undergo diffraction around obstacles of micrometre-dimension.

(ii) Sound waves are longitudinal waves whereas radio waves are transverse waves. Only transverse waves can be polarised as polarisation involves the transmission of vibrations in a plane perpendicular to the direction of the wave's propagation.

Sound waves, being longitudinal waves, do not have vibrations in the plane perpendicular to the direction of propagation. In longitudinal waves, the vibrations are parallel to the direction of propagation hence the entire wave will pass through the polariser. They cannot be polarised.

(iii) The figure is only a schematic graphical representation of the displacement of the air molecules in the sound wave. Since sound wave is a longitudinal wave, the air molecules vibrate parallel to the axis of the pipe.

The air molecules actually vibrate in a horizontal direction (left and right) but the schematic graphical representation is such that a rightward displacement is represented as positive while a leftward displacement negative.

(iv) The transmitted wave and the reflected wave travel in opposite directions. When two waves of the same type, amplitude, wavelength, frequency and speed travelling in opposite directions superpose with each other, a stationary wave is produced.

Along a stationary wave, there are locations where the two waves are 180° out-of-phase with each other. At these locations, the two waves undergo destructive interference resulting in a location with zero amplitude and intensity.

These are known as nodes and a detector placed at such locations will not detect a signal.

(c) (i) 600 nm.

(ii) Using the diffraction grating equation,

$$d \sin \theta = n\lambda$$
$$(1/p) \sin \theta = n\lambda$$
$$(1 / 200{,}000 \text{ m}^{-1}) \sin \theta = (1)(600 \times 10^{-9} \text{ m})$$
$$\sin \theta = 0.12$$
$$\theta = 6.892 = 6.9^\circ$$

where d is the grating spacing, θ the angle of diffraction, n the order and λ the wavelength of the light.

30 (a) Explain the following terms with reference to the light diffracted by a diffraction grating that is used with a monochromatic light source.

(i) *diffraction* [1]

(ii) *coherence* [1]

(iii) *superposition* [1]

(b) A parallel beam of white light that consists of wavelengths from 350 nm to 650 nm is incident normally on a diffraction grating that has 500 lines per millimetre.

(i) Calculate the maximum angle in the second order spectrum.
maximum angle = ...° [2]

(ii) Calculate the minimum angle in the third order spectrum.
minimum angle = ...° [2]

(iii) Explain a problem with viewing the second or third order maxima of the white light with this diffraction grating. [1]

N09/II/4

Solution

(a) (i) *Diffraction* refers to the spreading of light waves when they pass through each of the apertures of the diffraction grating. Diffraction is significant and observable if the width of the apertures is of the same of order of magnitude as the wavelength of the wave.

(ii) Two sources, such as two apertures in the diffraction grating through which the light waves emerge, possess *coherence* if the waves emerging from them have a constant phase difference (or phase relationship).

(iii) The principle of *superposition* states that when two or more waves of the same kind, such as those emerging from the apertures in the diffraction grating, exist at a point in space, the resultant displacement of the wave at that point is the vector sum of the displacements of the individual waves.

(b) The separation between adjacent slits in the diffraction grating $d = 1/p = 1/(500{,}000) = 2.0 \times 10^{-6}$ m.

(i) For the second order spectrum, $n = 2$, the maximum angle can be found using the longest wavelength $\lambda = 650 \times 10^{-9}$ m.

Applying the diffraction grating equation:

$$d \sin \theta = n\lambda$$
$$(2.0 \times 10^{-6} \text{ m}) \sin \theta = 2(650 \times 10^{-9} \text{ m})$$
$$\sin \theta = 0.65$$
$$\theta = 40.5^\circ$$

(ii) For the third order spectrum, $n = 3$, the minimum angle can be found using the shortest wavelength $\lambda = 350 \times 10^{-9}$ m.

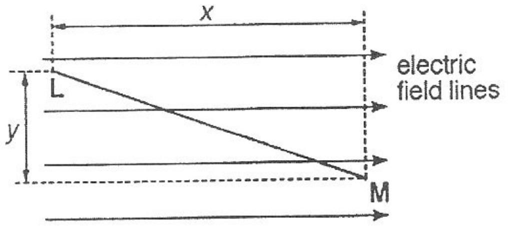
Applying the diffraction grating equation:

$$d \sin \theta = n\lambda$$
$$(2.0 \times 10^{-6} \text{ m}) \sin \theta = 3(350 \times 10^{-9} \text{ m})$$
$$\sin \theta = 0.525$$
$$\theta = 31.7^\circ$$

(iii) There is an angular overlap of 8.8° between the second and third order maxima. Within this 8.8°, the colours are mixed up. It is difficult to tell which colour belongs to which order of the spectrum.

TOPIC 12 Electric Fields

1 A small positive charge, placed at a point L inside a uniform electric field, experiences a force of magnitude F.



The charge is moved from point L to point M.

What is the change in the potential energy of the charge?

A gain of Fx

B gain of $F\sqrt{(x^2 + y^2)}$

C loss of Fx

D loss of $F\sqrt{(x^2 + y^2)}$ N02/I/9

Solution Answer: **C**

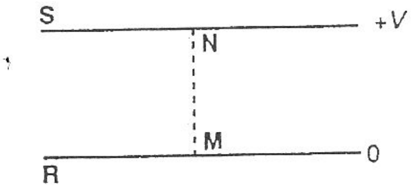
A positive charge's potential energy U increases with potential ($U = qV$).

The direction of electric field lines is from higher to lower potential hence the left side of the diagram is at a higher potential than the right.

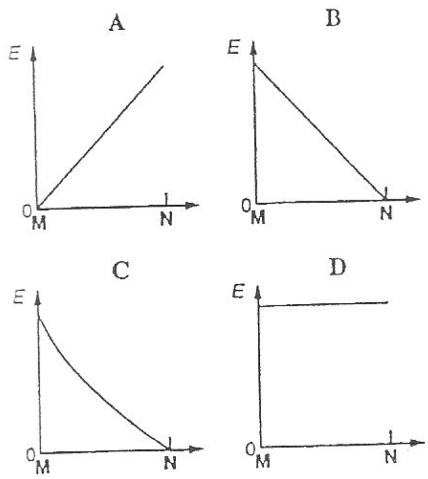
When the positive charge moves towards the right, it losses electric potential energy.

The amount of energy is lost is equal to the work done by the electric force $W = Fx$ where x is the distance moved along the direction of the field.

2 Two horizontal conducting plates R and S are a fixed distance apart. Plate S is at potential $+V$ with respect to plate R. MN is a line perpendicular to the plates.



Which graph shows how the magnitude E of the electric field strength varies along the line MN?



N02/I/18

Solution Answer: **D**

The electric field strength between two parallel plates is uniform and given by the equation $E = V/d$ where V is the potential difference between the two places and d the distance separating them.

*3 A gold nucleus of radius r is represented by the symbol ${}^{197}_{79}Au$. The charge on the nucleus may be considered to be concentrated at the centre of the nucleus.

Taking e as the elementary charge and ε_0 as the permittivity of free space, what is the electric field strength at the surface of an isolated gold nucleus?

A $\dfrac{79e}{4\pi\varepsilon_0 r^2}$

B $\dfrac{197e}{4\pi\varepsilon_0 r^2}$

C $\dfrac{79e^2}{4\pi\varepsilon_0 r^2}$

D $\dfrac{197e^2}{4\pi\varepsilon_0 r^2}$ N03/I/17

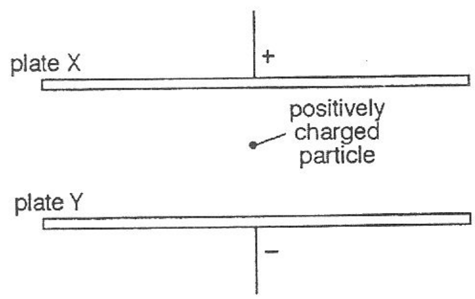
Solution Answer: **A**

The formula for the electric field strength at a distance r from a charge Q is given by $E = Q / 4\pi\varepsilon_0 r^2$.

The charge in a nucleus of proton number Z is Ze, where e is the elementary charge.

The proton number for gold $Z = 79$. Hence the electric field strength at a distance r from the nucleus is $E = 79e / 4\pi\varepsilon_0 r^2$.

4



Two large parallel metal plates X and Y are situated in a vacuum as shown.

Plates X and Y carry equal and opposite charges.

What happens to the force on a positively charged particle as it moves from plate X to plate Y?

A It decreases because the positively charged particle is moving away from the positively charged plate.

B It decreases because the positively charged particle is moving in the direction of the electric field between the plates.

C It increases because the positively charged particle is moving closer to a negatively charged plate.

D It remains constant because the positively charged particle is in the uniform electric field between the plates. N04/I/20

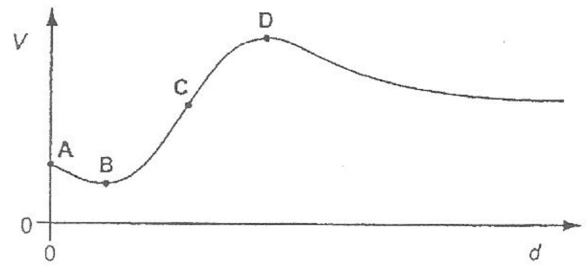
Solution Answer: **D**

The electric field strength between two parallel plates is uniform and given by the equation $E = V/d$ where V is the potential difference between the two places and d the distance separating them.

The electric force $F = qE = q(V/d)$ remains constant.

5 The diagram shows the variation of the electric potential V with distance d along a straight line in a particular electric field.

At which point is the magnitude of the electric field greatest?



N05/I/20

Solution Answer: **C**

The electric field strength E at a point in an electric field is the negative of the potential gradient at that point, $E = -dV/dx$.
The magnitude of the electric field strength is the greatest at the point where the gradient is the steepest, i.e. at point C.

6 An oil-drop of mass m, carrying a charge q, is in the region between two horizontal plates. When the potential difference between the upper and lower plates is V, the drop is stationary. The potential difference is then increased to 2 V.

What is the initial upward acceleration of the drop?

A g

B $2g$

C $\frac{2qV}{m} - g$

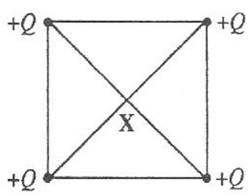
D $\frac{2qV}{m}$

N05/I/28

Solution Answer: **A**

Ignoring upthrust effects due the low density of the fluid medium (air), initially when the charge was stationary, the electric force is equal in magnitude and opposite in direction to the gravitational force ($qE = q(V/d) = mg$).
When the potential difference V is doubled, the electric force becomes $q(2V/d) = 2q(V/d) = 2mg$.
The resultant upward force $= 2mg - mg = mg$.
The initial upward acceleration $a = F/m = mg/m = g$.

7 Four identical point charges are arranged at the corners of a square as shown.



Which statement about the values of the electric field strength E and the electric potential V at point **X** in the middle of the square is true?

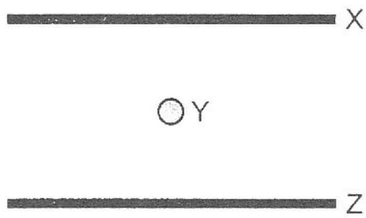
	E	V
A	not zero	zero
B	not zero	not zero
C	zero	not zero
D	zero	zero

N06/I/17

Solution Answer: **C**

Electric field strength is a vector quantity.
Electric field lines are directed away from positive charges, hence the field at X due to charge at the top-right corner cancels vectorially with that due to the charge at the bottom-left corner.
Similarly, the field due to the charge at the top-left corner cancels that due to the charge at the bottom-right corner.
The resultant electric field at X is thus zero.
Electric potential is a scalar quantity.
Since all four charges are positive, the potentials at X due to all charges are positive and add up numerically to produce a positive value for potential at X.

8 The diagram shows two charged horizontal metal plates X and Z. The magnitudes of the charges on the plates are equal. Between the plates, suspended stationary in the uniform electric field, is a charged sphere Y.



What could be the signs of the charges on the three components?

	plate X	sphere Y	plate Z
A	negative	positive	negative
B	negative	positive	positive
C	positive	positive	negative
D	positive	negative	positive

N07/I/24

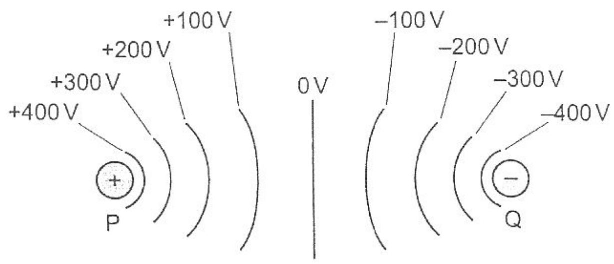
Solution Answer: B

To hold the sphere Y in mid air, the electric force must be upward (opposite to its weight).

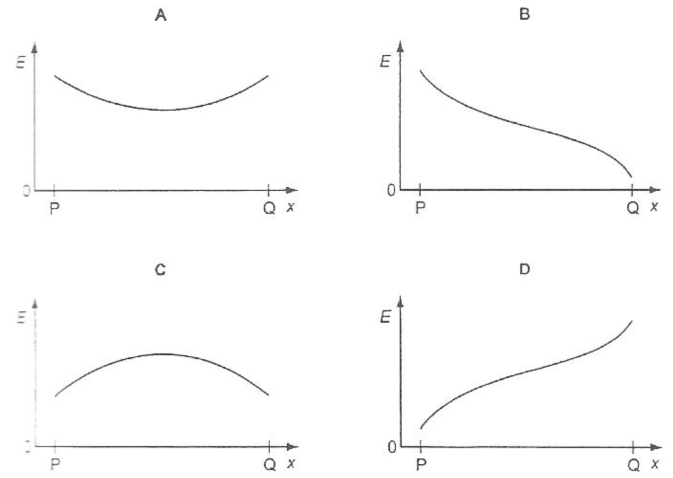
This can only be achieved if the top plate X and the sphere Y are of opposite polarity, while the bottom plate Z and the sphere Y are of the same polarity.

9 An object with a positive charge is placed at P and a similar object with a negative charge is placed at Q.

The diagram shows a number of lines along which the potential has a constant value.



Which graph shows the variation with distance x along line PQ of the electric field strength E?

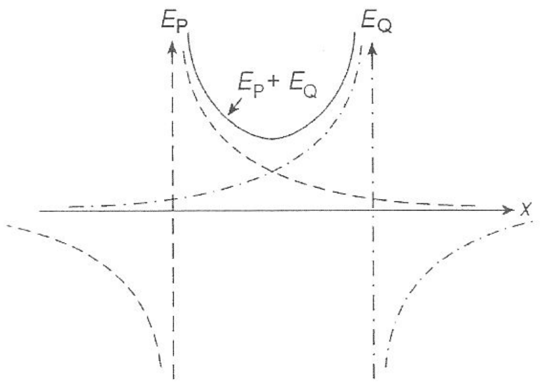


N07/I/25

Solution Answer: **A**

Refer to the following figure.

In the figure, the rightward direction is considered to be positive.



In the figure above, the dashed curve (and axis) _ _ _ _ represents E_P while the dot dashed curve (and axis) _._._._ represents E_Q.

The solid curve represents the resultant field.

10 Electric field strength is defined as force per unit positive charge on a small test charge.

Why is it necessary for the test charge to be small?

A so that the test charge does not distort the electric field
B so that the force on the test charge is small
C so that the test charge does not create any forces on nearby charges
D so that Coulomb's law for point charges is obeyed.

N08/I/24

Solution Answer: **A**

By 'small test charge', it means the magnitude of the charge is small (measured in coulombs), and not the physical dimension of the body holding the charge.

The test charge has to be small so that its presence in the electric field does not distort the field.

11 A point electric charge $-q$ is brought from a point P_1 in space to another point P_2 by an external agent. The electric potentials at points P_1 and P_2 are V_1 and V_2 respectively.

What is the work done on the charge?

A $q(V_2 + V_1)$ **B** $q(-V_2 - V_1)$ **C** $q(V_2 - V_1)$ **D** $q(V_1 - V_2)$

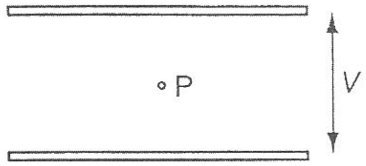
N08/I/25

Solution Answer: **D**

Work done by external agent is equal to the increase in the electrical potential energy of the charge.

$$W = \Delta U = U_2 - U_1 = -qV_2 - (-qV_1) = q(V_1 - V_2)$$

12 A small positively-charged particle P is balance halfway between two horizontal plates when a potential difference V is applied between the plates.



When V is increased, P rises towards the upper plate.
When V is decreased, P falls towards the lower plate.
Which statement is correct?

A Decreasing V decreases both the electric and the gravitational potential energy of the particle.

B Decreasing V increases the electric potential energy and decreases the gravitational potential energy of the particle.

C Increasing V increases both the electric and the gravitational potential energy of the particle.

D The change in electric potential energy of the particle must equal the change of gravitational potential energy of the particle.

N09/I/11

Solution Answer: **B**

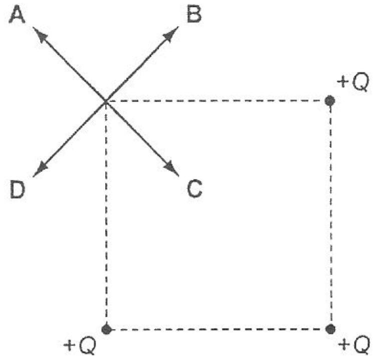
In order to hold the positively-charged particle P halfway between the two plates, the upper plate must be at a lower potential with respect to the lower plate.

By decreasing V and causing the positively-charged particle P to move towards the lower plate (which is at a higher potential), the electric potential energy of the particle increases. This is because a positive charge's electric potential energy increases with electric potential.

However, as the particle has fallen to a lower position, its gravitational potential energy decreases.

13 The diagram shows point charges, each of magnitude Q, placed at three corners of a square.

What is the direction of the resultant electric field at the fourth corner?

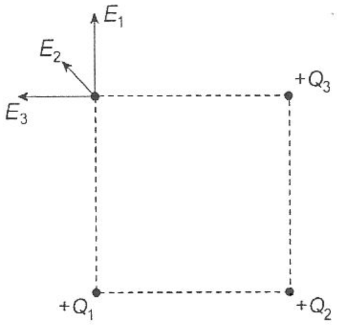


N09/I/25

Solution Answer: **A**

The electric field due to a positive charge is in a direction away from the charge.

Since all three charges are positive, their resultant fields are in the direction as shown below (with E_1 by charge Q_1, E_2 by Q_2 and E_3 to Q_3).

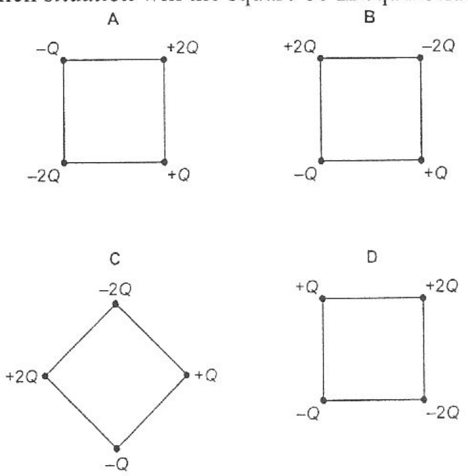


The three fields will add up vectorially towards the direction of A given in the question.

14 Four point charges of magnitude +2Q, +Q, –2Q and –Q are fixed at the corners of a rigid square, resting on a frictionless surface.

A uniform electric field is applied in the direction from left to right in the plane of the surface.

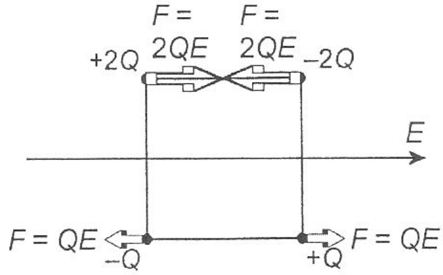
In which situation will the square be in equilibrium?



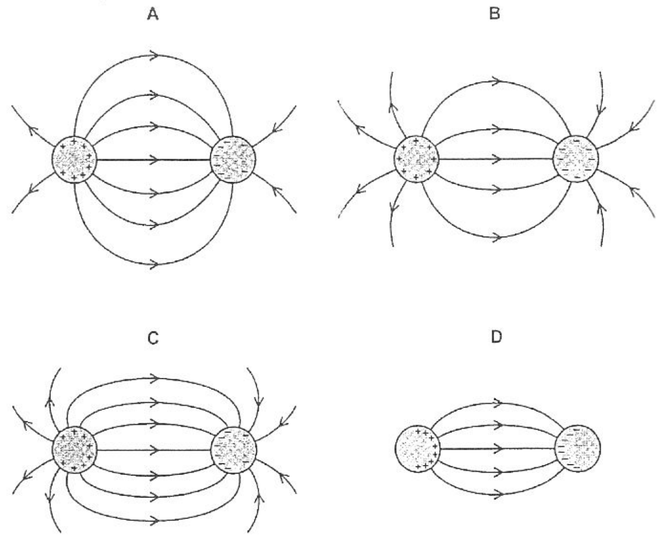
N10/I/24

Solution Answer: **B**

Positive charges experience an electric force F in the same direction as that of the electric field E where $F = qE$, while negative charges experience a force in the opposite direction to that of the field. In diagram B, the forces on the two charges of magnitude $2Q$ at the upper part of the square cancel out; while the forces on the two charges of magnitude Q at the lower part of the square cancel out, leaving no resultant force and no resultant moment.



15 Two small metal spheres have equal but opposite charges. Which diagram shows a possible distribution of charge on the sphere and the electric field near the spheres?



N10/I/25

Solution Answer: **A**

Since both are metal spheres, charges are able to move freely within each sphere. Since unlike charges attract, charges will move closer toward each other hence the charge concentration will be greater on the sides facing each other (eliminate answer B). However, it wouldn't be possible to have absolutely no charge (and zero field) on the far sides (eliminate answer D). The field strength around point charges decreases with distance hence the field lines cannot be evenly spaced (eliminate C). The field lines should be further apart at where the field strength is lower.

Short Questions

16 (a) Define *electric field strength*. [2]

(b) Fig. 5.1 shows a positively charged metal sphere.

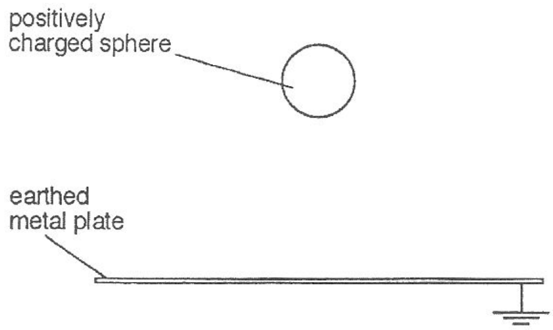


Fig. 5.1

As the sphere is moved close to the earthed plate, the plate becomes negatively charged.

(i) Suggest how the negative charge on the earthed plate arises. [2]

(ii) On Fig. 5.1, draw lines to represent the electric field between the sphere and the plate. [3]

(c) Two electrons are separated by a distance of 5.0×10^{-10} m (the approximate diameter of an atom). Determine the ratio

$$\frac{\text{electric force between the electrons}}{\text{weight of an electron}}$$ [4]

N04/II/5

Solution

(a) The *electric field strength* at a point in an electric field is defined as the electric force per unit positive charge on a small test charge placed at that point.

The direction of the electric field is the direction of the electric force on a positive test charge.

(b) (i) Unlike charges attract.

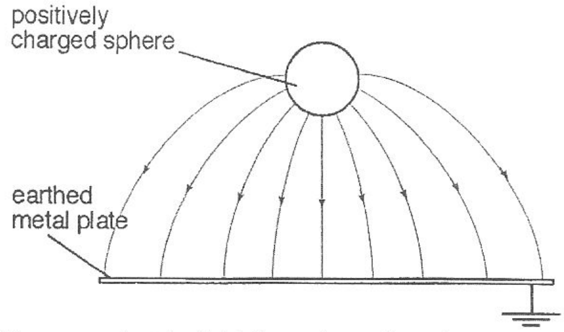
The positive charges on the sphere attract electrons to move up through the earth wire towards the upper surface of the metal plate.

The excess of the negatively charged electrons over the positive charged nuclei of the metal ions causes the upper surface of the metal plate to be negatively charged.

(ii) Electric field lines radiate away from positive charges and terminate at negative charges.

Furthermore, electric field lines must always be perpendicular to the equipotential lines, and both the charged sphere and the metal plate are equipotential surfaces.

Therefore, the electric field lines must be perpendicular to both surfaces, as shown in the figure below.



There are electric field lines above the sphere as well but the question only requires the field between the sphere and the plate.

(c) The electric force between two point charges q_1 and q_2 is give by the equation $F = q_1q_2/4\pi\varepsilon_0r^2$ where ε_0 is the permittivity of free space and r the distance of separation between the two charges.

For a pair of electrons,

$F = q_1q_2/4\pi\varepsilon_0r^2$
$= (1.6 \times 10^{-19}\ \text{C})^2 / 4\pi(8.85 \times 10^{-12}\ \text{F m}^{-1})(5.0 \times 10^{-10}\ \text{m})^2$
$= 9.208 \times 10^{-10}$ N

The weight of an electron
$W = mg = (9.11 \times 10^{-31}\ \text{kg})(9.81\ \text{N kg}^{-1}) = 8.937 \times 10^{-30}$ N

The ratio of F/W
$= 9.208 \times 10^{-10}\ \text{N} / 8.937 \times 10^{-30}\ \text{N}$
$= 1.030 \times 10^{20} = 1.0 \times 10^{20}$.

17 (a) Define *electric field strength* at a point.

....
.... [1]

(b) Electrons are emitted from a cathode C and are accelerated towards an anode A, as illustrated in Fig. 2.1.

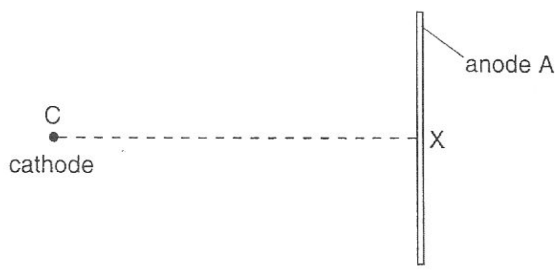


Fig. 2.1

The anode is earthed. CX is a line drawn from C normal to the anode A. The distance CX is 4.0 cm.

The variation with distance d from C along CX of the magnitude of the electric field strength E is shown in Fig. 2.2.

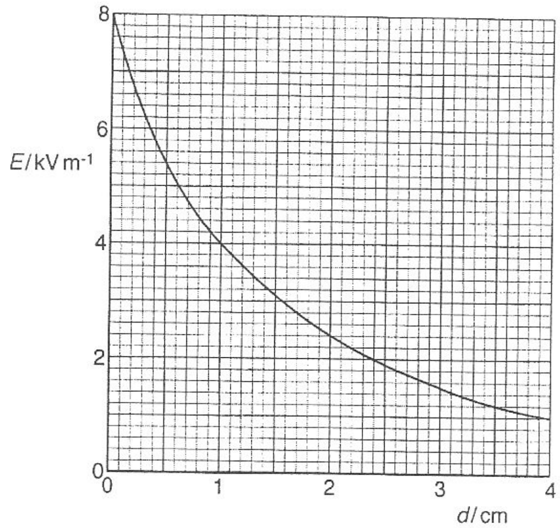


Fig. 2.2

(i) On Fig. 2.1, mark with an arrow the direction of the electric field along CX. [1]

(ii) Use Fig. 2.2 to determine the force F on an electron at a point mid-way between C and X.
F = ... N [2]

(c) (i) A student assumes that the force F on the electron remains constant as the electron moves from C to X.

Use the value of F calculated in (b)(ii) to estimate, on the basis of this assumption, the potential difference between C and X.
potential difference = ... V [2]

(ii) Suggest, with a reason, whether the magnitude of the potential difference calculated in (i) will be an over-estimate or an under-estimate of the actual potential difference.
....
.... [1]

N07/II/2

Solution

(a) The *electric field strength* at a point in an electric field is the electric force per unit positive charge on a small test charge placed at that point.

(b) (i) The arrow is indicated in the reproduced figure in Fig. 2.3 below.

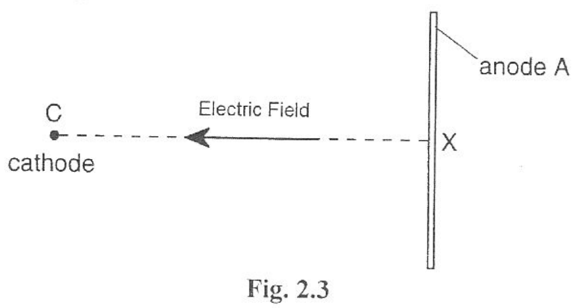


Fig. 2.3

Electrons, being negative charges, experience an electric force opposite in direction to that of the electric field.

(ii) The electric field strength at the mid-point between C and X is $E = 2.40$ kV m^{-1} or 2400 N C^{-1}.

The force $F = qE = eE$
$= 1.60 \times 10^{-19}$ C x 2400 N C^{-1}
$= 3.84 \times 10^{-16}$ N

(c) (i) Assuming a constant force F, the work done by th field to accelerated the electron from C to X is

$W = Fd$
$= (3.84 \times 10^{-16}$ N)(0.040 m)
$= 1.536 \times 10^{-17}$ J

The potential difference

$V = W/q$
$= (1.536 \times 10^{-17}$ J)/(1.60 x 10^{-19} C)
$= 96$ V.

Check: $V = Ed = (2400 \text{ V m}^{-1})(0.040 \text{ m}) = 96$ V.

(ii) Under-estimate.

The actual potential difference is the area under th field-distance graph ($V = -\int E\,dr$), which is greate than the area under the assumed constant-field graph.

The under-estimation to the left of the mid-point i greater than the over-estimation to the right of th mid-point. Refer to Fig. 2.4.

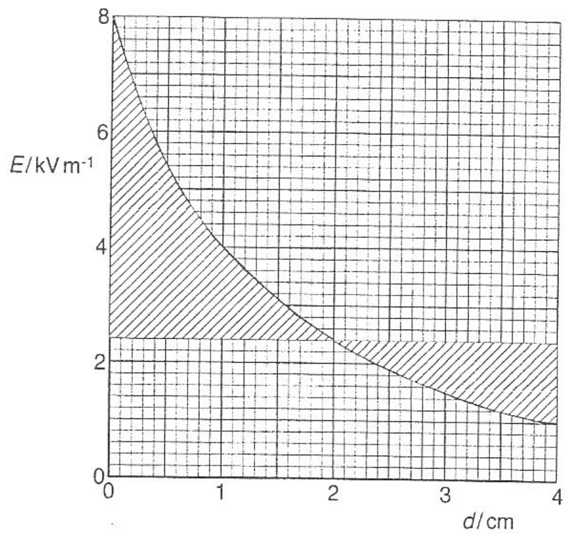
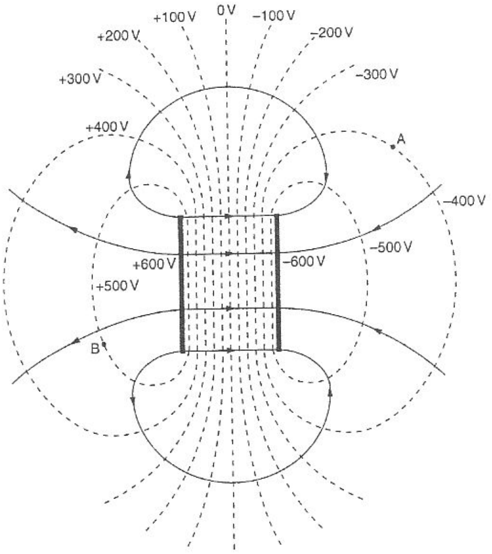


Fig. 2.4

18 Fig. 2.1 is drawn full scale and shows the pattern of th electric field (solid lines) in and around a pair of paralle charged metal plates. It also shows lines joining points a the same potential (dotted lines).

Note: the diagram in the original exam script was draw full scale but the diagram in this book has been reduce by 50%.



Scale 1 : 2

Fig. 2.1

(a) The plates have a separation of 2.5 cm.

From the diagram, deduce

(i) the value of the electric field strength between the plates,
electric field strength = ... [3]

(ii) the work that needs to be done to move a charge of 8.0×10^{-19} C from point A to point B.
work done = ... J [2]

(b) Explain why, in the absence of any other charged bodies, the potential will be zero along the centre line between the plates.
....
....
.... [2]

N07/III/2

Solution

(a) (i) Electric field between parallel plates,

$$E = V/d$$

$= (600 - (-600))\text{V} / (0.025 \text{ m}) = 4.8 \times 10^4 \text{ V m}^{-1}$

(ii) Work done W = change in potential energy U

$$W = U = qV$$

$= (8.0 \times 10^{-19} \text{ C})(500 - (-400))\text{V} = 7.2 \times 10^{-16}$ J

(b) The electric field strength between parallel plates is uniform.

Since electric field strength is the negative of the potential gradient ($E = -dV/dr$), the potential decreases uniformly from the left plate (+600 V) to the right plate (-600 V).

Therefore, the central equi-potential line, which is mid-way between the +600 V and -600 V plates, must be the zero-volt equipotential line.

19 A particle P_1 is moving with speed v directly towards a second particle P_2. Fig 5.1 represents P_1 when it is a distance 2.0×10^{-15} m from P_2. The charge on each particle is $+1.6 \times 10^{-19}$ C.

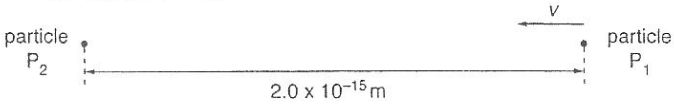


Fig. 5.1

(a) Calculate the electric force between the two particles at this separation.
electric force = ... N [3]

(b) Fig. 5.1 shows the variation with separation x of the electric force F as P_1 moves towards P_2.

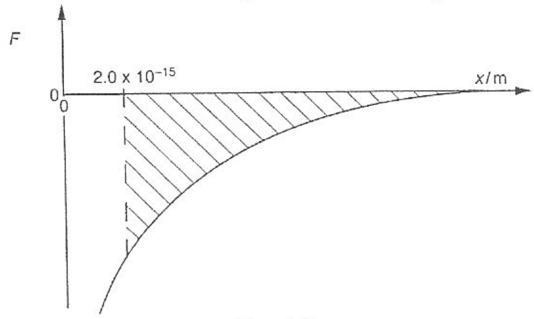


Fig. 5.2

Explain what the shaded area on the graph represents. [2]

(c) The two particles fuse when they are at a separation of 2.0×10^{-15} m. Explain the energy changes when fusion occurs. [2]

N09/II/5

Solution

(a) The electric force:

$$F = (1/4\pi\varepsilon_0)(Q_1Q_2) / (r^2)$$

$= (8.9918 \times 10^9)(1.6 \times 10^{-19})^2 / (2.0 \times 10^{-15})^2 = 57.55$ N

electric force = 58 N

(b) It represents the work done by an external agent in bringing the two particles from a distance of 2.0×10^{-15} m apart to an infinite distance apart (*negative because the two charges repel hence positive work is actually done by the electric forces*). It is also the negative of the electric potential energy stored in this system comprising the two particles when they are 2.0×10^{-15} m apart.

(c) When the particles fuse, the products have less mass than the reactants. The mass difference becomes the kinetic energy of the products and radiation energy of the gamma rays emitted.
An example of such a reaction would be proton-proton fusion to produce a deuterium (^2_1H), a positron (^0_1e) and a neutrino (ν_e).

$$^1_1H + ^1_1H \rightarrow ^2_1H + ^0_1e + \nu_e + \text{energy}$$

While the positron (^0_1e) then annihilates with an electron ($^0_{-1}e$) to further form 2 gamma ray photons (γ).

$$^0_1e + ^0_{-1}e \rightarrow 2\gamma + \text{energy}$$

Long Questions

20 *(a)* Define the term *electric field strength* and state the SI unit in which it is measured. [2]

(b) Two charged parallel plates are separated by a distance d and have a potential difference V between them. Using your definition in *(a)*, obtain an expression for the electric field strength of the uniform field between the plates [3]

(c) A stream of nuclear particles, travelling horizontally at 3.9 × 106 m s^{-1}, is deflected as it passes through a vacuum between two parallel plates as shown in Fig.4.1.

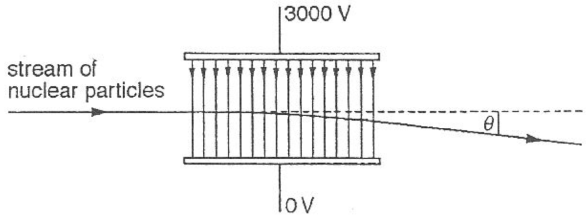


Fig. 4.1

The separation of the plates is 5.0 mm, their length is 40 mm and the p.d. between the plates is 3000 V. It is found that the angle θ of deflection is 4.3°.

Determine

(i) the field strength between the plates, [1]

(ii) the time for which a particle is between the plates. [2]

(d) The experiment in *(c)* is designed to determine the charge to mass ratio $\frac{q}{m}$ of the nuclear particle.

(i) Determine, in terms of $\frac{q}{m}$,

1. the acceleration of a particle vertically downwards,

2. the horizontal and vertical velocities of a particle as it leaves the electric field. [4]

(ii) Calculate $\frac{q}{m}$. [2]

(e) For the experiment in *(c)* suggest

(i) how θ may be measured, [2]

(ii) how the speed of 3.9 × 10^6 m s^{-1} could be determined, [2]

(iii) a particle that might have this value of $\frac{q}{m}$. [2]

N05/III/4

Solution

(a) The *electric field strength* at a point in an electric field is defined as the electric force per unit positive charge on a small test charge placed at that point.

The direction of the electric field is the direction of the electric force on a positive test charge.

The SI unit of electric field strength is newton per coulomb (N C^{-1}).

(b) Consider a small positive test charge $+q$ placed between the plates.

From the definition of electric field strength ($E = F/q$) the force acting on this small positive test charge is $F = qE$ in the direction of the field (since the direction of an electric field is the same as the direction of the force acting on a positive charge).

From the definition of work done by a force being the product of the force and the displacement in the direction of the force, the work done by this electric force to move the positive charge across the plates is $W = Fd$.

The potential difference V between two points is the work done per unit positive charge to move charge across the two points. Hence $V = W/q = Fd/q = (F/q)d = Ed$.

Hence, $E = V/d$.

(c) (i) Electric field strength
$E = V/d$
= (3000 V – 0 V) / (5.0 x 10^{-3} m)
= 6.0 x 10^5 V m^{-1} or N C^{-1}.

(ii) Horizontal displacement $s_x = u_x t + ½a_x t^2$ where u_x is the horizontal velocity (3.9 x 10^6 m s^{-1}) and a_x the horizontal acceleration which is zero.

$$40 \times 10^{-3}\text{ m} = (3.9 \times 10^6\text{ m s}^{-1})\,t$$

$$t = 1.026 \times 10^{-8} = 1.0 \times 10^{-8}\text{ s.}$$

(d) (i) 1. $F = qE$ and also $F = ma$.

$$ma = qE$$

$$a = (q/m)E = 6.0 \times 10^5\ q/m\text{ m s}^{-2}.$$

2. The horizontal velocity remains unchanged at 3.9 x 10^6 m s^{-1} since the horizontal acceleration a_x is zero.

The vertical velocity can be found from the equation $v_y = u_y + a_y t$ where u_y is the initial vertical velocity (which was zero since the particles were travelling horizontally) and a_y the vertical acceleration 6.0 x 10^5 q/m.

v_y = (6.0 x 10^5 q/m m s^{-2})(1.026 x 10^{-8} s)
= 6.153 x 10^{-3} q/m
= 6.2 x 10^{-3} q/m m s^{-1}.

(ii) The deflection is 4.3° hence,

$$v_y / v_x = \tan 4.3° = 0.07519$$

$$6.153 \times 10^{-3}\ (q/m) / 3.9 \times 10^6 = 0.07519$$

$$q/m = 4.766 \times 10^7 = 4.8 \times 10^7\text{ C kg}^{-1}$$

(e) (i) Place a zinc sulphide-coated fluorescent screen along the path of the particles. Where the particles hit the screen, a glow will be detected.

The angle θ may be found by measuring the vertical deflection (y) on the screen and also the distance from the centre of the parallel plates to the screen (x), as illustrated in Fig. 4.2.

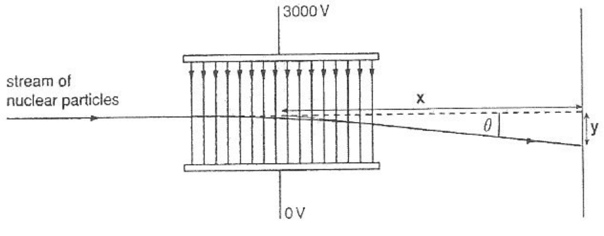


Fig. 4.2

From the figure, $\tan\theta = y/x$ and $\theta = \tan^{-1}(y/x)$.

It has to be noted that this method of measurement is approximate as the tangent to the curve does not exactly meet the central axis at the centre of the parallel plates, but slightly to the right of it.

Nevertheless, for small angles of less than 5°, as in this example, the error is negligible.

(ii) The speed could be determined by applying the concept of cross-fields velocity selector.

Firstly, since the particles deflect in the same direction as the electric field E, it can be deduced that they are positively charged.

A magnetic field B is then applied between the parallel plates in a direction pointing into the plane of the paper.
This will produce a magnetic force F_B acting upwards (Fleming's left hand rule), in the opposite direction to the electric force F_E.

Adjust the magnetic field until the path of the particles is straight (i.e. no deflection), which occurs when:

$$F_B = F_E$$

$$Bqv = qE$$

$$v = E/B = V/Bd$$

where v is the velocity, V is the potential difference between the plates and d the separation between the plates.

(iii) Since the particles deflect in the same direction as the electric field, they must be positively charged.

The value for q/m satisfies that for α-particles ($q = +2e = 2 \times 1.6 \times 10^{-19}$ C $= 3.2 \times 10^{-19}$ C and $m = 4u = 4 \times 1.66 \times 10^{-27}$ kg $= 6.64 \times 10^{-27}$ kg).

$$q/m = 4.8 \times 10^{7}\ \text{C kg}^{-1}.$$

21 (a) (i) Defined *electric field strength.*

....

....

.... [1]

(ii) Fig. 5.1 shows a charge $+q$ at X in a uniform electric field of electric field strength E.

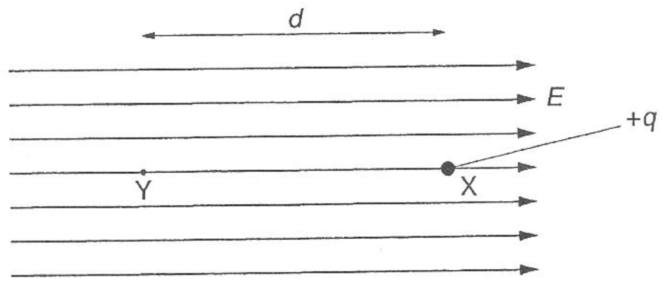


Fig. 5.1

The charge at X is moved to Y through a distance d. Using your definition in (i), deduce an expression for the work done.

....

....

.... [1]

(iii) The potential difference between X and Y is V. Using your answer from (ii), deduce an expression for V in terms of E and d.

....

....

.... [2]

(b) In the vacuum of an X-ray tube, electrons are accelerated from rest through a potential difference of 60 kV between the cathode and the anode. The current in the tube is 8.6 mA.
Calculate

(i) the number of electrons passing through the tube in one second,
number = ... [2]

(ii) the speed of electrons arriving at the anode,
speed = ... m s^{-1} [4]

(iii) the power supplied by the electrons hitting the anode.
power = ... W [2]

(c) X-ray production uses a negligible fraction of the power reaching the anode in (b), so it has to be cooled by passing a coolant through the anode. The specific heat capacity of the coolant is 3500 J kg^{-1} K^{-1} and the temperature rise of the coolant is 30 K.
Calculate the rate at which the coolant must be pumped through the anode.
rate = ... kg s^{-1} [3]

(d) A conducting sphere of radius 0.10 m carries a charge of +0.060 μC. The electric field around the sphere is shown in Fig. 5.2.

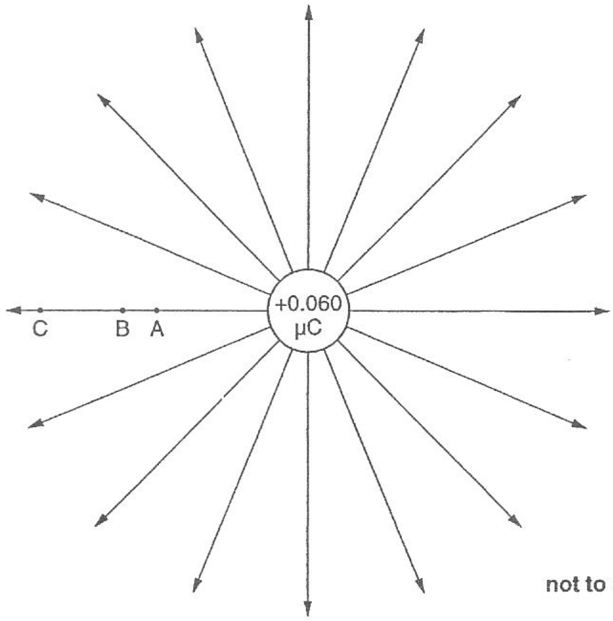


Fig. 5.2

(i) By reference to Fig. 5.2, suggest why it appears as if the charge is concentrated at the centre of the sphere.

....

....

.... [1]

(ii) Calculate the magnitude of the electric field strength at the surface of the sphere.

electric field strength = ... N C^{-1} [2]

(iii) On Fig. 5.2, A is 0.40 m from the centre of the sphere and B is 0.50 m from the centre of the sphere. The potential difference between A and B is equal to the potential difference between B and C.

Calculate the distance from the centre of the sphere to C.

distance = ... m [2]

N08/III/5

Solution

(a) (i) The *electric field strength* at a point in an electric field is the electric force per unit positive charge on a small test charge placed at that point.

(ii) Since electric field strength $E = F/q$, the force acting on the charge +q is

$$F = +qE$$

in the direction of the field since it is a positive charge.

Work done $W = Fd = +qEd$.

(iii) The potential difference V between two points is the work done per unit positive charge when a small test charge is moved between the two points.

$$V = W/q = +qEd / q = Ed$$

(b) (i) $n = (8.6 \times 10^{-3}\ \text{C s}^{-1}) / (1.60 \times 10^{-19}\ \text{C electron}^{-1})$

$$= 5.38 \times 10^{16}\ \text{electrons s}^{-1}$$

(ii) Gain in kinetic energy = loss in electrical potential energy

$$\tfrac{1}{2}mv^2 = eV$$

$$v = \sqrt{(2eV/m)}$$

$$= \sqrt{(2 \times 1.60 \times 10^{-19}\ \text{C} \times 60 \times 10^3\ \text{V} / 9.11 \times 10^{-31}\ \text{kg})}$$

$$= 1.45 \times 10^8\ \text{m s}^{-1}$$

(iii) The energy of each electron = 60 keV.

The electrons arrive at a rate of $5.38 \times 10^{16}\ \text{s}^{-1}$.

Power supplied

$$P = 60\ \text{keV} \times 5.38 \times 10^{16}\ \text{s}^{-1}$$

$$= 3.23 \times 10^{21}\ \text{eV s}^{-1}$$

$$= 3.23 \times 10^{21}\ \text{eV s}^{-1} \times 1.60 \times 10^{-19}\ \text{J eV}^{-1}$$

$$= 517\ \text{W}$$

(c) Since X-ray production uses negligible fraction of the power, the 517 W has to be dissipated by the coolant.

$$dQ/dt = (dm/dt)c\Delta\theta$$

$$517\ \text{W} = (dm/dt)(3500\ \text{J kg}^{-1}\ \text{K}^{-1})(30\ \text{K})$$

$$dm/dt = 4.92 \times 10^{-3}\ \text{kg s}^{-1}$$

(d) (i) Extrapolating the field lines backwards, they meet at, and appear to originate from, the centre of the sphere. At distances $r \geq R$ (radius), the sphere's electric field is indistinguishable from that of a point charge at its centre.

(ii) Electric field strength

$$E = q/4\pi\varepsilon_0 r^2$$

$$= 0.060 \times 10^{-6}\ \text{C} / 4\pi(8.85 \times 10^{-12}\ \text{F m}^{-1})(0.10\ \text{m})^2$$

$$= 5.4 \times 10^4\ \text{N C}^{-1}$$

(iii) $V_{BC} = V_{AB}$

$$V_B - V_C = V_A - V_B$$

$$q/4\pi\varepsilon_0\ [1/r_B - 1/r_C] = q/4\pi\varepsilon_0\ [1/r_A - 1/r_B]$$

$$1/r_B - 1/r_C = 1/r_A - 1/r_B$$

$$1/(0.50\ \text{m}) - 1/r_C = 1/(0.40\ \text{m}) - 1/(0.50\ \text{m})$$

$$r_C = 0.67\ \text{m}$$

TOPIC 13 Current of Electricity

1 The resistance of a thermistor depends on its temperature, and the resistance of a light-dependent resistor (LDR) depends on the illumination.

Under what conditions will the resistance of both a thermistor and a light-dependent resistor be highest?

	thermistor	light-dependent resistor
A	highest temperature	highest illumination
B	highest temperature	lowest illumination
C	lowest temperature	highest illumination
D	lowest temperature	lowest illumination

N02/I/21

Solution Answer: **D**

Both the thermistor and light-dependent resistor (LDR) are semiconductor-based devices.
Both rely on energy input (either in the form of heat as in the thermistor or light as in the LDR) to free electrons from their bonds and hence increase the charge carrier density.
This then results in a drop in resistance.
In the absence of energy sources, the resistance of both the thermistor and LDR will be high.

2 The electron beam current in a cathode-ray oscilloscope is 50 μA. The time-base causes the beam to sweep horizontally across the screen at a speed of 1.0×10^4 cm s^{-1}.

What is the number of electrons arriving at the screen in one centimetre length of the horizontal trace?

A 3.1×10^{10}

B 3.1×10^{13}

C 1.3×10^{13}

D 8.0×10^{14}

N02/I/22

Solution Answer: **A**

The time taken for the electron beam to sweep a distance of one cm is:
(1 cm / 1.0×10^4 cm s^{-1}) = 1.0×10^{-4} s.
The amount of charge arriving at the screen within this time is:
$Q = It = (50 \times 10^{-6}\text{ A})(1.0 \times 10^{-4}\text{ s}) = 5.0 \times 10^{-9}$ C.
The number of electrons arriving is:
$Q/e = (5.0 \times 10^{-9}\text{ C})/(1.60 \times 10^{-19}\text{ C}) = 3.1 \times 10^{10}$ electrons.

3 Two lamps X and Y are rated as follows.

X	230 V, 100 W
Y	230 V, 60 W

When operating normally, which has the larger current and which has the larger resistance?

	larger current	larger resistance
A	X	X
B	X	Y
C	Y	X
D	Y	Y

N03/I/19

Solution Answer: **B**

For lamp X, the normal operating current
$I = P/V = (100\text{ W})/(230\text{ V}) = 0.43$ A
and its resistance
$R = V^2/P = (230\text{ V})^2/(100\text{ W}) = 530\ \Omega$.

For lamp Y, the normal operating current
$I = P/V = (60\text{ W})/(230\text{ V}) = 0.26$ A
and its resistance
$R = V^2/P = (230\text{ V})^2/(60\text{ W}) = 880\ \Omega$.

4 Two wires **P** and **Q**, each of the same length and the same material, are connected in parallel to a battery. The diameter of **P** is half that of **Q**.

What fraction of the total current passes through **P**?

A 0.20 B 0.25 C 0.33 D 0.50

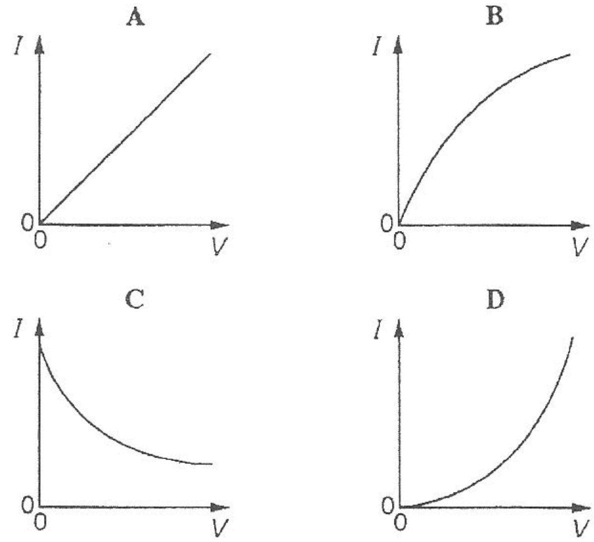
N04/I/21

Solution Answer: **A**

The resistance of a wire $R = \rho l/A$, where ρ is the resistivity of the liquid, l its length and A the cross-sectional area perpendicular to the current flow.
Since P's diameter is half that of Q's, its area A is one-quarter that of Q's hence its resistance is four times that of Q's.
The current $I = E/R$ is inversely proportional to R hence P's current is one-quarter that of Q's.
Therefore, the fraction of the total current passing through P is $1/(1+4) = 1/5 = 0.20$.

5 The potential difference across a tungsten filament lamp is slowly raised from zero to its normal operating value.

Which graph represents the variation of current I in the bulb with potential difference?

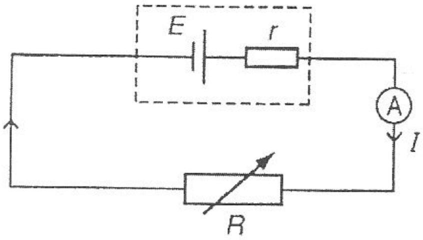


N04/I/22

Solution Answer: **B**

Resistance R is the ratio of potential difference V to current I.
The resistance of a metallic conductor such as tungsten increases with temperature.
As the potential difference V increases and the filament heats up, its resistance R increases and so does the ratio of its V to I.

6 A battery of e.m.f. E and internal resistance r delivers a current I through a variable resistance R.



R is set at two different values and the corresponding currents I are measured using an ammeter of negligible resistance.

R/Ω	I/A
1.0	3.0
2.0	2.0

What is the value of the e.m.f. E?

A 3.0 V

B 3.5 V

C 4.0 V

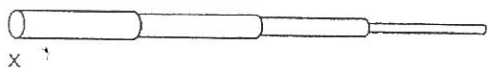
D 6.0 V

N04/I/23

Solution Answer: **D**

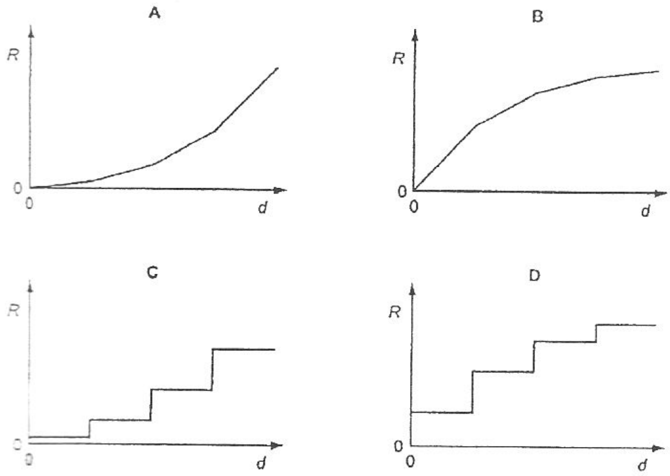
The terminal potential difference $V = IR = E - Ir$ where r is the internal resistance of the battery.
In the first setting,
$(3.0\ \text{A})(1.0\ \Omega) = E - (3.0\ \text{A})r$ thus $3.0 = E - 3.0\ r$.
In the second setting,
$(2.0\ \text{A})(2.0\ \Omega) = E - (2.0\ \text{A})r$ thus $4.0 = E - 2.0\ r$.
Eliminating r, $0.5\ E = 3.0$ V hence $E = 6.0$ V.

7 A composite wire is made by connecting in series four uniform wires made of the same material but having different diameters.



The resistance R of this composite wire is measured between X and other points on the wire at distances d from X.

Which graph best represents the relationship between R and d?



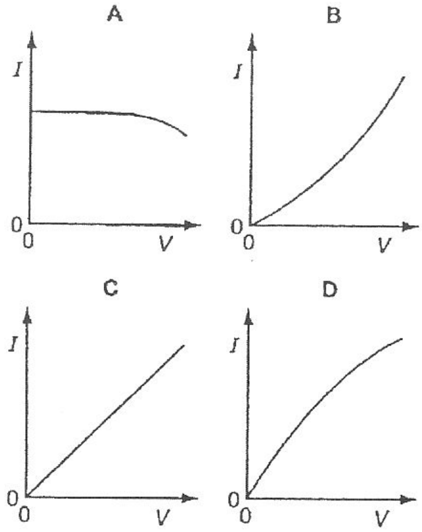
N05/I/22

Solution Answer: A

The resistance R of a conductor is related to its length l by the equation $R = \rho l/A$, where in this question l is replaced with d hence the equation becomes $R = \rho d/A$ or $(\rho/A)d$. A graph of R against d is a straight line with a gradient of ρ/A.
Since the resistivity ρ of the material is the same throughout, its resistance is inversely proportional to the cross sectional area A of the wire. The smaller the cross-sectional wire, the steeper the gradient of the graph $R = (\rho/A)d$.
With each subsequent section, the wire's cross-sectional area decreases hence the graph of R against d becomes steeper.

8 Some early electric lamps used carbon filaments. The resistance of these filaments decreases as their temperature increases.

Which graph shows how the current I in the filament varies with the potential difference V across it?

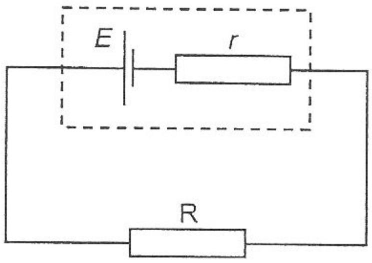


N06/I/21

Solution Answer: **B**

The resistance of a conductor is the ratio of the potential difference V cross it to the current I flowing through it.
As the voltage increases, the power generated in the carbon filament increases hence its temperature increases. Since this results in a decrease in its resistance, the ratio of V to I should decrease.
This implies that the increase in current should outpace the increase in potential difference, as shown in answer B.

9 A cell of e.m.f. E is connected in series with a resistor R.



The potential difference across R is V_R. The potential difference across the internal resistance r of the cell is V_r.

What is the energy transferred by the cell in driving unit charge round the complete circuit?

A $V_R - V_r$ **B** V_R **C** $E - V_r$ **D** E

N07/I/26

Solution Answer: **D**

Energy transferred by the cell in driving unit charge round the complete circuit is the electromotive force (e.m.f.) of the cell. This is the definition of e.m.f.

10 The resistivity of aluminium is 2.0 times that of silver. An aluminium wire of length L and diameter d has resistance R.

What is the diameter of a silver wire, also of length L and resistance R?

A 0.50 d **B** 0.71 d **C** 1.4 d **D** 2.0 d

N07/I/27

Solution Answer: **B**

For a wire of length l and cross-sectional area A, the formula relating resistance R to resistivity ρ is

$$R = \rho l/A = \rho l/(\pi d^2/4) = 4\rho l/(\pi d^2)$$

where d is the diameter of the wire.

Rearranging, $d = 2[\rho l/(\pi R)]^{1/2}$

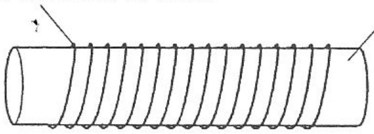
$$d_{\text{Silver}} / d_{\text{Alum}} = [\rho_{\text{Silver}} l_{\text{Silver}} R_{\text{Alum}} / \rho_{\text{Alum}} l_{\text{Alum}} R_{\text{Silver}}]^{1/2}$$

Since both wires have the same length and resistance,

$$d_{\text{Silver}} / d_{\text{Alum}} = [\rho_{\text{Silver}} / \rho_{\text{Alum}}]^{1/2} = [1/2]^{1/2} = 0.71$$

$$d_{\text{Silver}} = 0.71\ d_{\text{Alum}}$$

11 The material of a wire has resistivity 1.3 x 10^{-8} Ω m. The wire has diameter 0.50 mm and its length is just enough to enable it to be wound tightly round an insulating rod 30 times. The rod has diameter 1.5 cm.



What is the resistance of the wire?

A 1.1 x 10^1 Ω **B** 9.4 x 10^{-2} Ω **C** 7.0 x 10^{-4} Ω **D** 4.7 x 10^{-5} Ω

N08/I/26

Solution Answer: **B**

Resistance $R = \rho l/A$.

Resistivity ρ = 1.3 x 10^{-8} Ω m.

Length $l = 30\ (2\pi R_{\text{rod}}) = 30\ (2\pi \times 0.0075\text{ m}) = 1.414$ m.

Area $A = \pi R_{\text{wire}}^2 = \pi(0.25 \times 10^{-3}\text{ m})^2 = 1.963 \times 10^{-7}\text{ m}^2$.

$$R = \rho l/A$$

$$= (1.3 \times 10^{-8}\ \Omega\text{ m})(1.414\text{ m})/(1.963 \times 10^{-7}\text{ m}^2) = 0.094\ \Omega$$

12 A battery of e.m.f. 6.0 V and internal resistance 0.40 Ω is connected to an external resistor of resistance 2.9 Ω.

What is the power supplied to the external resistor?

A 1.3 W **B** 5.3 W **C** 9.6 W **D** 12.4 W

N08/I/27

Solution Answer: **C**

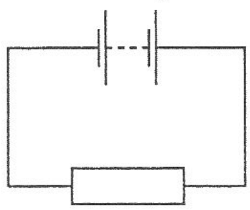
Using the potential divider principle, the potential difference across the external resistance is:

$$V = [R / (R+r)] \times E = [2.9\ \Omega / (2.9 + 0.40)\Omega] \times 6.0\text{ V} = 5.273\text{ V}$$

The power generated in the external resistor:

$$P = V^2/R = (5.273\text{ V})^2/2.9\ \Omega = 9.6\text{ W}$$

13 The diagram shows a battery connected in series with a resistor R.



The current in the circuit is I. Power P is dissipated in the resistor R. Power p is dissipated in the battery itself.

What is the electromotive force (e.m.f.) of the battery?

A $\dfrac{P}{I}$ **B** $\dfrac{P+p}{I}$ **C** $\dfrac{P}{I^2}$ **D** $\dfrac{P+p}{I^2}$

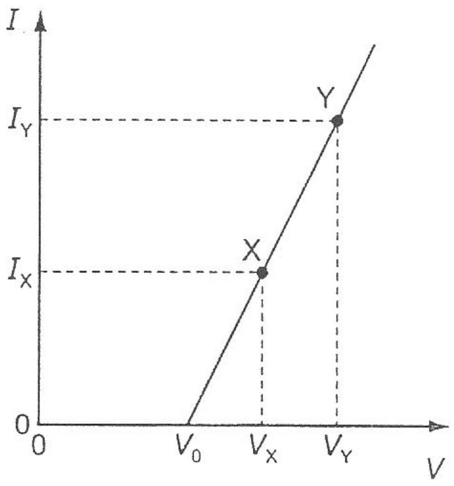
N09/I/26

Solution Answer: **B**

The electromotive force E is the total energy delivered per unit positive charge by the cell around the circuit, and must be equal to the total energy dissipated per unit positive charge flowing around the circuit, both in the load itself (W) and in the internal resistance of the battery (w).

Hence $E = W/Q + w/Q = P/I + p/I = (P + p) / I$.

14 The graph shows the variation with potential difference V of the current I in an electrical component.



The resistance is measured for current I_Y and for current I_X.

What is the change in the resistance of the component?

A zero **B** $\dfrac{V_Y - V_X}{I_Y - I_X}$ **C** $\dfrac{V_X}{I_X} - \dfrac{V_Y}{I_Y}$ **D** $\dfrac{V_Y - V_0}{I_Y} - \dfrac{V_X - V_0}{I_X}$

N09/I/27

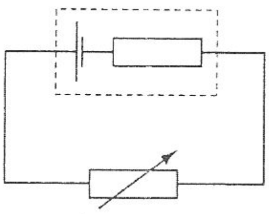
Solution Answer: **C**

The resistance of a device, at any point on the graph, is the ratio of the potential difference across the device to the current in it.

Hence $R_X = V_X/I_X$ and $R_Y = V_Y/I_Y$

The change in resistance $R_Y - R_X = V_Y/I_Y - V_X/I_X$.

15 A variable resistor is connected across the terminals of a cell as shown.



The cell has constant internal resistance.
The resistance of the variable resistor is gradually reduced.
What happens to the terminal potential difference and the power wasted in the internal resistance of the cell?

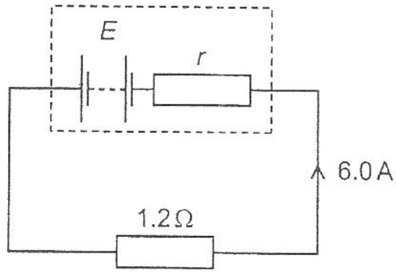
	terminal potential difference	power wasted in the cell
A	decreases	decreases
B	decreases	increases
C	increases	decreases
D	increases	increases

N10/1/26

Solution Answer: **B**

The terminal potential difference $V = E - Ir$ where E is the cell's electromotive force (which is constant), I the current drawn from the cell and r its internal resistance. As the variable resistor's resistance is reduced, the current drawn from the cell increases hence the terminal potential difference V falls, while the power wasted in the internal resistance I^2r increases.

16 A battery of internal resistance r and e.m.f. E can supply a current of 6.0 A to a resistor of resistance 1.2 Ω. The circuit is shown in the diagram.



When the resistor is changed to one having a value of 1.6 Ω, the current becomes 5.0 A.
What are the values of the e.m.f. E and internal resistance r?

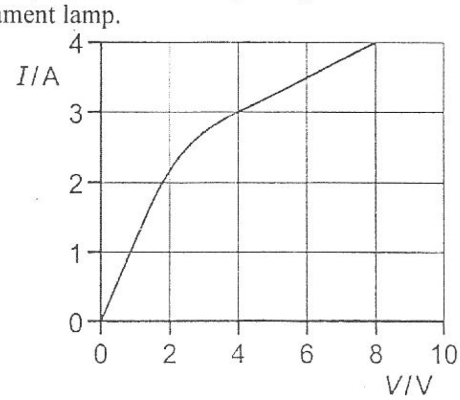
	E/V	r/Ω
A	7.6	0.073
B	12	2.0
C	12	0.80
D	15	8.0

N10/I/27

Solution Answer: **C**

Initially, $6.0 = E/(1.2 + r)$;
Later, $5.0 = E/(1.6 + r)$.
Solving simultaneously, $E = 12$ V and $r = 0.80$ Ω.

17 The graph plots current I against potential difference V for a filament lamp.



What is the resistance when the potential difference across the lamp is 6 V?

A 0.25 Ω **B** 0.58 Ω **C** 1.7 Ω **D** 4.0 Ω

N10/I/28

Solution Answer: **C**

Resistance R is defined as V/I = 6 V / 3.5 A = 1.7 Ω.

Short Questions

18 (a) An electrical component C has an *I-V* characteristic as shown in Fig. 3.1.

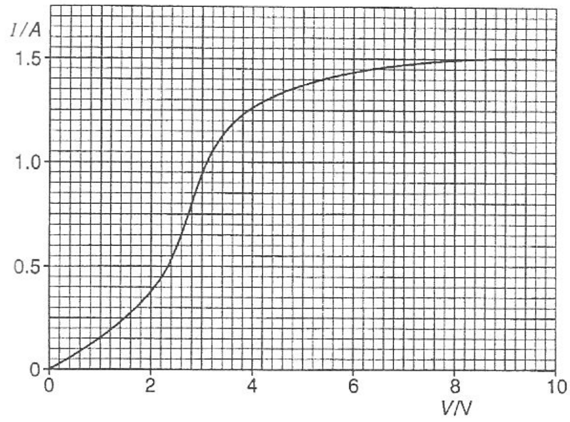


Fig. 3.1

(i) Calculate the resistance of component C when a p.d. of 6.0 V is applied across it.
resistance = ... Ω [1]

(ii) Deduce the minimum value of the resistance of component C over the range 0 – 10 V.
resistance = ... Ω [2]

(b) Component C is then connected into a circuit with a supply of internal resistance 0.80 Ω and a resistor of constant resistance 5.0 Ω, as shown in Fig. 3.2.

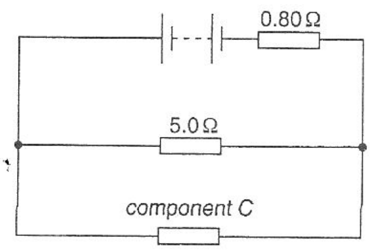


Fig. 3.2

The current through the 5.0 Ω resistor is found to be 0.85 A.

Calculate

(i) the p.d. across component C,
p.d. = ... V [2]

(ii) the total current from the supply,
current = ... A [2]

(iii) the e.m.f. of the supply,
e.m.f. = ... V [2]

(iv) the energy supplied to component C in 20 minutes.
energy = ... J [3]

N07/III/3

Solution

(a) (i) Resistance R is the ratio of potential difference V to current I.

$R = V/I = 6.0$ V / 1.43 A = 4.2 Ω

This is illustrated in Fig. 3.3.

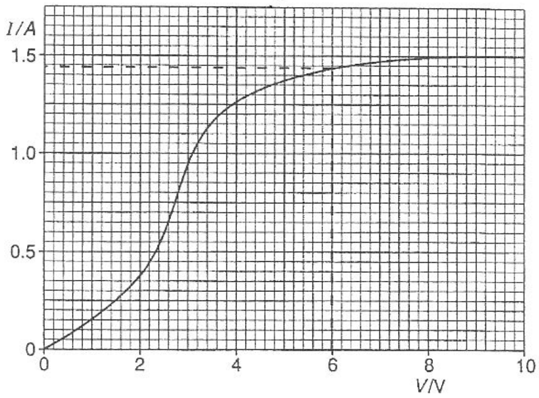


Fig. 3.3

(ii) The minimum value occurs at a point where the ratio of V to I is lowest.
This occurs when $V = 3.3$ V and $I = 1.1$ A.
The resistance R = 3.3 V / 1.1 A = 3.0 Ω. Refer to Fig. 3.4.

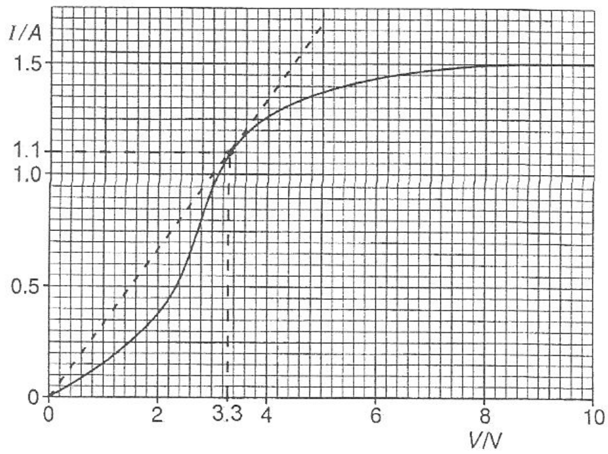


Fig. 3.4

(b) (i) The p.d. across the component C is the same as that across the 5.0 Ω resistor, since both of them are connected in parallel.

$V = IR = (0.85\text{ A})(5.0\ \Omega) = 4.25 = 4.3$ V

(ii) The total current from the supply is the sum of the current through the 5.0 Ω resistor (which is 0.85 A) and the current through C (from the graph, when the p.d. across C is 4.3 V, the current through it is 1.3 A).
Total current = 0.85 A + 1.3 A = 2.15 = 2.2 A.

(iii) Terminal p.d. $V = E - Ir$ hence
$E = V + Ir = 4.25\text{ V} + (2.15\text{ A})(0.80\ \Omega) = 5.97 = 6.0$ V.

(iv) Energy

$$W = Pt = IVt$$
$$= (1.3\text{ A})(4.25\text{ V})(20 \times 60\text{ s})$$
$$= 6.63 \times 10^3 = 6.6 \times 10^3\text{ J.}$$

19 A variable resistor R is connected between the terminals of a battery of e.m.f. E and internal resistance r, as shown in Fig. 4.1.

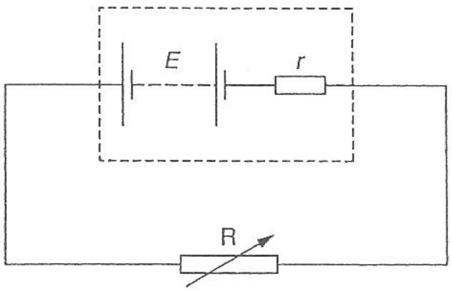


Fig. 4.1

The resistance of resistor R is varied.
The potential difference across R is V and the power dissipated in R is P.
The variation with V of P is shown in Fig. 4.2.

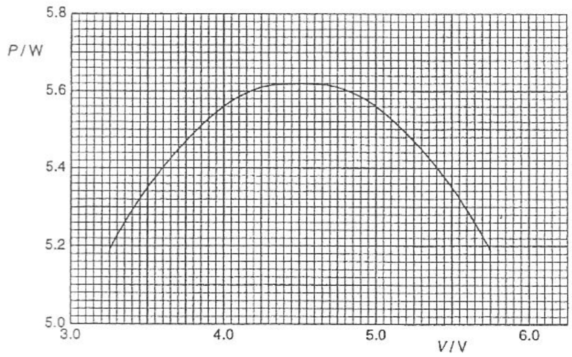


Fig. 4.2

(a) For the maximum value of P, use Fig. 4.2 to

(i) calculate the current in the circuit,
current = ... A [2]

(ii) show that the resistance of R is 3.6 Ω. [1]

(b) When R has resistance 2.03 Ω, the current in the circuit is 1.60 A.
Use these data and your answers to (a) to determine the internal resistance r of the battery.
r = ... Ω [3]

N08/II/2

Solution

(a) (i) The maximum power $P = 5.62$ W when $V = 4.5$ V.
Current $I = P/V = 5.62 \text{ W} / 4.5 \text{ V} = 1.25$ A

(ii) Resistance $R = V/I = 4.5 \text{ V} / 1.25 \text{ A} = 3.6\ \Omega$.

(b) The terminal potential difference V across the battery is related to its e.m.f. E and internal resistance r by the equation:

$$V = E - Ir$$

$$4.5 \text{ V} = E - (1.25 \text{ A})r \ldots (1)$$

When $R = 2.03\ \Omega$ and $I = 1.60$ A, the potential difference across the resistance (which is also the terminal potential difference) $V = IR = (1.60 \text{ A})(2.03\ \Omega) = 3.25$ V.

Using the same equation as above,

$$V = E - Ir$$

$$3.25 \text{ V} = E - (1.60 \text{ A})r \ldots (2)$$

(1) – (2)

$$1.25 \text{ V} = (0.35 \text{ A})\, r$$

$$r = 3.6\ \Omega$$

Remark: This result agrees with the Maximum Power Theorem, which states that the power generated in a load resistor has the maximum value when it is equal to the internal resistance of the cell (refer to Fig. 4.2 and (a)(ii)).

20 A car headlamp, rated at 12 V and 24 W, is used with a constant 12 V supply for 1800 s.

(a) Calculate, for the lamp used in this way,

(i) the resistance of the lamp,
resistance = ... Ω [2]

(ii) the energy transferred in the lamp,
energy = ... J [1]

(iii) the number of electrons passing through the lamp.
number = ... [2]

(b) The I-V characteristics of the lamp are investigated using the circuit shown in Fig. 2.1.

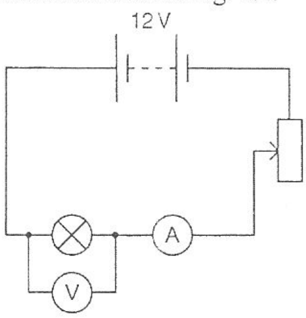


Fig. 2.1

The variable resistor can be adjusted to have resistance values between 0 and 10 Ω. Readings of potential difference (p.d.) V across the lamp and the current I in the circuit are taken. The results obtained are shown in Fig. 2.2.

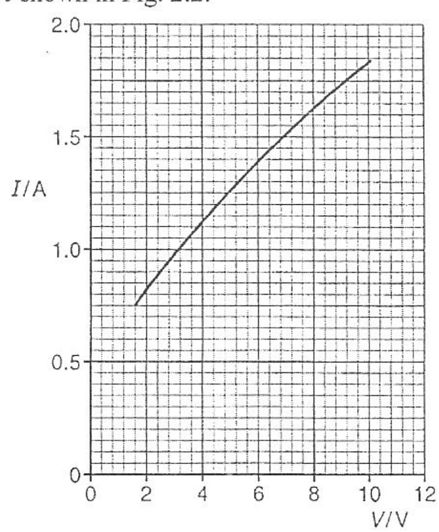


Fig. 2.2

(i) Explain how the resistance of the lamp can be obtained from Fig. 2.2. [1]

(ii) On Fig. 2.3 sketch the variation in resistance of the lamp when the p.d. across it is varied over the range of 2 V to 10 V. (Numerical values for the resistance are not expected.)

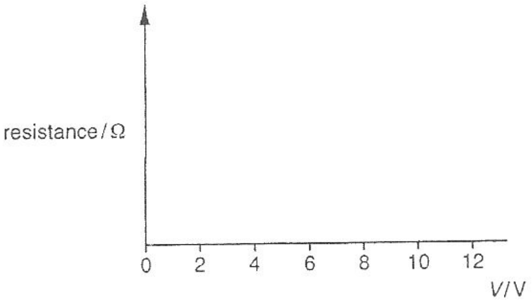


Fig. 2.3

[2]

(iii) Explain why, in the circuit of Fig. 2.1, the p.d. across the lamp cannot be varied from 0 to 12 V. [2]

N10/II/2

Solution

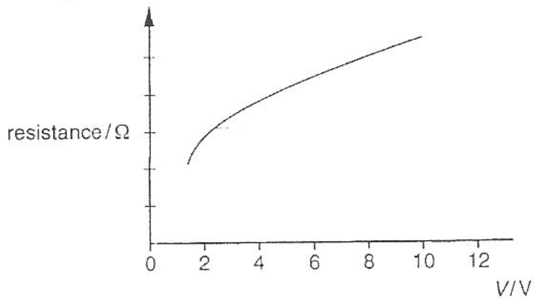
(a) (i) Power $P = V^2/R$ hence resistance $R = V^2/P = (12)^2/24 = 6.0\ \Omega$.

(ii) Energy transferred in the lamp $E = Pt = (24)(1800) = 4.32 \times 10^4$ J $= 4.3 \times 10^4$ J (2 s.f.).

(iii) Potential difference $V = E/Q$ hence the amount of charge passing through the lamp $Q = E/V = (4.32 \times 10^4\ \text{J})/(12) = 3600$ C.
The charge carried by one electron $= 1.60 \times 10^{-19}$ C.
Hence the number of electrons passing through $= (3600)/(1.60 \times 10^{-19}) = 2.25 \times 10^{22} = 2.3 \times 10^{22}$ (2 s.f.)

(b) (i) The resistance (R) at any particular set of values of potential difference (V) and current (I) is the ratio of the potential difference to current, $R = V/I$.

(ii) The variation of resistance with p.d. is shown in the diagram below.



(iii) The p.d. across the lamp cannot be 0 V even with the variable resistor set to its maximum value (10 Ω), as the circuit resistance in not infinite and there will still be current through the lamp.

The p.d. across the lamp cannot be 12 V even with the variable resistor set to its minimum value (0 Ω) because there could be p.d. across the internal resistance of the cell and the ammeter, which may not be ideal.

Long Questions

21 Suggest why, in the SI system, it was **not chosen** to define potential difference as the product of electric current and resistance. [2]

N04/III/1 (part)

Solution

In the SI system, the resistance R of a material is defined as the ratio of the potential difference V across it to the current I flowing through it.

Since R is defined as a function of V, V cannot be defined as a function of R, as this will lead to V being used to define itself, an invalid definition under the SI system.

22 Two resistors of resistance R_1 and R_2 are connected in parallel. The equivalent single resistance is R.

(a) Show, using Kirchhoff's laws, that R is given by

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

In your deduction of this equation, state clearly when each of Kirchhoff's laws is used. [5]

N06/III/6 part

Solution

(a) Refer to Fig. 6.2.

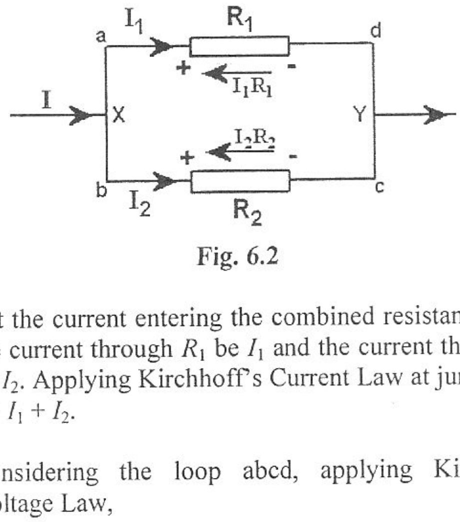


Fig. 6.2

Let the current entering the combined resistances be I, the current through R_1 be I_1 and the current through R_2 be I_2. Applying Kirchhoff's Current Law at junction X, $I = I_1 + I_2$.

Considering the loop abcd, applying Kirchhoff's Voltage Law,

$$\Sigma \text{ e.m.f.} = \Sigma \text{ p.d.}$$
$$0 = I_2R_2 - I_1R_1$$
$$I_2R_2 = I_1R_1$$
$$R_1 = I_2R_2/I_1 \ldots (1)$$

The effective resistance of the combined resistance,

$$R = V/I \ldots (2)$$

Where V is the potential difference across both resistors.

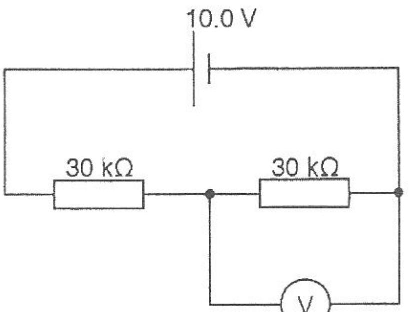
Also, $V = I_2R_2 = I_1R_1$

From equations (1) and (2),

$$1/R = I/V$$
$$= (I_1 + I_2)/I_2R_2$$
$$= I_1/I_2R_2 + 1/R_2$$
$$= 1/R_1 + 1/R_2$$
$$= (R_1 R_2)/(R_1 + R_2)$$

TOPIC 14 D.C. Circuits

1 In the circuit, a 10.0 V supply of negligible internal resistance is joined to two 30 kΩ resistors in series. A voltmeter of resistance 60 kΩ is connected in parallel with one of the resistors.



What is the reading on the voltmeter?

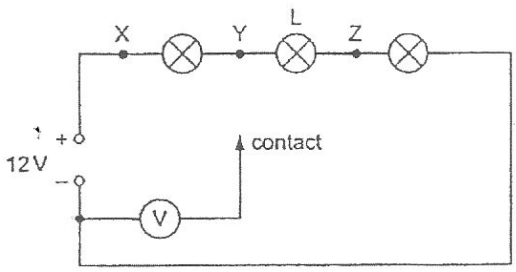
A 4.0 V
B 5.0 V
C 6.0 V
D 6.7 V

N02/I/23

Solution Answer: A

The effective resistance across the voltmeter (which is in parallel with the 30 kΩ load that it is measuring) is $(1 / 60 + 1 / 30)^{-1}$ kΩ = 20 kΩ.
Using the potential divider principle, the potential difference across this parallel combination is
(20 kΩ / (20 kΩ + 30 kΩ)) x 10.0 V = 4.0 V.

2 The diagram shows three lamps in series with a 12 V supply.



To test the circuit, the contact is connected in turn to points X, Y and Z. The lamps **do not light** because lamp L has a broken filament.

Which line of the table below shows the readings of the voltmeter?

	reading at X	reading at Y	reading at Z
A	12 V	8 V	4 V
B	8 V	8 V	0 V
C	12 V	12 V	0 V
D	8 V	12 V	4 V

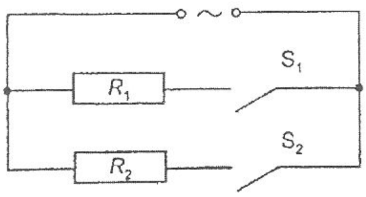
N05/I/23

Solution Answer: **C**

Since there is no current flowing through the lamp between X and Y, the potential difference across it V_{XY} is zero ($V_{XY} = IR$ = (0 A) R = 0 V) hence placing the contact at Y is same as placing it at X. In both cases, the voltmeter is just reading the terminal potential difference across the 12 V supply.

Since lamp L (the middle one) is broken, there is no current flowing through the lamp on the right hence the potential difference across it V_{Z-} is zero ($V_{Z-} = IR$ = (0 A) R = 0 V) where 'Z' refers to point Z and '–' refers to the negative terminal of the battery.
Placing the contact at Z is the same as placing it at the negative terminal. The voltmeter is reading the potential difference between the negative terminal and itself which is, of course, zero.

3 An electric heater can be represented as two resistors of resistances R_1 and R_2 and two switches S_1 and S_2. The resistance R_2 is greater than that of R_1.



Which switches must be closed so that the heater produces the maximum possible power and the minimum non-zero power?

	maximum possible power	minimum non-zero power
A	S_1 and S_2	S_2
B	S_1 and S_2	S_1
C	S_1	S_2
D	S_2	S_1

N05/I/24

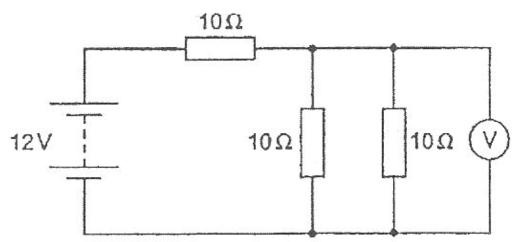
Solution Answer: A

Since the potential difference V across both resistances are the same (parallel combination), use the formula power $P = V^2/R$ to compare their power.

The resistor with the higher resistance (R_2) will produce the smaller amount of power (minimum non-zero power).

For maximum power, both resistances should be operating hence both switches should be closed.

4 In the circuit shown, the voltmeter has infinite resistance.



What is the voltmeter reading?

A 3V B 4V C 6V D 8V

N06/I/20

Solution Answer: **B**

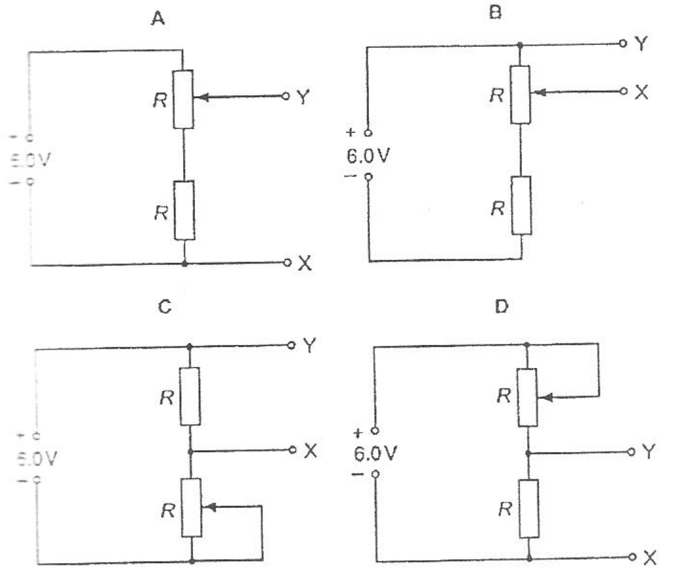
The effective resistance of the two parallel 10 Ω resistors is $(1/10 + 1/10)^{-1} = 5\ \Omega$.

Using the potential divider principle, the potential difference across them (i.e. the voltmeter reading) is:

$$[5\ \Omega / (10\ \Omega + 5\ \Omega)] \times 12\ \text{V} = 12\ \text{V} / 3 = 4\ \text{V}.$$

5 A potential divider has a constant supply of 6.0V as shown in the diagrams.

Which circuit will provide a potential difference between X and Y that can be varied between zero and 3.0V?



N06/I/19

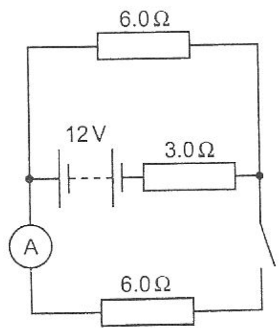
Solution Answer: **B**

In answer B, when the movable contact X is at the topmost position (so that it is at the same position as Y), the potential difference between Y and X is zero.

When the movable contact X is at its lowermost position, it is positioned exactly midway between the two resistors. Using the potential divider principle, $V_{YX} = R / (R + R) \times 6.0 = 3.0$ V.

In answers A, C and D, the range is between 3.0 V and 6.0 V.

6 In the circuit, the battery has an e.m.f. of 12 V and an internal resistance of 3.0 Ω. The ammeter has negligible resistance.



The switch is closed.

What is the reading on the ammeter?

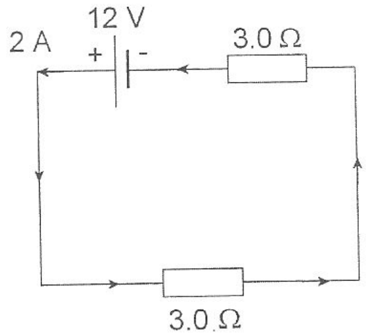
A 0.50 A **B** 1.0 A **C** 1.3 A **D** 2.0 A

N07/I/28

Solution Answer: **B**

The two 6.0 Ω resistors are in parallel and can be combined to form a 3.0 Ω effective load resistance.

The circuit can be redrawn as in the following figure.

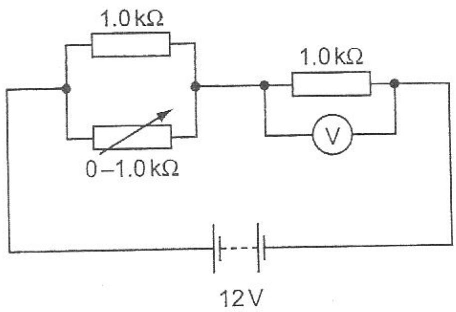


The current delivered by the battery is

$$I = 12\ \text{V} / (3.0 + 3.0)\ \Omega = 2.0\ \text{A}.$$

However this current is shared equally by the two 6.0 Ω load resistors hence the current flowing through each is 1.0 A.

7 The diagram shows a resistor network connected to a 12 V battery of negligible internal resistance. The variable resistor has the range indicated, and the voltmeter has infinite resistance.



What are the maximum and minimum possible values of the voltmeter reading as the variable resistor is altered?

	maximum / V	minimum / V
A	4	0
B	8	4
C	8	6
D	12	8

N07/I/29

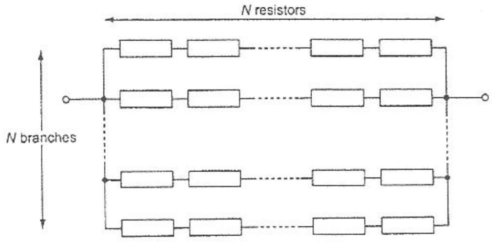
Solution Answer: **D**

When the variable resistor is set to 0 Ω, the effective resistance across the two parallel resistors is also 0 Ω. The entire terminal potential of 12 V across the battery is applied across the voltmeter hence the voltmeter reads 12 V.

When the variable resistor is set to 1.0 kΩ, the effective resistance across the two parallel 1.0 kΩ resistors is 500 Ω. The potential difference across the voltmeter can be found using the potential divider principle,

$$[1.0\ \text{k}\Omega / 1.5\ \text{k}\Omega] \times 12\ \text{V} = 8\ \text{V}$$

8 An array of resistors, each of resistance R, consists of N parallel branches. Each branch contains N resistors connected in series.



What is the total resistance of the array?

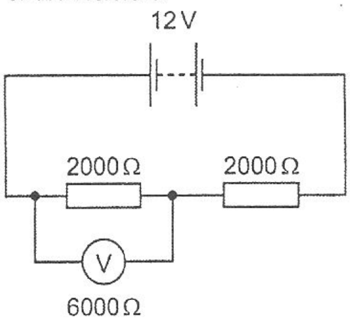
A $\frac{1}{RN}$ **B** $\frac{R}{N}$ **C** R **D** NR

N09/I/28

Solution Answer: **C**

For N identical resistors each of value R connected in series, the total resistance is NR.
For N identical resistors each of value R connected in parallel, the total resistance is R/N.
Hence for N resistors connected in series and then in parallel for N branches, the total resistance is $(NR)/N = R$.

9 A 12 V battery of negligible internal resistance is connected across two 2000 Ω resistors in series. A voltmeter of resistance 6000 Ω is connected in parallel across one of the resistors.



What will be the reading of the voltmeter?

A 5.14 V **B** 6.00 V **C** 6.86 V **D** 7.20 V

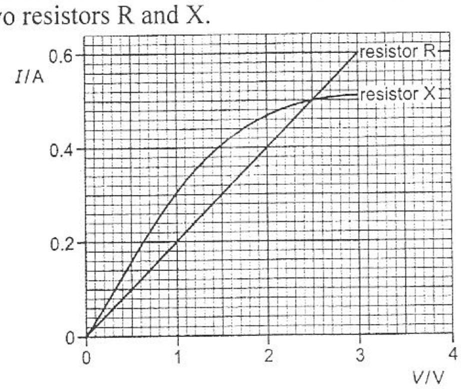
N09/I/29

Solution Answer: **A**

The effective resistance across the voltmeter is $(1/2000 + 1/6000)^{-1} = 1500\ \Omega$.
Applying the potential divider principle, the potential difference across the voltmeter is:

$$(1500 / 3500) \times 12\text{ V} = 5.14\text{ V}$$

10 The graph shows the current-voltage (I-V) characteristics of two resistors R and X.



The resistor R and X are connected in series with a cell of negligible resistance. The current in the circuit is 0.3 A.
The resistors R and X are then connected in parallel with the same cell.
What is the e.m.f. of the cell and the current in the cell when the resistors are connected in parallel?

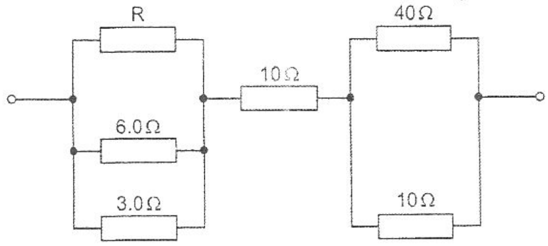
	e.m.f. / V	current / A
A	1.0	0.3
B	1.5	0.7
C	2.5	0.5
D	2.5	1.0

N10/I/29

Solution Answer: **D**

When R and X are connected in series, they have the same current of 0.3 A in them. From the graph, when the current is 0.3 A, the potential difference across R is 1.5 V while that across X is 1.0 V. The e.m.f. is equal to the sum of these two potential differences (since internal resistance is negligible). When both resistors are connected in parallel, the p.d. across them is the e.m.f. (2.5 V). From the graph, when e.m.f. is 2.5 V, R draws 0.50 A and X draws 0.50 A hence the current drawn is 1.0 A.

11 In the arrangement shown, R is a resistor that has an unknown resistance between zero and infinity.



Between what limits must the resistance of the whole arrangement lie?

	minimum resistance / Ω	maximum resistance / Ω
A	18	20
B	20	20
C	20	infinity
D	23	56

N10/I/30

Solution Answer: **A**

When R is zero, it effectively short circuits the left section of the circuit, which becomes zero ohm. The resistance of the whole arrangement is thus left with the middle section in series with the right section $= 10 + (1/10 + 1/40)^{-1} = 18\ \Omega$. When R is infinity, it can be ignored hence the resistance of the whole arrangement is $(1/6.0 + 1/3.0)^{-1} + 10 + (1/10 + 1/40)^{-1} = 20\ \Omega$.

12 The variation with current of the potential difference (p.d.) across a component X is shown in Fig. 2.1.

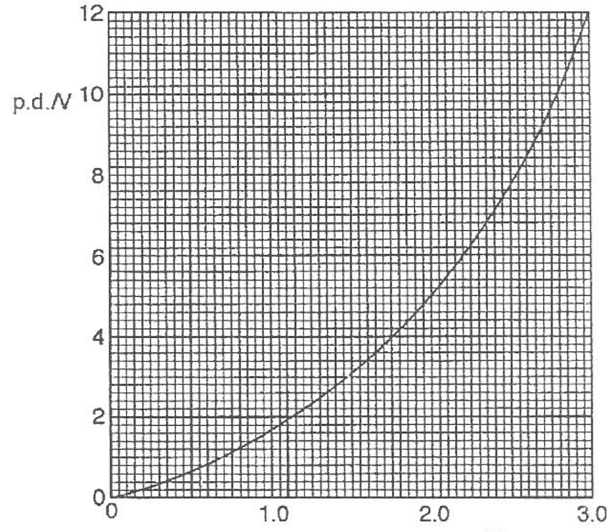


Fig. 2.1

(a) (i) State how the resistance of component X varies, if at all, with increase of current. [1]

(ii) On Fig. 2.1, draw a line to show the variation with current of the p.d. across a resistor R of constant resistance 3.0 Ω. [2]

(b)

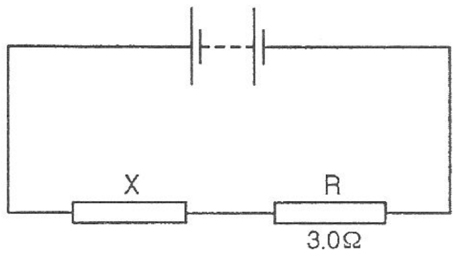


Fig. 2.2

The component X and the resistor R of resistance 3.0 Ω are connected in series with a battery of negligible internal resistance, as shown in Fig. 2.2.

The current in the circuit is found to be 2.0 A.

(i) Use Fig. 2.1 to determine the p.d. across component X.

(ii) Determine
1. the p.d. across R,
2. the e.m.f. of the battery. [3]

(c) The resistor R and the component X are now connected in parallel with the battery, as shown in Fig. 2.3.

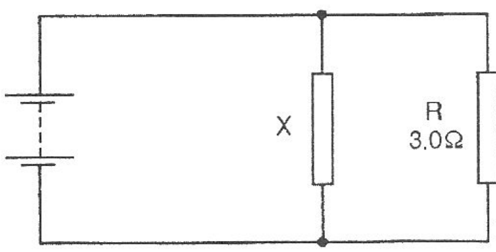


Fig. 2.3

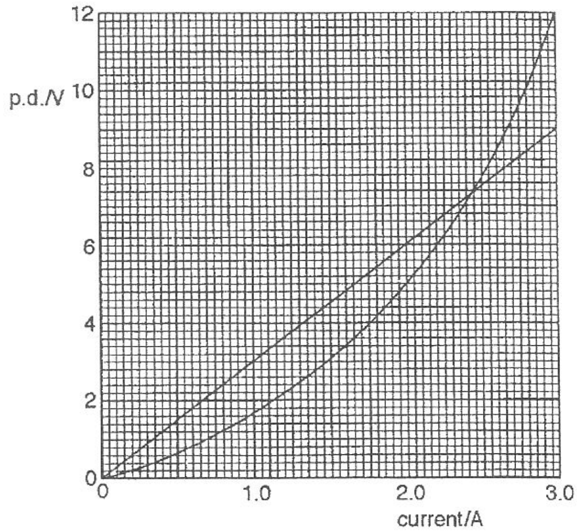
Using your answer to *(b)*(ii) and the graph of Fig. 2.1, determine the current from the battery. [4]

N03/II/2

Solution

(a) (i) Resistance R is the ratio of potential difference V to current I.
As the current I increases, the ratio of V to I increases, thus the resistance R increases with current.

(ii) The line is drawn as shown in the figure below.



(b) (i) From Fig. 2.1, when the current is 2.0 A, the p.d. across X is 5.0 V.

(ii) 1. P.d. across R is 6.0 V when I is 2.0 A (read from the straight line graph above).

Alternatively, $V = IR = (2.0\ \text{A})(3.0\ \Omega) = 6.0\ \text{V}$.

2. Sum of e.m.f. = sum of p.d. hence
$E = 5.0\ \text{V} + 6.0\ \text{V} = 11.0\ \text{V}$.

(c) Since both X and R are connected in parallel across the battery, their p.d. are both equal to the e.m.f. of 11.0 V.

The current through X can be read from the graph:
when $V = 11.0$ V, the current through X is 2.9 A.

The current through R can be calculated from
$I = V/R = 11.0\ \text{V} / 3.0\ \Omega = 3.667\ \text{A} = 3.7\ \text{A}$.

The total current drawn from the battery is thus
$I_{total} = 2.9\ \text{A} + 3.667\ \text{A} = 6.567 = 6.6\ \text{A}$.

13 A circuit consists of three resistors R_1, R_2 and R_3, and two switches A and B, as shown in Fig. 4.1.

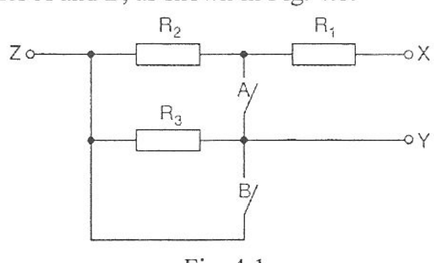


Fig. 4.1

The resistance between terminals X and Y is measured for different settings of the switches A and B. The results are shown in Fig. 4.2.

switch A	switch B	resistance between X and Y /Ω
open	open	12
open	closed	10
closed	open	6
closed	closed	6

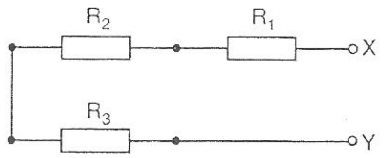
Fig. 4.2

(a) Determine the resistance of
 (i) resistor R_1,
 resistance = ... Ω [1]
 (ii) resistor R_2,
 resistance = ... Ω [1]
 (ii) resistor R_3,
 resistance = ... Ω [1]
(b) Switch A is now closed and switch B is open.
 Calculate the resistance between terminals X and Z.
 resistance = ... Ω [2]

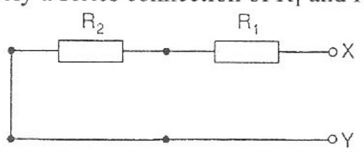
N10/III/4

Solution

(a) Working:
With A & B open, the circuit is effectively a series connection of the 3 resistors:

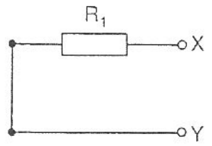


$R_1 + R_2 + R_3 = 12\ \Omega$.
With A open & B closed, resistor R_3 is shorted, the circuit is effectively a series connection of R_1 and R_2:



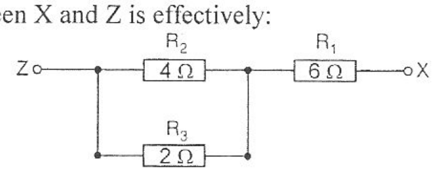
$R_1 + R_2 = 10\ \Omega$ hence $R_3 = 2\ \Omega$.

With A closed, both R_2 & R_3 are shorted (regardless of whether B is open or closed), the circuit is effectively just R_1 alone:



$R_1 = 6\ \Omega$.
Hence $R_2 = 4\ \Omega$.
(i) $R_1 = 6\ \Omega$.
(ii) $R_2 = 4\ \Omega$.
(iii) $R_3 = 2\ \Omega$.

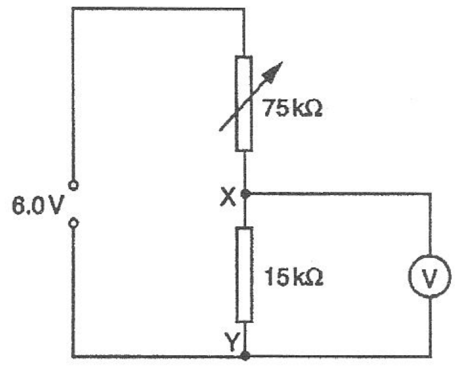
(b) With A closed and B open, the effective resistance between X and Z is effectively:



$R_{eff} = (1/4 + 1/2)^{-1} + 6 = 7.33\ \Omega$.

Potential Divider

14 The diagram shows two resistors connected in a circuit with a power supply and a voltmeter.



What range of voltages can be obtained between points X and Y?

A zero to 1.0 V
B zero to 6.0 V
C 1.0 V to 5.0 V
D 1.0 V to 6.0 V

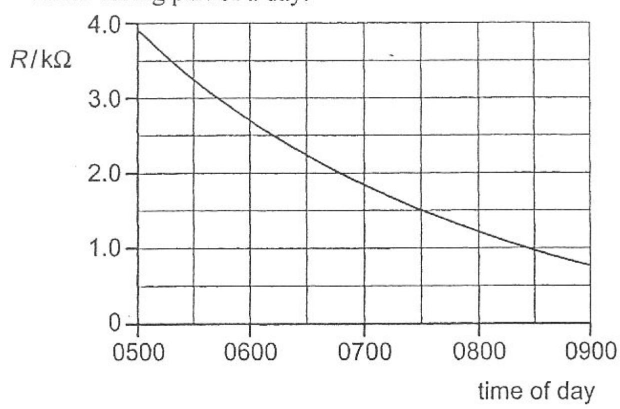
N03/I/20

Solution Answer: D

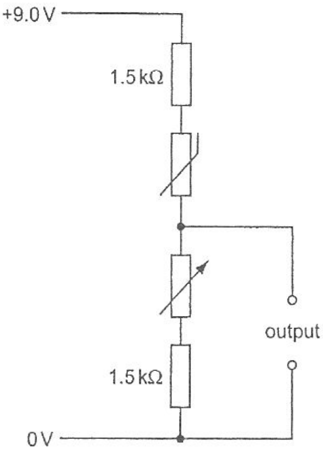
When the variable resistor is set to zero, the potential difference across the 15 kΩ is
$V = ((15\text{ k}\Omega) / (15\text{ k}\Omega + 0\ \Omega)) \times 6.0\text{ V} = 6.0\text{ V}$.

When the variable resistor is set to 75 kΩ, the potential difference across the 15 kΩ is
$V = ((15\text{ k}\Omega) / (15\text{ k}\Omega + 75\text{ k}\Omega)) \times 6.0\text{ V} = 1.0\text{ V}$.

15 The graph shows how the resistance R of a thermistor varies during part of a day.



The thermistor is connected in the potential divider circuit shown.



In order to obtain an output of 6.0 V at 0730, to which value should the variable resistor be set?

A 1.5 kΩ B 3.0 kΩ C 4.5 kΩ D 6.0 kΩ

N08/I/28

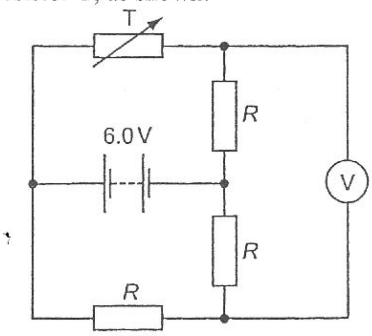
Solution Answer: **C**

At 0730, the resistance of the thermistor is 1.5 kΩ.
Let $R_{variable}$ be the resistance of the variable resistor.
Apply the potential divider principle,

$$[(R_{variable} + 1.5\text{ k}\Omega) / (R_{variable} + 4.5\text{ k}\Omega)] \times 9.0\text{ V} = 6.0\text{ V}$$

$$R_{variable} = 4.5\text{ k}\Omega$$

16 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to three resistors, each of resistance R, and a variable resistor T, as shown.



The resistance of T changes from R to $5R$.

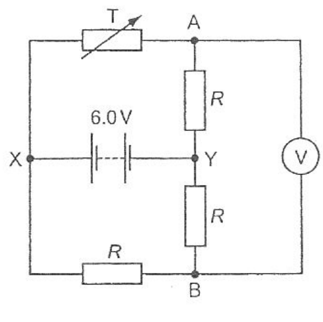
What is the change in the reading of the high resistance voltmeter?

A zero B 2 V C 4 V D 5 V

N08/I/29

Solution Answer: **B**

The four junctions are labelled A, B, X and Y, as shown.



Initially, the circuit was symmetrical above and below the battery.

Using the potential divider principle, the potential difference across AY was $R/2R$ x 6.0 V = 3.0 V; and the potential difference across BY was $R/2R$ x 6.0 V = 3.0 V.

The potential difference between A and B (as measured by the voltmeter) was 3.0 V – 3.0 V = 0.0 V.

When T changes from R to $5R$, the potential difference across AY changes to $R/6R$ x 6.0 V = 1.0 V; while the potential difference across BY remains unchanged at 3.0 V.

The potential difference between A and B (as measured by the voltmeter) is now 1.0 V – 3.0 V = -2.0 V.

The reading of the voltmeter has changed from 0.0 V to -2.0 V. The change in reading is 2.0 V.

Long Questions

17 (a) Electromotive force (e.m.f.) and potential difference (p.d.) may both have the volt as a unit.

(i) Define the *volt*. [1]

(ii) By reference to energy transfers, distinguish between e.m.f. and p.d.

e.m.f.

p.d. [2]

(b) A cell of e.m.f. 1.5 V and internal resistance 0.25 Ω is connected in series with a resistor R, as shown in Fig. 7.1.

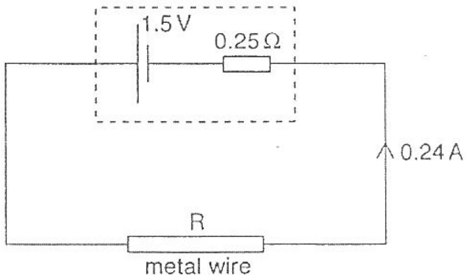


Fig. 7.1

The resistor R is made of metal wire.
A current of 0.24 A passes through R for a time of 5.0 minutes.
Calculate

(i) the charge that passes through the cell,
charge = ...C [1]

(ii) the total energy transferred by the cell,
energy = ... J [2]

(iii) the energy transferred in the resistor R,
energy = ... J [3]

(iv) the resistance of R.
resistance = ... Ω [2]

(c) A second similar cell is now connected in series with the cell in (b) and the resistor R.
The current in the circuit is 0.41 A and the resistance of R changes.

(i) Calculate the new resistance of R.
resistance = ... Ω [2]

(ii) Resistor R is made of metal wire. Suggest why the answers in (b)(iv) and (c)(i) are different. [1]

(d) The cells in (c) are now connected in series with a fixed resistor of resistance 2000 Ω and a thermistor, as shown in Fig. 7.2.

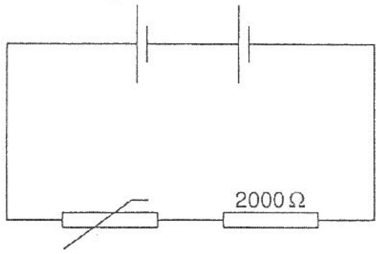


Fig. 7.2

The thermistor has resistance 4000 Ω at 0 °C and 1800 Ω at 20 °C.

(i) Explain why, in this circuit, the internal resistance of the cells may be considered to be negligible.

....

.... [1]

(ii) Determine the potential difference across the thermistor

1. at 0 °C,
potential difference = ... V [1]

2. at 20 °C.
potential difference = ... V [1]

(iii) In one particular application of the circuit of Fig. 7.2., it is desired that the potential difference across the **fixed** resistor should range from 1.2 V at 0 °C to 2.4 V at 20 °C.
Determine whether, by substituting a different fixed resistor in the circuit of Fig. 7.2, it is possible to achieve this range of potential differences. [3]

N09/III/7

Solution

(a) (i) One volt is the potential difference between two points in a circuit if one joule of energy is converted from electrical to non-electrical form per coulomb of positive charge flowing between the two points.

(ii) 1. e.m.f. of a source is the non-electrical energy converted into electrical energy and transferred from the source to the circuit, when unit positive charge is driven by the source round a complete circuit.

2. p.d. across a load is the electrical energy converted into non-electrical energy and transferred from the circuit to the load, when unit positive charge flows through the load.

(b) (i) Charge = electric current x time.

$Q = I\,t$ = (0.24 A)(5.0 x 60 s) = 72 C.

charge = 72 C

(ii) Energy transferred = EQ = (1.5 V)(72 C) = 108 J.
energy = 108 J

(iii) Energy loss in the internal resistance
= Pt = I^2Rt = (0.24 A)^2(0.25 Ω)(5.0 x 60 s) = 4.32 J

Energy transferred in the resistor R = 108 J – 4.32 J = 103.68 J

energy = 104 J

(iv) Since the current through R is the same as that of through the internal resistance, the ratio of their resistances is the same as that of their power and also that of their energy transferred.

Hence R = (103.68 / 4.32) x 0.25 Ω = 6.0 Ω.

resistance = 6.0 Ω

(c) (i) The total resistance in the new circuit

R_{total} = (1.5 V + 1.5 V) / (0.41 A) = 7.317 Ω

New R = R_{total} – (0.25 Ω + 0.25 Ω) = 6.817 Ω.

resistance = 6.8 Ω

(ii) A higher current causes more heating and an increase in the wire's ionic lattice vibration, which results in more frequent collisions between drifting electrons and the lattice. This increases the wire's resistivity.

(d) (i) The external loads will have a minimum total resistance of 3800 Ω (at 20 °C), transferring energy at least 7600 times as much as in the combined internal resistance (0.50 Ω) of the two cells.

(ii) 1. At 0 °C, V = (4000 / 6000) x 3.0 V = 2.0 V.
potential difference = 2.0 V

2. At 0 °C, V = (1800 / 3800) x 3.0 V = 1.421 V.
potential difference = 1.4 V

(iii) In order for the potential difference across the fixed resistor to be 1.2 V at 0 °C, its resistance R should be such that:

[R / (4000 + R)] x 3.0 V = 1.2 V
R = 2670 Ω

In order for the potential difference across the fixed resistor to be 2.4 V at 20 °C, its resistance R should be such that:

[R / (1800 + R)] x 3.0 V = 2.4 V
R = 7200 Ω

There is no single value of R that satisfies both conditions.

Potentiometer

Short Questions

18 *(a)* A uniform wire XY of length 120 cm and radius 0.55 mm is connected in series with a cell of e.m.f. 3.0 V and internal resistance 0.70 Ω, as shown in Fig. 6.1.

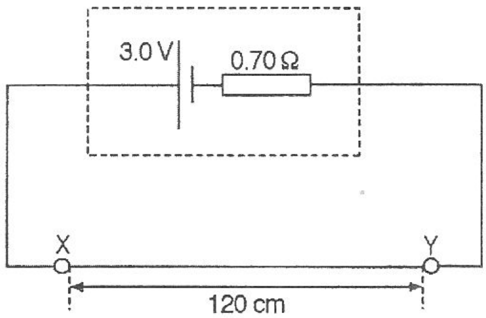


Fig. 6.1

(i) The resistivity of the material of the wire is 1.1×10^{-6} Ω m. Show that the resistance of the wire XY is 1.4 Ω.

(ii) Calculate the potential difference (p.d.) per unit length of XY.

potential difference per unit length = ... V m^{-1}

[5]

(b) A cell C of e.m.f. 1.5 V and internal resistance 0.50 Ω is connected to the circuit of Fig. 6.1, as shown in Fig. 6.2.

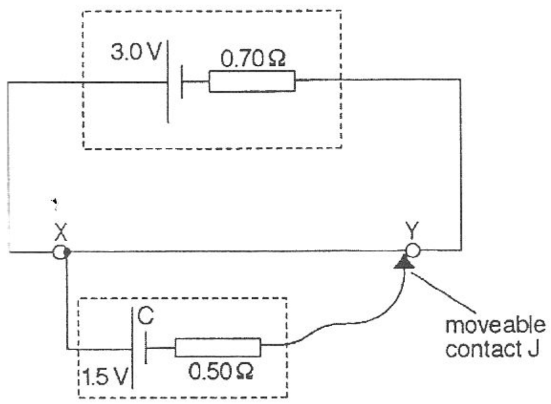


Fig. 6.2

The moveable contact J can be connected to any point along the wire XY.

(i) Initially, the contact J is connected to end Y. The p.d. across the wire XY is then equal to the p.d. applied across cell C. On Fig. 6.2, mark with an arrow the direction of the current through the cell C.

(ii) Determine the position of the contact J on XY such that there is no current through the cell C.

position: ....

(iii) Suggest one way in which the circuit in Fig. 6.2 may be modified so that the position found in (ii) is nearer to end Y. [5]

N02/II/6

Solution

(a) (i) Resistance

$R = \rho l/A$

$= (1.1 \times 10^{-6}\ \Omega\ m)(1.2\ m)/(\pi(0.55 \times 10^{-3}\ m)^2)$

$= 1.389$

$= 1.4\ \Omega$.

(ii) Applying the potential divider principle,

$V_{XY} = (1.4\ \Omega / (1.4\ \Omega + 0.70\ \Omega)) \times 3.0\ V = 2.0\ V$.

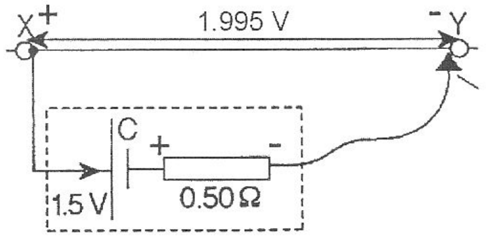
The p.d. per unit length

$= 2.0\ V / 1.2\ m$

$= 1.667$

$= 1.7\ V\ m^{-1}$.

(b) (i) The direction of the current through C is shown in the figure below.



Current is forced into the positive terminal of cell C because its e.m.f. (1.5 V) is less than V_{XY} (2.0 V).

(ii) For there to be no current through cell C, the potential difference across the wire must be equal to its e.m.f. (1.5 V).

The balance length is

$l = (1.5\ V)/(1.667\ V\ m^{-1}) = 0.90\ m$

(measured from X).

(iii) To shift the balance point closer to Y, the p.d. across XY (i.e. V_{XY}) should be reduced.

This can be achieved by adding a resistor in series with the 3.0 V driver cell.

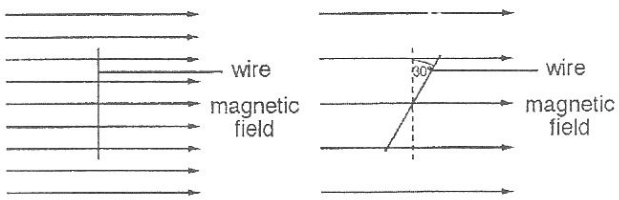
Alternatively, use a driver cell with an e.m.f. of lower than 3.0 V.

TOPIC 15 Electromagnetism

Force on Current-Carrying Conductor

1 A straight, horizontal, current-carrying wire lies at right angles to a horizontal magnetic field. The field exerts a vertical force of 8.0 mN on the wire.

The wire is rotated, in its horizontal plane, through 30° as shown. The flux density of the magnetic field is halved.



What is the vertical force on the wire?

A 2.0 mN   C 4.6 mN
B 3.5 mN   D 8.0 mN   N02/I/25

Solution Answer: **B**

The magnetic force $F = BIL \sin \theta$ where B is the magnetic flux density, I the current, L the length of the conductor and θ the angle between the field and the current.

Initially, the angle $\theta = 90^\circ$ and $\sin 90^\circ = 1$.

After being rotated by 30°, the angle $\theta = 60^\circ$ and $\sin 60^\circ = 0.866$.

Furthermore, the magnetic flux density is halved.

The magnetic force

$$\begin{aligned} F &= (0.5\,B)(I)(L) \sin 60^\circ \\ &= 0.433\,BIL \\ &= 0.433 \times 8.0 \text{ mN} \\ &= 3.5 \text{ mN.} \end{aligned}$$

2 A horizontal wire carries current at right angles to a horizontal magnetic field. The wire is then turned through 90° and so becomes parallel with the magnetic field.

What describes the initial and final force on the wire?

	initial force	final force
A	vertical	vertical
B	vertical	zero
C	zero	horizontal
D	zero	vertical

N03/I/22

Solution Answer: **B**

The magnetic force $F = BIL \sin \theta$ where B is the magnetic flux density, I the current, L the length of the conductor and θ the angle between the field and the current.

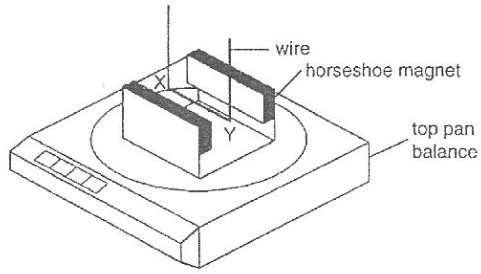
Initially, the angle $\theta = 90^\circ$ and $\sin 90^\circ = 1$ and the magnetic force is maximum.

Using Fleming's left hand rule, the direction of the magnetic force (the thumb) should be perpendicular to both the field (first finger) and current (middle finger).

Since both the current and the field lie on the horizontal plane, the force should be vertical.

After being rotated by 90°, the angle $\theta = 0^\circ$ and $\sin 0^\circ = 0$. The force $F = BIL \sin 0^\circ = 0$.

3 A horseshoe magnet rests on a top-pan balance with a wire situated between the poles of the magnet.



With no current in the wire, the reading on the balance is 142.0 g.

With a current of 2.0 A in the wire in the direction XY, the reading on the balance changes to 144.6 g.

What is the reading on the balance, when there is a current of 3.0 A in the wire in the direction YX?

A 138.1 g
B 140.7 g
C 145.9 g
D 148.5 g   N04/I/24

Solution Answer: **A**

The magnetic force $F = BIL \sin \theta$ where B is the magnetic flux density, I the current, L the length of the conductor and θ the angle between the field and the current (in this case 90° and $\sin 90^\circ = 1$).

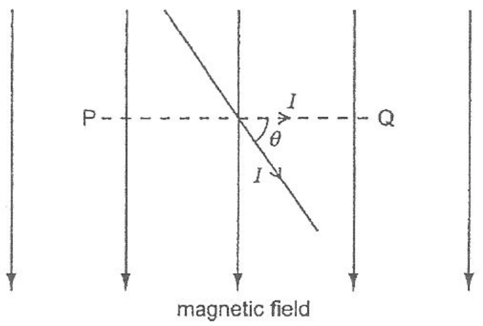
Initially, the downward magnetic force exerted by the wire on the balance is 144.6 g – 142.0 g = 2.6 g.

When the current is increased from 2.0 A to 3.0 A, the magnetic force, being directly proportional to the current, will also increase by a factor of 1.5, from 2.6 g to 3.9 g.

However, since the direction is reversed, an upward magnetic force is exerted by the wire on the balance.

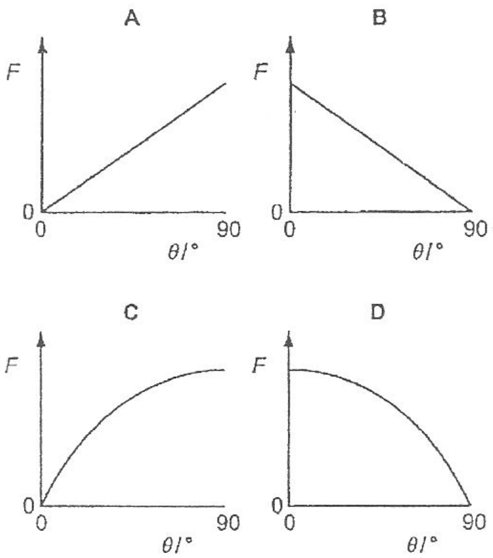
The balance thus reads 142.0 g – 3.9 g = 138.1 g.

4 A straight wire PQ carrying a constant current I is placed at right angles to a uniform magnetic field, as shown by the dotted line in the diagram.



The wire is then rotated through an angle θ about an axis perpendicular to the plane of the diagram.

Which graph shows how the magnitude of the magnetic force F on the wire varies with θ in the range 0° to 90°?



N06/I/23

Solution Answer: **D**

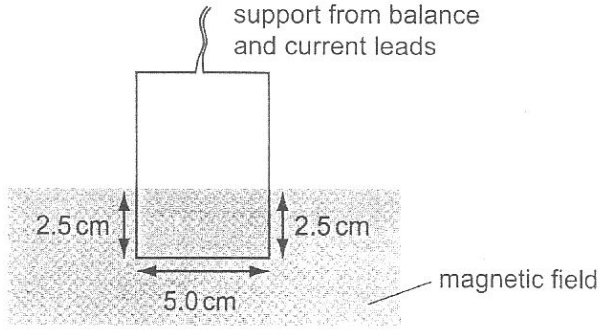
The magnetic force $F = BIL \sin \alpha$ where B is the magnetic flux density, I the current, L the length of the wire and α the angle **between the current and the field**, and α was initially 90°.

Since $\alpha = 90° - \theta$, the equation can be rewritten as:

$F = BIL \sin \alpha = F = BIL \sin (90° - \theta) = F = BIL \cos \theta$.

Answer D is a cosine graph starting from $\theta = 0°$ (the initial position of the wire).

5 A single-turn rectangular wire loop hangs from a balance reading in grams so that its lower part is in a region of uniform magnetic field. The direction of the field is at right-angles to the plane of the loop. The arrangement is as shown in the diagram.



When there is no current in the loop, the reading of the balance is 10.060 g. When the current in the loop is 3.0 A, the balance reading is 10.040 g.

What is the magnitude of the flux density of the field?

A 6.5×10^{-4} T
B 1.3×10^{-3} T
C 1.3×10^{-2} T
D 6.6×10^{-1} T

N07/I/30

Solution Answer: **B**

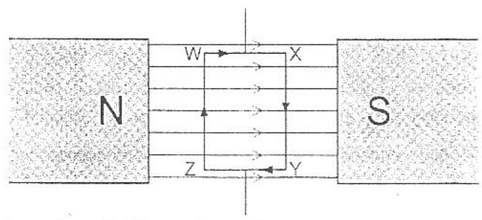
The difference between the two readings is due to the magnetic force hence

$$BIL = mg$$
$$BIL = (0.020 \times 10^{-3}\ \text{kg})(9.81\ \text{N kg}^{-1})$$
$$BIL = 1.962 \times 10^{-4}\ \text{N}$$

Magnetic flux density

$B = F/IL = (1.962 \times 10^{-4}\ \text{N})/(3.0\ \text{A} \times 0.05\ \text{m}) = 1.3 \times 10^{-3}$ T

6 In an electric motor, a rectangular coil WXYZ has 20 turns and is in a uniform magnetic field of flux density 0.83 T.



The lengths of sides XY and ZW are 0.17 m and of sides WX and YZ are 0.11 m. The current in the coil is 4.5 A.

What is the maximum torque provided by the motor?

A 0.070 Nm B 0.63 Nm C 1.4 Nm D 2.8 Nm

N10/I/31

Solution Answer: **C**

The forces acting on XY and ZW have the same magnitude (although opposite directions) and can be found using the equation $F = NBIL = (20)(0.83)(4.5)(0.17) = 12.7$ N.

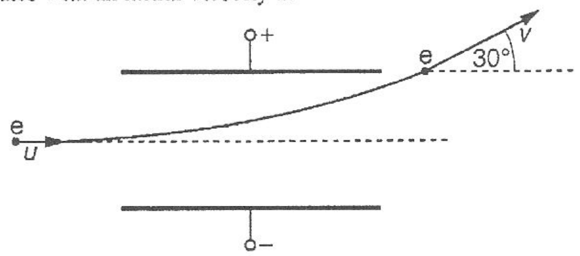
These two forces form a couple with a torque $T = Fd = (12.7)(0.11) = 1.4$ Nm (where F is the force of one of the couple and d the perpendicular distance between them).

The derived formula for the torque of a rectangular coil is:

$T = Fd = (NBIL)d = BANI$ where A is the area of the coil.

Force on Moving Charges

7 An electron enters the space between two parallel charged plates with an initial velocity u.



While in the electric field its direction changes by 30° and it emerges with a velocity v. What is the relation between v and u?

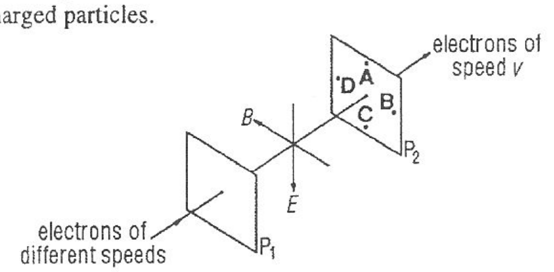
A $v = \dfrac{u}{\cos 30^\circ}$ **C** $v = \dfrac{u}{\sin 30^\circ}$

B $v = u \cos 30^\circ$ **D** $v = u \sin 30^\circ$ N02/I/20

Solution Answer: **A**

Since the electric field acts perpendicular to the electron's initial velocity u, the electric force does not alter the horizontal component of the electron's velocity.
Therefore, by the time the electron leaves the parallel plates, the horizontal component of its velocity $v \cos \theta$ is still equal to u. Therefore, $v = u / \cos 30^\circ$.

8 The diagram illustrates a method of velocity selection of charged particles.



A beam containing electrons of different speeds passes through a hole in plate P_1. P_2 is a plate, parallel to P_1, also with a hole in it. In the region between the plates a uniform electric field of intensity E and a uniform magnetic field of flux density B are applied at right angles to each other, as shown. The magnitudes of E and B are adjusted so that only electrons of speed v pass undeviated through the holes in P_1 and P_2.

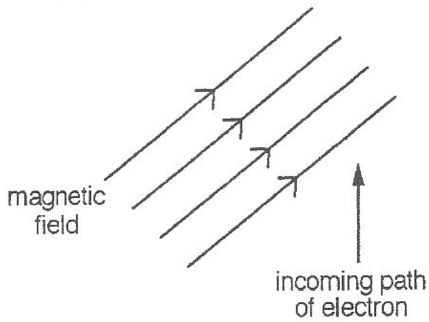
Which point indicates where electrons of a speed greater than v could strike P_2?

N02/I/28

Solution Answer: **C**

The electric field exerts an upward force qE on the electrons (opposite to the direction of the electric field, since electrons are negative charges).
The magnetic field exerts a downward force Bqv on the electrons (Fleming's left hand rule).
Those electrons with a particular speed v whereby $Bqv = qE$ or $v = E/B$ will pass straight through undeflected through hole P_2.
Electrons with speeds greater than v would experience a greater downward magnetic force Bqv while its electric force qE remains unchanged.
Hence these electrons would be deflected downwards.

9 The diagram shows an electron as it enters a magnetic field. The path of the electron and the magnetic field are in the plane of the paper.



In which direction is the electron initially deflected?

A into the plane of the paper

B out of the plane of the paper

C to the left of its incoming path

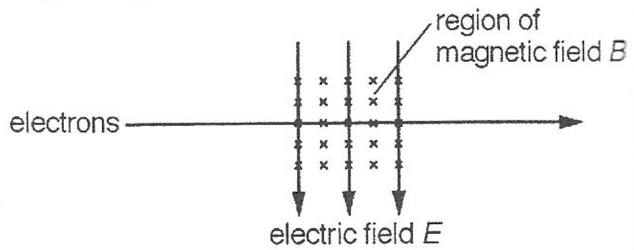
D to the right of its incoming path N03/I/21

Solution Answer: **B**

The movement of negative charges such as electrons produces an electric current I in the opposite direction to their motion.

Applying Fleming's left hand rule, with the first finger pointing in the direction of the horizontal component of the magnetic field (towards the right, since only this component of the magnetic field, which is perpendicular to the current's direction, will result in a magnetic force); the second finger in the direction of the current (downwards, i.e. opposite to the direction of the electron beam), the thumb points in the direction of the force, out of the plane of the paper.

10 A beam of electrons enters a region in which there are magnetic and electric fields directed at right angles. It passes straight through without deviation.



A second beam of electrons travelling twice as fast as the first is then directed along the same line.

How is this second beam deviated?

A downwards in the plane of the paper

B upwards in the plane of the paper

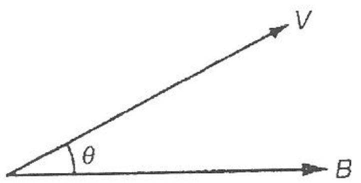
C out of the plane of the paper

D into the plane of the paper N03/I/27

Solution Answer: **A**

The electric field exerts an upward force qE on the electrons (opposite to the direction of the electric field, since electrons are negative charges).
The magnetic field exerts a downward force Bqv on the electrons (Fleming's left hand rule).
Those electrons with a particular speed v whereby $Bqv = qE$ or $v = E/B$ will pass straight through undeflected.
Electrons with speeds greater than v would experience a greater downward magnetic force Bqv while its electric force qE remains unchanged.
Hence these electrons would be deflected downwards, in the plane of the paper.

11 An electron of charge e and mass m_e is injected into a uniform magnetic field of flux density B in a vacuum. Its initial velocity v makes an angle θ with the direction of the field as shown.



Which of the following correctly describes the component of the electron velocity parallel to B in the subsequent motion?

A constant speed equal to $v \sin\theta$

B constant speed equal to $v \cos\theta$

C speed increasing with acceleration $\dfrac{Bev}{m_e}$

D speed increasing with acceleration $\dfrac{Bev\cos\theta}{m_e}$

N04/I/28

Solution Answer: **B**

A magnetic field exerts a force on charged particles in a direction perpendicular to both the particle's velocity and the field (Fleming's left hand rule).
There is no force component in the direction parallel to the magnetic field B.
Therefore, the parallel component of its velocity $v \cos\theta$ remains unchanged.

12 The acceleration of an electron of mass m and charge e, moving with uniform speed v at right angles to a magnetic field of flux density B, is given by

A $\dfrac{Bev}{m}$ B $\dfrac{Be}{m}$ C $\dfrac{Bv}{m}$ D $Bevm$.

N06/I/27

Solution Answer: **A**

The magnetic force provides the centripetal force:

$$Bev = mv^2/r$$

The centripetal acceleration $v^2/r = Bev/m$.

13 A charged particle, initially travelling in a vacuum in a straight line, enters a uniform field. This causes the particle to travel in a curved path that is not the arc of a circle.

Which type of field, and which initial direction of the particle with reference to the field, causes this to happen?

	field type	initial direction of the particle compared to the field
A	electric	parallel
B	electric	perpendicular
C	magnetic	parallel
D	magnetic	perpendicular

N08/I/30

Solution Answer: **B**

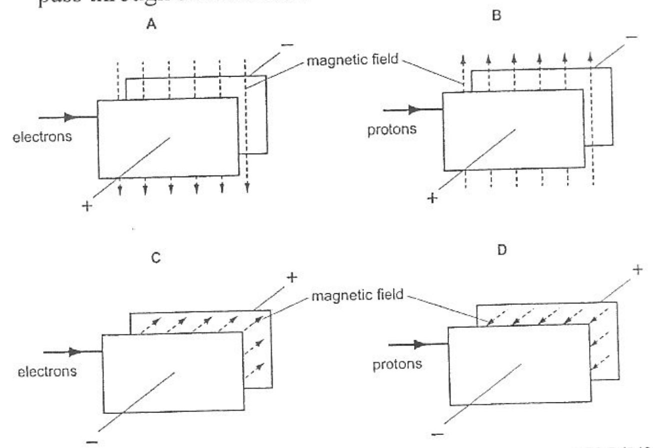
A magnetic field causes a charged particle to move in either circular (or helical) motion. Since the curved path is not the arc of a circle, the field type is not magnetic but likely to be electric, which causes charged particles to move in parabolic motion within it.

For the path to be curved along a parabola, the initial velocity must be perpendicular to the field (similar to a projectile in a uniform gravitational field).

On the other hand, a charged particle with initial velocity parallel to an electric field will either be accelerated or decelerated along the direction of its original path. The motion will then be that of a straight line and not a curve.

14 The diagram show different particle beams entering a region between two metal plates in which there are uniform electric and magnetic fields.

In which arrangement would it be possible for the beam to pass through undeflected?



N08/I/31

Solution Answer: **B**

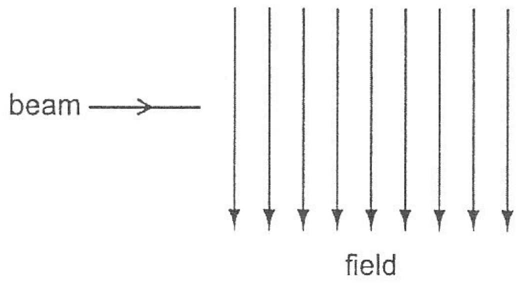
For the beam of particles to pass through undeflected, the electric and magnetic forces must be equal in magnitude and opposite in direction.

In diagram B, the beam of protons will experience an electric force into the plane of the diagram. This is because positive charges such as protons experience an electric force in the same direction as that of the electric field (which is from the positive to the negative plate, into the diagram).

The magnetic force on the beam of protons is out of the plane of the diagram, applying Fleming's left hand rule with the current finger pointing in the same direction as the protons' velocity (since protons have a positive charge, the direction of the current is the same as that of their velocity).

The electric and magnetic forces are hence in opposite directions and could possibly cancel out.
In diagram A, both the electric and magnetic forces are out of the plane of the diagram.
In diagram C, the electric force is into the plane of the diagram while the magnetic force is downwards.
In diagram D, the electric force is out of the plane of the diagram while the magnetic force is downwards.

15 A beam of particles or radiation is directed horizontally into vertically gravitational, electric and magnetic fields in turn.



The table shows features of the force on the beam and the shape of the beam in each case.

	gravitational field	electric field	magnetic field
force	negligible	upwards	out of the page
shape of beam	a horizontal line	a curve	a circle

Which particles or radiation are used in the beam?

A alpha particles
B beta particles
C gamma rays
D protons

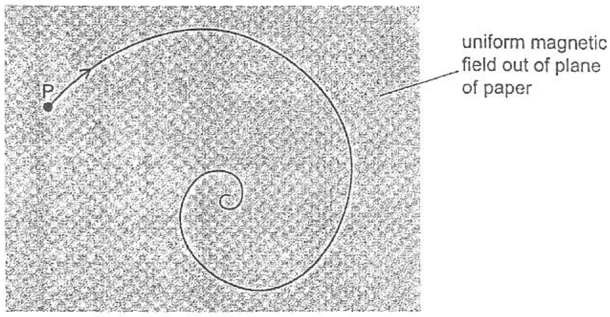
N09/I/9

Solution Answer: **B**

Beta particles are high speed electrons with a negative charge.

They thus experience an electric force in a direction opposite to that of the electric field (i.e. upwards); and a magnetic force in a direction perpendicular to that of the magnetic field, and out of the page (applying Fleming's left hand rule).

16 A charged particle is moving in a region where there is a uniform magnetic field acting out of the plane of the paper. The path of the particle begins at P and is as shown.



What can be deduced about the charge on the particle and its speed?

	charge	speed
A	negative	decreasing
B	negative	increasing
C	positive	decreasing
D	positive	increasing

N09/I/30

Solution Answer: **C**

Applying Fleming's left hand rule, it can be seen that, with the magnetic force providing the centripetal force acting towards the centre of rotation, the direction of the current is the same as that of the charge's velocity. Hence the charge on the particle must be positive.

From the equation: radius of rotation $r = mv/Bq$ (derived by equating the magnetic force to the centripetal force) where m is the mass of the particle, v its velocity, q its charge and B the magnetic flux density, it can be deduced that a decreasing radius of curvature implies a decreasing speed.

Short Questions

17 There are two situations in which a charged particle in a magnetic field does not experience a magnetic force. State these two situations. [2]

N02/II/7 (part)

Solution

Situation 1: The charge is stationary
($F = Bqv \sin\theta = Bq(0) \sin\theta = 0$).

Situation 2: The charge is travelling in a direction parallel to the magnetic field.
($F = Bqv \sin\theta = Bqv \sin 0 = 0$).

18 *(a)* A small mass experiences a force when placed in a field of force. The field may be electric, magnetic or gravitational. State the type of field when the mass is

(i) uncharged and the force is in the direction of the field,

(ii) charged and the force is in the opposite direction to the field,

(iii) charged and a force is experienced only when it is moving. [3]

(b) An electron is travelling at right angles to a uniform magnetic field of flux density 1.5 mT, as illustrated in Fig. 6.1.

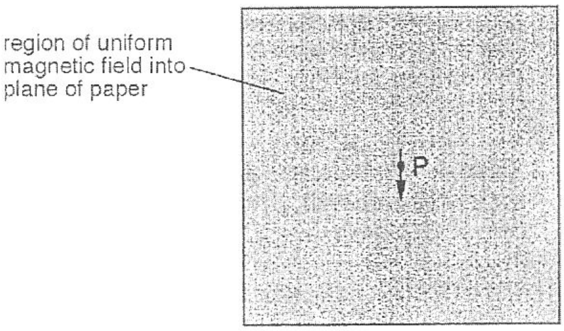


Fig. 6.1

The magnetic field is directed into the plane of the paper.

When the electron is at P, its velocity is 2.9×10^7 m s^{-1} in the direction shown. This is normal to the magnetic field.

(i) On Fig. 6.1, sketch the path of the electron, assuming that it does not leave the region of the magnetic field. [2]

(ii) Calculate, for this electron,

1. the force on it due to the magnetic field,

force = ... N

2. the radius of its path.

radius = ... m
[4]

(c) A uniform electric field is now produced in the same region and in the same direction as the magnetic field. Suggest the shape of the resultant path of the electron. You may draw a sketch to illustrate the path, if you wish. [2]

N03/II/6

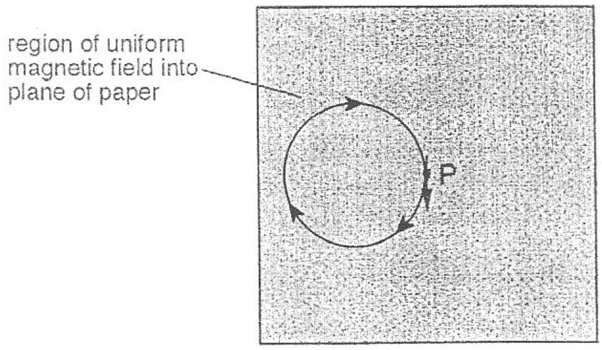
Solution

(a) (i) Gravitational field.

(ii) Electric field.

(iii) Magnetic field.

(b) (i) The path of the electron is shown in the figure below. The direction of the magnetic force (which provides the centripetal force) can be found from Fleming's left hand rule.



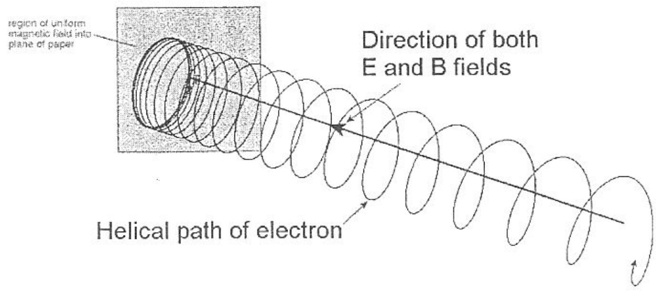
(ii) 1. $F = Bqv \sin 90^\circ$
$= (1.5 \times 10^{-3}\ \text{T})(1.60 \times 10^{-19}\ \text{C})(2.9 \times 10^7\ \text{m s}^{-1})$
$= 6.96 \times 10^{-15}$
$= 7.0 \times 10^{-15}$ N.

2. Since $Bqv = mv^2/r$, the radius
$r = mv/Bq$

$= (9.11 \times 10^{-31}\ \text{kg})(2.9 \times 10^7\ \text{m s}^{-1})$
$/(1.5 \times 10^{-3}\ \text{T} \times 1.60 \times 10^{-19}\ \text{C})$

$= 0.1101$
$= 0.11$ m.

(c) A uniform electric field E directed into the plane of the paper will cause an electric force ($F = eE$) to act on the electron in the opposite direction (i.e. out of the plane of the paper).
This causes the electron to accelerate out of the plane of the paper in a helical path with progressively larger pitch between successive helices, as shown in the figure below.



19 *(a)* Define the *radian*. [2]

(b) The path of an α-particle travelling at a linear speed of 2.8×10^7 ms^{-1} is an arc of a circle of radius 1.4 m, as shown in Fig. 2.1.

(i) Calculate the angular speed of the α-particle about the centre C of the circle.

angular speed = ... rad s^{-1} [2]

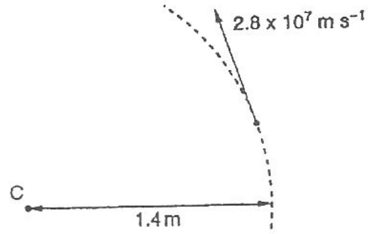


Fig. 2.1

(ii) The α-particle is moving in a uniform force-field.

1. State the name of the force-field that would produce this motion. [1]
2. State the direction of the force-field. [1]
3. Calculate the magnitude of the field strength.

field strength = ... [4]

(c) The track of a second α-particle in the same force-field is seen to spiral, as shown in Fig. 2.2.

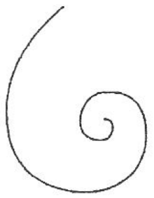


Fig. 2.2

Suggest a reason for the spiral path and hence, on Fig. 2.2, mark the direction of motion of the α-particle. [3]

N06/II/2

Solution

(a) The *radian* is a unit of angular measure defined as the angle subtended at the centre of a circle by an arc of a length equal to the radius of the circle.

(b) (i) Angular speed $\omega = v/r$ where v is the linear speed and r the radius of the orbit.

$$\omega = v/r$$
$$= (2.8 \times 10^7 \text{ m s}^{-1})/(1.4 \text{ m})$$
$$= 2.0 \times 10^7 \text{ rad s}^{-1}$$

(ii) 1. Magnetic field.

2. Into plane of the paper (Fleming's left hand rule).

3. The magnetic force provides the centripetal force hence,

$$Bqv = mv^2/r$$
$$r = mv/Bq$$
$$B = mv/rq$$
$$= (4\,u \times 2.8 \times 10^7 \text{ m s}^{-1})/(1.4 \text{ m} \times 2\,e)$$

$$= \frac{4 \times 1.66 \times 10^{-27} \text{ kg} \times 2.8 \times 10^7 \text{ m s}^{-1}}{1.4 \text{ m} \times 2 \times 1.60 \times 10^{-19} \text{ C}}$$

$$= 0.415 = 0.42 \text{ T}$$

where B is the magnetic flux density, q the charge of the α-particle, v its speed, m its mass and r the radius of its orbit.

(c) The particle is losing kinetic energy as it collides with fluid particles in the medium.

As its speed decreases, the radius of curvature r of its motion also decreases, as seen from the formula $r = mv/Bq$ derived in (b)(ii)3.

The direction of motion of the α-particle is marked by the arrow in Fig. 2.3 as shown below.

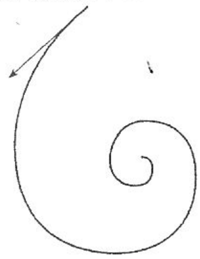


Fig. 2.3

20 α-particles, each of speed v and mass m, are travelling in a narrow beam in a vacuum, as shown in Fig. 4.1.

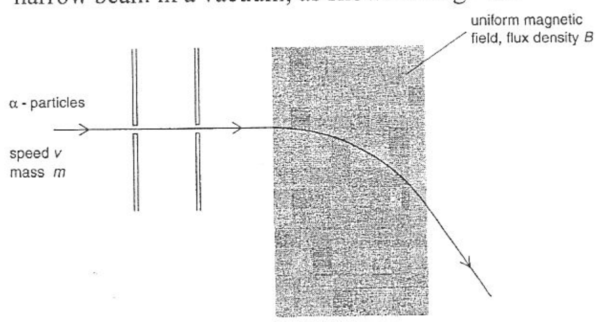


Fig. 4.1

The α-particles enter a region where there is a uniform magnetic field of flux density B. The α-particles in the magnetic field move along the arc of a circle of radius r.

(a) State the direction of the magnetic field.

.... [1]

(b) Show that, for an α-particle in this beam,

(i) the magnitude p of its momentum is given by the expression

$$p = 2Ber,$$

where e is the elementary charge. [2]

(ii) the kinetic energy E_K is given by the expression

$$E_K = \frac{2(Ber)^2}{m}.$$

(c) On Fig. 4.1, sketch the path of an α-particle that has a speed greater than v. [2]

N07/II/4

Solution

(a) Out of the plane of the paper.

Use Fleming's left hand rule. The direction of the current is the same as the direction of the velocity of the α-particles (since α-particles have a positive charge).

(b) (i) The magnetic force acting on the α-particle provides the centripetal force for its circular motion.

$$Bqv = mv^2/r$$

Where q is the charge of the α-particle ($q = 2e$).

$$B(2e)v = mv^2/r$$
$$2Be = mv/r$$
$$2Be = p/r$$
$$p = 2Ber$$

(ii) Kinetic energy

$$E_K = ½mv^2 = (mv)^2/2m = p^2/2m = (2Ber)^2/2m$$
$$= 4(Ber)^2 / 2m = 2(Ber)^2 / m$$

(c) Rearranging the equation in (b)(i), the radius of curvature

$$r = p / 2Be$$

An α-particle with a greater speed will have a greater momentum p and hence a greater radius of curvature r within the magnetic field.

Once outside the magnetic field, the resultant force acting on the α-particle is zero. Newton's first law of motion applies and the α-particle travels in a straight line with uniform speed.

Its path is shown in Fig. 4.2.

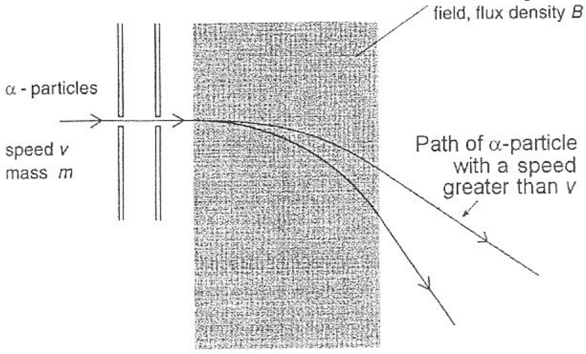


Fig. 4.2

Long Questions

21 (b) Fig. 5.1 shows a beam of electrons entering a magnetic field of magnetic flux density 8.4 mT. The electrons are travelling with velocity 3.4×10^7 m s^{-1} at right angles to the field.

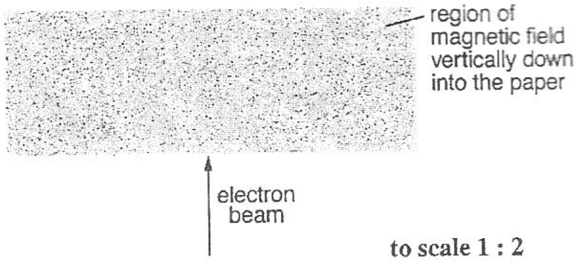


Fig. 5.1

Calculate, for the electrons in the field, the magnitude of

(i) the force on an individual electron, [2]

(ii) the acceleration of the electron. [1]

(c) (i) Calculate the radius of the path of the electrons within the magnetic field. [2]

(ii) Sketch the path travelled within and beyond the field. [2]

(d) Charged particles from the Sun, on approaching the Earth, may become trapped in the Earth's magnetic field near the poles, as shown in Fig. 5.2. This can cause the sky to glow. The phenomenon is called the *aurora borealis*.

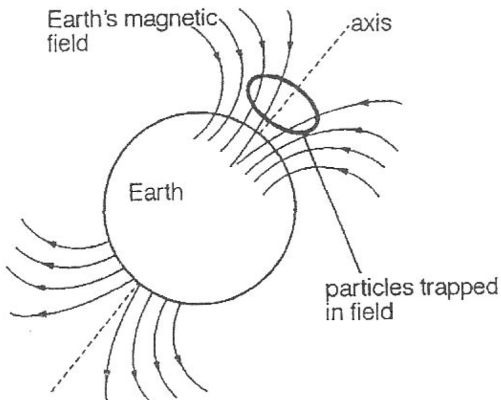


Fig. 5.2

Some of the charged particles travel in a circle of radius 50 km in a region where the magnetic flux density is 6.0×10^{-5} T.

(i) For a charged particle of charge to mass ratio e/m, deduce an expression for its speed v when travelling in a circle of radius r within a magnetic field of flux density B. [2]

(ii) Use your answer to (i) and the information about the path of the particles to show that the charged particles causing the aurora **cannot** be electrons. [2]

(iii) Suggest a particle that could cause the aurora and, for your suggested particle, calculate its speed as it travels in a circle. [3]

(e) The Earth has an electric field as well as a magnetic field. The electric field is shown in Fig. 5.3.

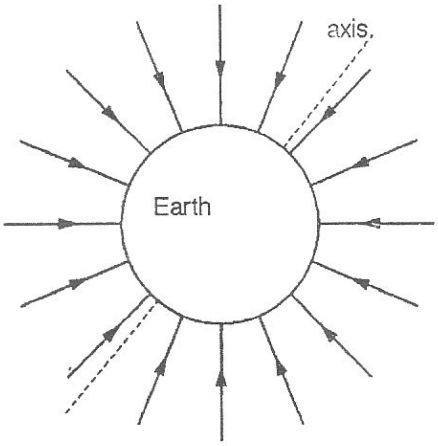


Fig. 5.3

Describe the effect that the electric field has on charged particles approaching the Earth from the Sun. [3]

N04/III/5 (part)

Solution

(b) (i) Magnetic force $F = Bqv$ where B is the magnetic flux density, q the charge and v its velocity.
$F = (8.4 \times 10^{-3}\ T)(1.6 \times 10^{-19}\ C)(3.4 \times 10^{7}\ m\ s^{-1})$
$= 4.570 \times 10^{-14}$
$= 4.6 \times 10^{-14}$ N.

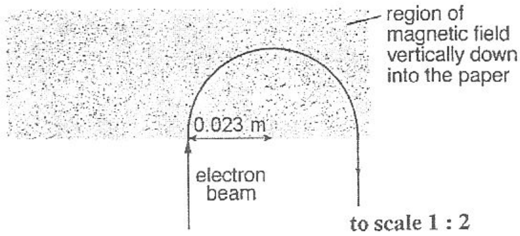
(ii) Acceleration
$a = F/m$
$= (4.570 \times 10^{-14}\ N) / (9.11 \times 10^{-31}\ kg)$
$= 5.016 \times 10^{16}$
$= 5.0 \times 10^{16}\ m\ s^{-2}$.

(c) (i) The centripetal acceleration $a = v^2/r$, hence

$$5.016 \times 10^{16}\ m\ s^{-2} = (3.4 \times 10^{7}\ m\ s^{-1})^2 / r$$

$$r = 0.02305 = 0.023\ m$$

(ii) The path of the electron is sketched in the figure below as the solid curve. The dotted part of the circle shows the circle with radius of curvature $r = 0.023$ m on which the path lies.
(Note: the scale is 1 : 2).



to scale 1 : 2

(d) (i) The magnetic force provides the centripetal force for the circular motion of the charged particle.

$$Bev = mv^2/r$$

$$v = Br(e/m)$$

(ii) Using the equation $v = Br(e/m)$ and substituting known values for the charge and mass of electrons,

$v = Br(e/m)$

$= (6.0 \times 10^{-5}\ T)(50{,}000\ m)(1.6 \times 10^{-19}\ C)$
$/(9.11 \times 10^{-31}\ kg)$

$= 5.269 \times 10^{11}$
$= 5.3 \times 10^{11}\ m\ s^{-1}$

It is not possible for a particle to travel faster than the speed of light in free space ($3.00 \times 10^{8}\ m\ s^{-1}$).
Hence the particle cannot be an electron.

(iii) The solar wind which causes the aurora borealis comprises a mixture of protons and electrons.

If the particle is not an electron then it is a proton.

$v = Br(e/m)$

$= (6.0 \times 10^{-5}\ T)(50{,}000\ m)(1.6 \times 10^{-19}\ C)$
$/(1.67 \times 10^{-27}\ kg)$

$= 2.874 \times 10^{8}$
$= 2.9 \times 10^{8}\ m\ s^{-1}$

(e) Positive charges experience an electric force in the same direction as the electric field. This causes them to accelerate towards the Earth.
Negative charges experience an electric force in the opposite direction to the electric field. This causes them to decelerate or, for slower particles, accelerate away from the Earth.
In the case of charged particles trapped in circular motion by the Earth's magnetic field, the electric field will cause their circular motion to turn into a helical motion either towards the Earth (proton) or away from the Earth (electron).

22 A student makes a solenoid with insulated copper wire. The solenoid has length 12.0 cm and the average length of one turn of wire on the solenoid is 8.8 cm, as shown in Fig. 2.1.

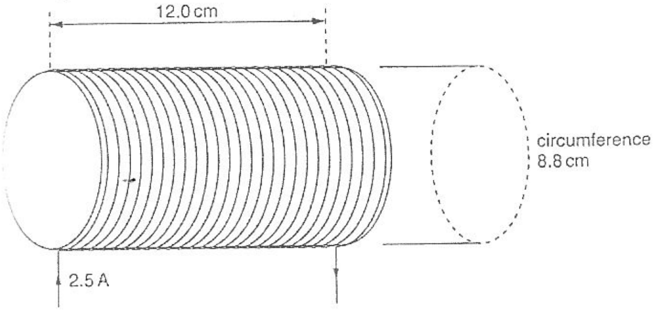


Fig. 2.1

The copper wire has a circular cross-section of diameter 0.60 mm. The resistivity of copper is $1.6 \times 10^{-8}\ \Omega$ m. It is found that the current in the solenoid is 2.5 A when the potential difference across its terminals is 4.5 V.

(a) (i) Calculate, for the solenoid, the resistance of the wire.
resistance = ... Ω [1]

(ii) Use your answer in (i) in order to calculate the total number of turns of wire on the solenoid.
number = ... [3]

(iii) Use your answer in (ii) to show that the number of turns per metre length of the solenoid is 3000. [1]

(b) The magnetic flux density B (in tesla) inside the solenoid and parallel to its axis is given by the expression

$$B = \mu_0 nI,$$

Where n is the number of turns per metre length of the solenoid, I is the current in the solenoid expressed in amperes and μ_0 is the permeability of free space.

(i) Define the tesla. [3]

(ii) Calculate the magnetic flux density in the solenoid.
flux density = ... T [1]

(c) The solenoid in (b) is in a vacuum.
An electron is injected into the magnetic field of the solenoid with a speed of 4.0×10^7 m s^{-1} at an angle of 30° to its axis, as shown in Fig. 2.2.

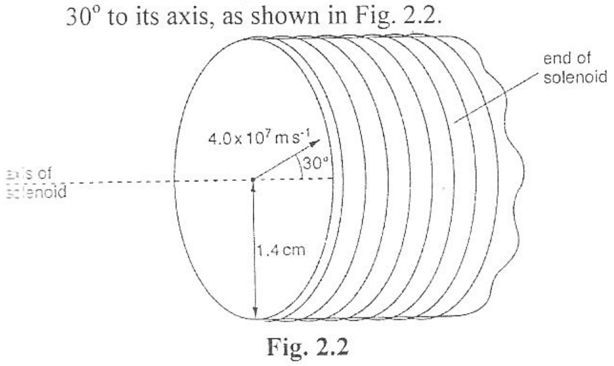


Fig. 2.2

Calculate the magnitude of the component of the electron's velocity

(i) along the axis of the solenoid,
magnitude = ... m s^{-1} [1]

(ii) normal to the axis of the solenoid.
magnitude = ... m s^{-1} [1]

(d) A particle of mass m and charge q is moving with speed v normal to a magnetic field of flux density B. Show that the particle will move in a circular path of radius r given by the expression

$$r = \frac{mv}{Bq}.$$

Explain your working. [3]

(e) The radius of the cross-section of the solenoid in (c) is 1.4 cm.
Use data from (c) and (d) to determine quantitatively whether the electron will travel down the length of the solenoid or will collide with its wall. [3]

N09/III/2

Solution

(a) (i) The resistance of the wire $R = V/I$ where V is the potential difference across its terminals and I the current in it.

$$R = (4.5\text{ V}) / (2.5\text{ A}) = 1.8\ \Omega$$

resistance = <u>1.8 Ω</u>

(ii) The resistance R of a wire is related to its resistivity ρ, length l and cross-sectional area A by the equation:

$$R = \rho l / A$$

Hence the total length of the copper wire

$$l = RA/\rho = (1.8)(\pi(0.30 \times 10^{-3})^2) / (1.6 \times 10^{-8}) = 31.8\text{ m}$$

The number of turns = 31.8 m / (0.088 m per turn) = 361.4 turns.

turns = <u>360</u>

(iii) The number of turns per metre length = 360 turns / 0.12 m = 3000 turns m^{-1}.

(b) (i) One tesla is the magnetic flux density of a magnetic field in which a magnetic force of one newton acts on a metre-long conductor carrying a current of one ampere, placed at right angles to the magnetic field.

This is based on the definition of magnetic flux density, which is the magnetic force per unit length, per unit current, on a current-carrying conductor at right angles to the magnetic field.

(ii) Using the formula given,

$B = \mu_0 nI = (4\pi \times 10^{-7})(3000)(2.5) = 9.425 \times 10^{-3}$ T

flux density = $\underline{9.4 \times 10^{-3} \text{ T}}$

(c) (i) Along the axis,

$v = 4.0 \times 10^7 \cos 30° = 3.464 \times 10^7$ m s^{-1}

magnitude = $\underline{3.5 \times 10^7 \text{ m s}^{-1}}$

(ii) Normal to the axis,

$v = 4.0 \times 10^7 \sin 30° = 2.0 \times 10^7$ m s^{-1}

magnitude = $\underline{2.0 \times 10^7 \text{ m s}^{-1}}$

(d) The magnetic force acting on the particle $F = Bqv$.

This force provides the centripetal force (mv^2/r) required to keep the particle in a circular path. Hence,

$$Bqv = mv^2/r$$

$$r = mv/Bq$$

(e) It will collide.

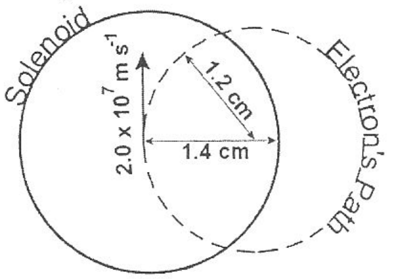
The normal component of the velocity determines the radius of the electron's helical path, which is:

$$r = mv/Bq$$

$= (9.11 \times 10^{-31} \times 2.0 \times 10^7)/(9.425 \times 10^{-3} \times 1.6 \times 10^{-19})$

$= 0.012$ m

The diameter of the electron's helical path (starting at the axis of the solenoid) is thus 0.024 m, greater than the radius of the solenoid (0.014 m), as shown below (cross-sectional view).



From the above figure, it can be worked out geometrically that the two circles intersect when the electron has travelled 0.015 m with respect to a plane perpendicular to the solenoid's axis. The time taken for this to happen is 0.015 m / (2.0×10^7 m s^{-1}) = 7.5×10^{-10} s.

In 7.5×10^{-10} s, the electron would have travelled only (3.5×10^7 m s^{-1})(7.5×10^{-10} s) = 0.026 m along the axis of the solenoid, still far from the end of the tunnel (which is 12.0 cm away) and hence is still within the solenoid.

Hence the electron will collide with the solenoid's wall.

23 Force-fields may be represented using lines that have direction.

(a) State
   (i) what is meant by a field of force, [1]
   (ii) how, using lines of force, changes in the strength of a force-field are represented. [2]

(b) Conventionally, arrows on field lines define the direction of a force acting on an object.
State the property of the object that experiences a force in this direction for
   (i) a gravitational field, [1]
   (ii) an electric field, [1]
   (iii) a magnetic field. [1]

(c) Explain how an electric field and a magnetic field may be used for the velocity selection of charged particles. You may draw a diagram if you wish. [4]

N10/III/3

Solution

(a) (i) A field of force is a region of space where a body experiences a force due to the presence of another of similar type.

(ii) The denser and closer the field lines are to one another, the greater the strength of the force-field.

(b) (i) Mass.

(ii) Positive charge.

(iii) Nothing. (The force is perpendicular to both the field and the direction of motion of the moving charge).

(c) The electric and magnetic cross fields (i.e. mutually perpendicular fields) act as a velocity selector which ensures that particles passing straight through them have a certain velocity.

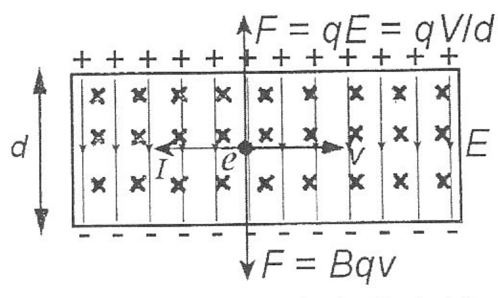
The electric field exerts a force qE on the charges while the magnetic field exerts a force Bqv in the opposite direction.

Only in the case of charged particles with a particular speed v will these two forces cancel out so that the particles pass straight through undeflected.

The condition is that $Bqv = qE$ or $v = E/B$.

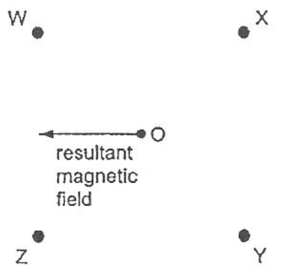
The speed v of particles to be let straight through into the mass spectrometer can be adjusted by varying the ratio E/B.

This is illustrated in the figure below for an electron, which carries a negative charge.



Magnetic Fields due to Currents

24 Parallel conductors WXYZ, carrying equal currents, pass vertically through the four corners of a square. In two conductors, the current is flowing into the page, and in the other two out of the page.



What are the directions of the current in order to produce a resultant magnetic field in the direction shown at O, the centre of the square?

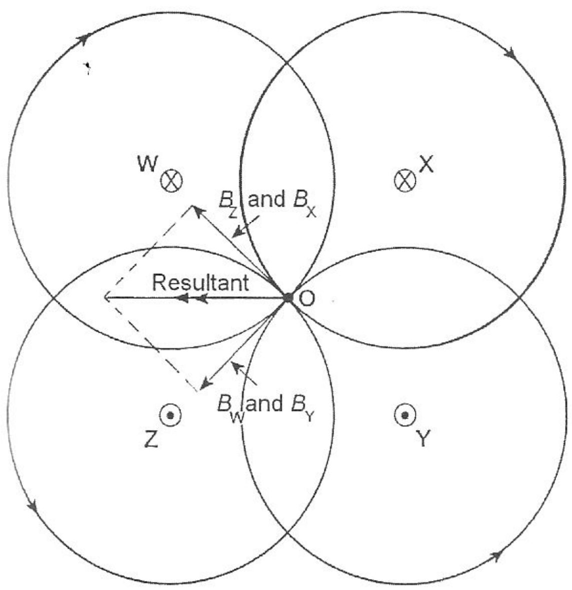
	into the page	out of the page
A	W and X	Y and Z
B	W and Y	X and Z
C	X and Z	W and Y
D	Y and Z	W and X

N05/I/25

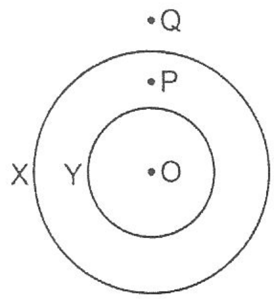
Solution Answer: **A**

The direction of the magnetic field produced by a long straight conductor can be found using the right-hand grip rule with the thumb pointing in the direction of the current and the fingers curling in the direction of the field.

The application of the right-hand grip rule using the combination in answer A is shown below, producing a resultant magnetic field towards the left.



25 X and Y are two coaxial circular coils lying on a table. O, P and Q are three points on the table.



Initially, there is a constant current in coil X and no current in coil Y.

A small current is now passed through coil Y, which decreases the magnitude of the magnetic flux density at O.

How does the magnitude of the flux density change at P and Q?

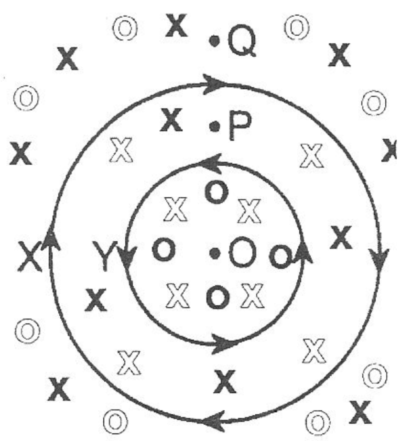
	P	Q
A	decreases	decreases
B	decreases	increases
C	increases	decreases
D	increases	increases

N09/I/31

Solution Answer: **C**

Assuming that the current in X is clockwise and that in Y anticlockwise (it could also be the other way round as long as they are in opposite directions).

The magnetic field due to X is shown below as hollowed crosses and dots; while that due to Y is shown as dark solid crosses and dots (using the right-hand grip rule version with the thumb as the current and fingers as the field).



It can be seen that in region P, the two fields are in the same direction, whereas in the region Q (as well as centre O), the two fields are in opposite directions.

Short Questions

26 *(b)* A loosely-coiled spring is suspended from a fixed point as illustrated in Fig. 7.1.

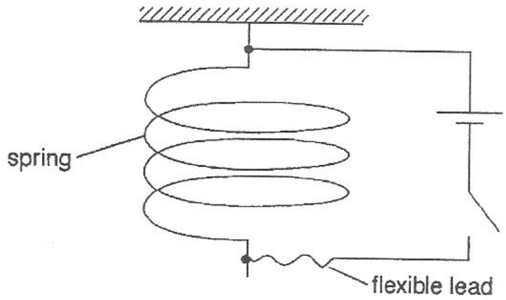


Fig. 7.1

Electrical connections are made to the ends of the spring. When a current is switched on in the spring, there is a small change in the length of the spring.

(i) On Fig. 7.1, mark the direction of the current in the individual turns of the spring.

(ii) On Fig. 7.2, draw the pattern of the magnetic field due to a straight current-carrying wire.

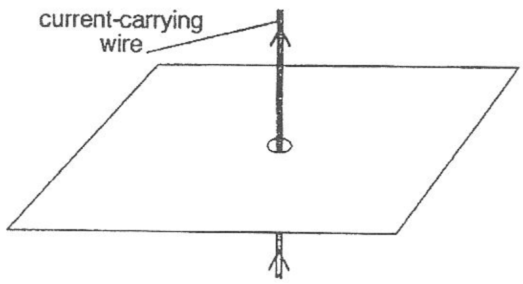


Fig. 7.2

(iii) Using your answers to (i) and (ii), explain why the spring changes in length when the current is switched on.

(iv) State whether the spring expands or is compressed.

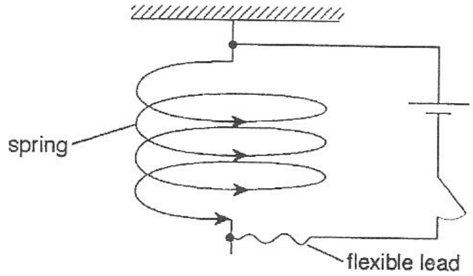
[8]

*(c) The free end of the spring in *(b)* is made to oscillate vertically. Explain why small fluctuations in the current in the spring are found to occur. [3]

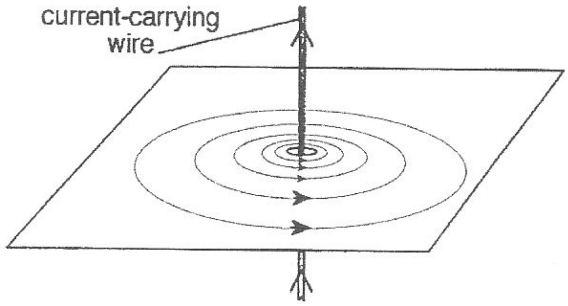
N02/II/7 (part)

Solution

(b) (i) The direction of the current is marked in the figure below.



(ii) The magnetic field pattern is shown in the figure below.



(iii) When the current is switched on, each coil of the spring produces a circular magnetic field that is intercepted by the currents in the neighbouring coils.

The neighbour above will experience a magnetic field radially outwards from the centre of the coil; while the neighbour below will experience a magnetic field radially towards the centre of the coil.

Using Fleming's left hand rule, each coil will experience a force which is perpendicular to the plane containing both the current and the magnetic field.

The force has to be vertical and in such a direction as to cause the neighbouring coils to be attracted towards one another.

(iv) Compressed.

(c) When the spring oscillates, each coil cuts the magnetic field produced by its neighbouring coil hence there is a change in the magnetic flux linking all the coils.

According to Faraday's law of electromagnetic induction, an e.m.f. will be induced in the coils the magnitude of which is directly proportional to the rate of change of magnetic flux linkage.

According to Lenz's law, the polarity of this e.m.f. will be such as to oppose the change in flux linkage.

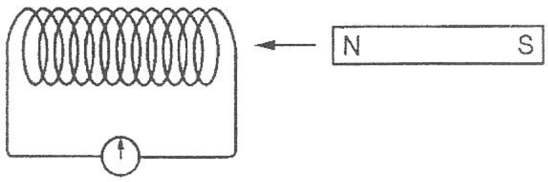
This induced e.m.f. opposes the battery's e.m.f. and causes slight reductions in the net e.m.f. around the circuit, which in turn reduces the current delivered by the battery.

Since the velocity of the oscillating coil fluctuates sinusoidally in simple harmonic motion, so too does this induced e.m.f. which opposes the battery's e.m.f.

Thus there are small sinusoidal fluctuations in the current in the spring.

TOPIC 16 Electromagnetic Induction

1 The North pole of a bar magnet is pushed into the end of a coil of wire. The maximum movement of the meter needle is 10 units to the left.



The South Pole of the magnet is then pushed into the other end of the coil at twice the speed.

What is the maximum movement of the meter needle?

A less than 10 units to the left
B less than 10 units to the right
C more than 10 units to the left
D more than 10 units to the right N02/I/24

Solution Answer: **C**

Faraday's law of electromagnetic induction states that the electromotive force (e.m.f.) E induced in a circuit is directly proportional to the rate of change of its magnetic flux linkage.
When the magnet is pushed in at twice the speed, the coil will experience twice the rate of change of magnetic flux linkage and hence twice the e.m.f.
In both cases, the polarity of the induced e.m.f. is the same because in both cases a north pole will be induced at the right end of the coil while a south pole at the left end (apply Lenz's law and the right-hand grip rule).

2 Which of the following correctly relates the units for magnetic flux and magnetic flux density?

A $1\ \text{Wb} = 1\ \text{T m}^{-3}$
B $1\ \text{Wb} = 1\ \text{T m}^{-2}$
C $1\ \text{Wb} = 1\ \text{T m}^{2}$
D $1\ \text{Wb} = 1\ \text{T m}^{3}$ N02/I/26

Solution Answer: **C**

The magnetic flux Φ through a surface is the product of its area A and the magnetic flux density B perpendicular to the surface. Hence $\Phi = BA$.
The unit for Φ is Wb, the unit for B is T and the unit for A is m^2. Hence 1 Wb = 1 T m^2.

3 P, Q, R and S are SI units.

P volt second^{-1}
Q volt second
R tesla metre^{-2}
S tesla metre^{2}

Which two are equivalent to the weber?

A P and R
B Q and S
C P and S
D Q and R N03/I/23

Solution Answer: **B**

The magnetic flux Φ through a surface is the product of its area A and the magnetic flux density B perpendicular to the surface. Hence $\Phi = BA$. The unit for Φ is Wb, the unit for B is T and the unit for A is m^2. Hence 1 Wb = 1 T m^2.
Also, Faraday's law of electromagnetic induction states that the electromotive force (e.m.f.) E induced in a circuit is directly proportional to the rate of change of its magnetic flux linkage $N\Phi$.
($E = -d(N\Phi)/dt$ where the negative sign complies with Lenz's law that the e.m.f. opposes the change).
The unit for E is V, the unit for Φ is Wb and the unit for t is s (while N is dimensionless). Hence 1V = 1 Wb s^{-1} and rearranging, 1 Wb = 1 V s.

4 A flat circular coil of diameter 30 mm has 500 turns and is situated so that the plane of the coil is perpendicular to a uniform magnetic field of flux density 20 mT. The flux density is reduced to zero and then increased to 20 mT in the opposite direction at a constant rate. The time taken for the whole operation is 60 ms.

What is the average value of the e.m.f. induced in the coil?

A 0
B 0.12 V
C 0.24 V
D 0.94 V N04/I/25

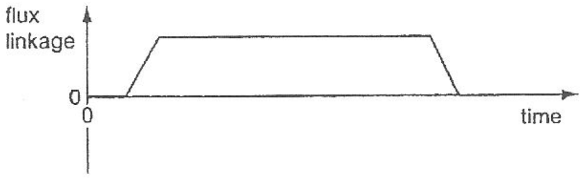
Solution Answer: **C**

E.m.f.
$|E| = d(N\Phi)/dt$
$= \Delta NBA/t$

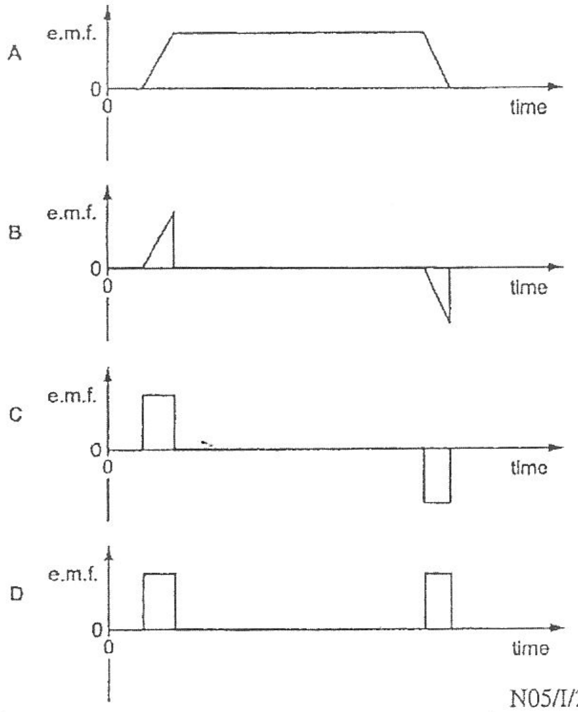
$= (500)(20 \times 10^{-3}\ \text{T} - (-20 \times 10^{-3}\ \text{T})(\pi\ (15 \times 10^{-3}\ \text{m})^2)$
$/(60 \times 10^{-3}\ \text{s})$

$= 0.2356$
$= 0.24$ V.

5 The magnetic flux linkage through a coil varies with time as shown.



Which graph shows the variation with time of the e.m.f. generated by the coil?



N05/I/26

Solution Answer: **C**

According to Faraday's law of electromagnetic induction, the e.m.f. E generated by a circuit is directly proportional to the rate of change of its magnetic flux linkage $N\Phi$.

$E \alpha -d(N\Phi)/dt$ or $E = -d(N\Phi)/dt$ if SI units are used, with the negative sign taking into account Lenz's law.

Therefore, it can be seen that the e.m.f. generated is the negative gradient of the flux linkage-time graph.

(Although answer C shows the e.m.f. as the positive gradient of the graph, this is acceptable since whether the measured e.m.f. is positive or negative depends on how the oscilloscope probes are connected to the coil. Just interchange the probe with the ground clip and the waveform will be reversed).

6 The e.m.f. induced in a coil by a changing magnetic flux is equal to the rate of change of flux with time.

Which is a unit for magnetic flux?

A $kg\ m^2\ s^{-2}\ A^{-1}$

B $kg\ m^2\ s^{-2}\ A$

C $kg\ m\ s^2\ A^{-1}$

D $ms^{-2}\ A^{-1}$

N06/I/1

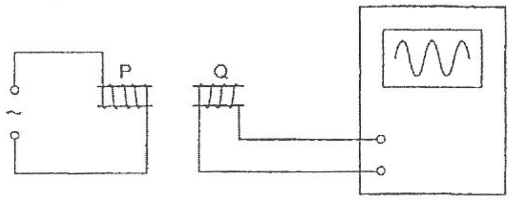
Solution Answer: **A**

$E = d\Phi/dt$ where E is e.m.f., Φ magnetic flux and t the time.

$$[\Phi] = [E][t] = V\ s = (J\ C^{-1})(s) = (N\ m)(C^{-1})(s)$$

$$= (kg\ m\ s^{-2})(m)(A\ s)^{-1}(s) = kg\ m^2\ s^{-2}\ A^{-1}.$$

7 A coil P is connected to a 50 Hz alternating supply of constant peak voltage. Coil P lies close to a separate coil Q which is connected to the Y-input terminals of a cathode-ray oscilloscope. A sinusoidal trace appears on the screen of the oscilloscope.



What would be the effect on the trace of linking the coils by a soft-iron core?

	height of trace	number of cycles on screen
A	increases	increases
B	increases	stays the same
C	stays the same	increases
D	stays the same	stays the same

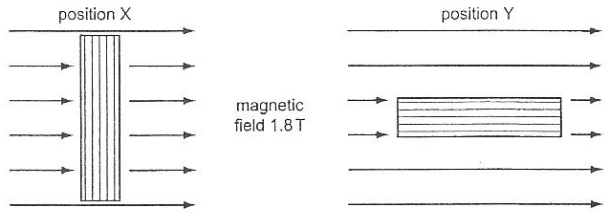
N06/I/24

Solution Answer: **B**

The soft-iron core concentrates the magnetic flux lines and increases the rate of change of magnetic flux linking coil Q. Hence the induced e.m.f. increases (Faraday's law of electromagnetic induction).

However, the increase in magnetic flux linkage does not alter the frequency of the induced e.m.f. hence the number of cycles on the screen remains unchanged.

8 A circular coil of diameter 0.020 m has 3000 turns. It is rotated in a magnetic field of 1.8 T from position X to position Y in 0.060 s.



What is the average e.m.f. induced in the coil during the rotation?

A 28 V **B** 36 V **C** 113 V **D** 1800 V

N07/I/31

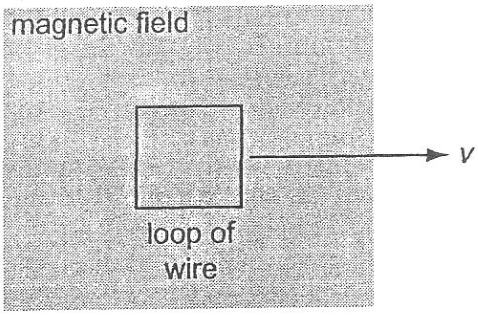
Solution Answer: **A**

Apply Faraday's law of electromagnetic induction,

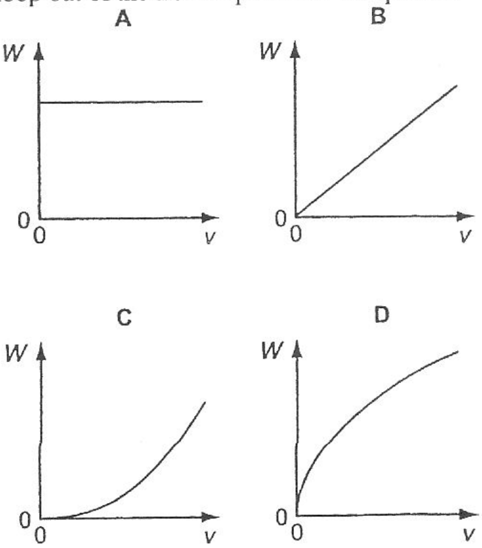
$$|E| = dN\Phi/dt = NBA/t$$

$$= (3000)(1.8\ T)(\pi\ (0.010\ m)^2)\ /\ (0.060\ s) = 28\ V$$

9 A square loop of wire is placed in a region of uniform magnetic field. The direction of the field is perpendicular to the plane of the loop of wire.



The loop is pulled out of the field at a uniform speed v. During this operation, the loop remains in the same plane. Which graph shows how the total work W done in pulling the loop out of the field depends on the speed v?



N08/I/10

Solution Answer: **B**

Let the dimension of the square loop be L by L.
Work done = force x distance moved = FL.
The distance moved L is constant and equal to the length of the side of the square loop parallel to the velocity.

Force $F = BIL = B(E/R)L = B(BLv/R)L = B^2L^2v/R$

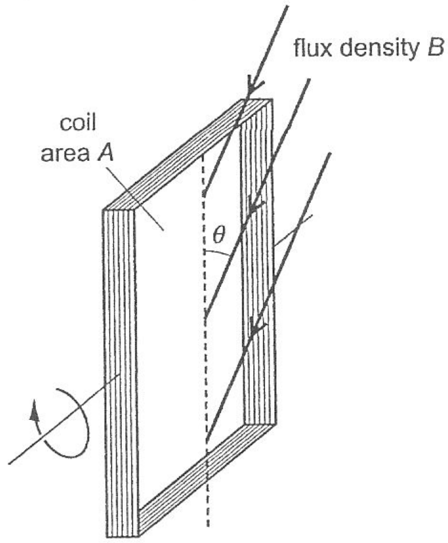
B is magnetic flux density, E the induced e.m.f., R the resistance of the wire loop, I the induced current and L the length of one side of the wire loop.

Hence work done $W = (B^2L^2v/R)(L) = B^2L^3v/R$

Work done W is directly proportional to speed v.

10 A coil has area A and n turns.

A uniform magnetic field of flux density B acts at an angle θ to the plane of the coil, as shown.



What is the change in magnetic flux linkage when the coil rotates so that the angle θ is reduced to zero?

A $BAn\cos\theta$ **B** $BAn\sin\theta$ **C** $2BAn\cos\theta$ **D** $2BAn\sin\theta$

N08/I/32

Solution Answer: **B**

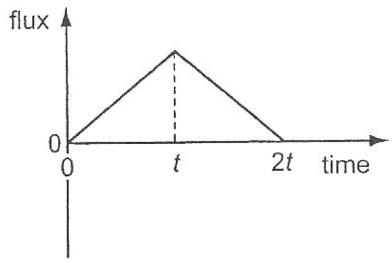
Magnetic flux Φ through a surface is the product of the component of the magnetic flux density $B\sin\theta$ perpendicular to the surface, and the area A of the surface, i.e. $\Phi = BA\sin\theta$ where θ is the angle between the magnetic flux density and the plane of the area.

Magnetic flux linkage = number of turns of the coil x magnetic flux = $nBA\sin\theta$. The change in magnetic flux linkage is hence:

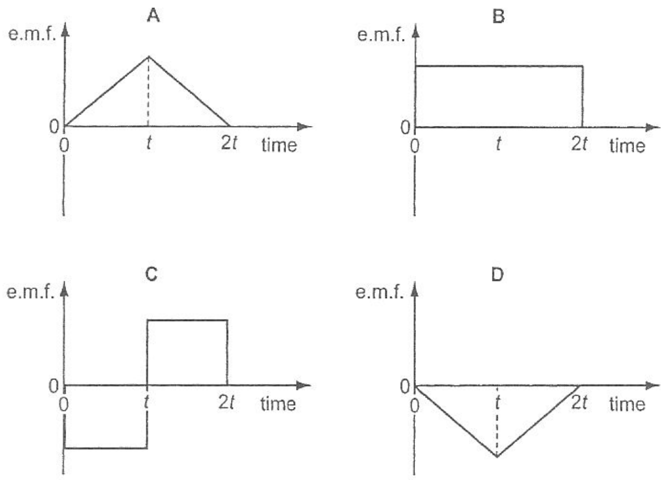
$$\Delta n\Phi = nBA\sin\theta - nBA\sin 0 = nBA\sin\theta$$

Note: If the angle φ is defined as that between the magnetic flux density and the normal to the area of the coil, then the corresponding formula will instead be $nBA\cos\varphi$.

11 The graph shows the variation with time of the magnetic flux linking a coil.



Which graph shows the variation with time of the e.m.f. induced in the coil?



N08/I/33

Solution Answer: **C**

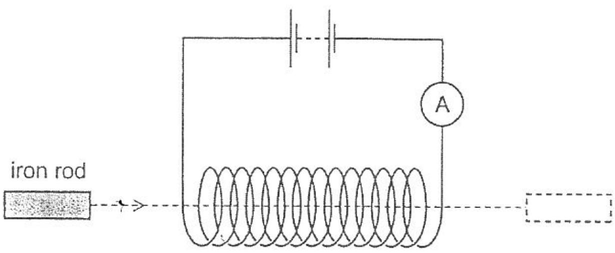
According to Faraday's law of electromagnetic induction, the e.m.f. induced in a circuit is directly proportional to the rate of change of magnetic flux linking it.

$$E = -d\Phi/dt$$

Therefore, the e.m.f. induced is the negative gradient of the magnetic flux linkage graph.

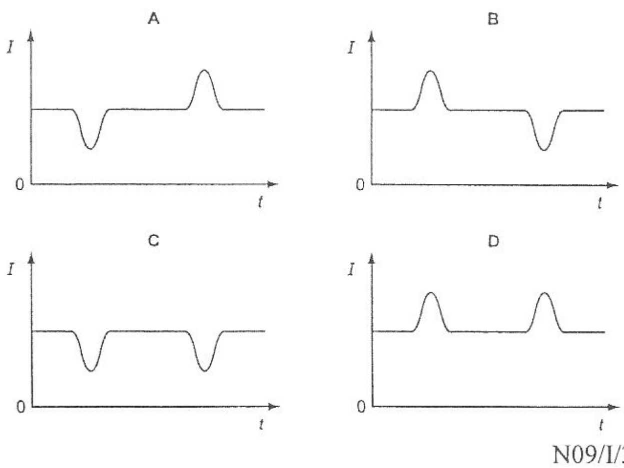
(The negative sign is from Lenz's law, to demonstrate that the induced e.m.f. opposes the change in magnetic flux linkage).

12 A solenoid with a large number of turns of wire is connected in series with an ammeter and a battery.



An iron rod is passed through the solenoid at constant speed.

Which graph shows the variation with time t of the reading I of the ammeter?



N09/I/32

Solution Answer: **A**

Iron has the ability to concentrate magnetic field, thereby causing the magnetic flux linkage in the solenoid to increase when the rod enters, and decrease when it leaves.

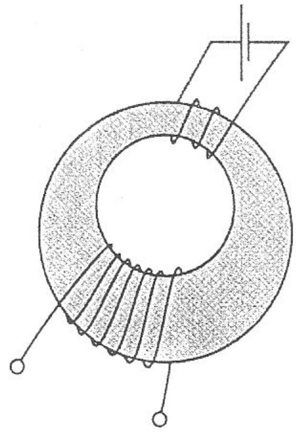
According to Faraday's law of electromagnetic induction, there will be an e.m.f. induced in the solenoid when there is a rate of change in its magnetic flux linkage.

According to Lenz's law, the induced e.m.f. opposes the change in magnetic flux linkage.

Therefore, when the iron rod is entering the solenoid, causing its magnetic flux linkage to increase, the induced e.m.f. will be of an opposite polarity, to oppose this increase in magnetic flux linkage.

This induced e.m.f. thus opposes the battery's e.m.f., causing a temporary drop in the current delivered by the battery when the iron rod enters the solenoid, and vice versa when it leaves.

13 A soft iron ring of variable cross-section has two coils wound around it. One coil has 7 turns. The other, of 3 turns, is connected to a d.c. supply.



Which quantity is the same inside each coil?

- **A** magnetic field
- **B** magnetic flux
- **C** magnetic flux density
- **D** magnetic flux linkage

N09/I/33

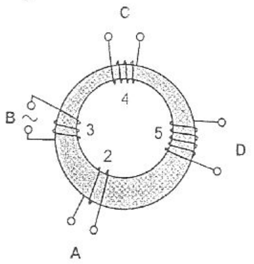
Solution Answer: **B**

The 3-turn coil produces a magnetic field of flux density B which, when multiplied by the cross sectional area A of the coil, is the magnetic flux Φ in the core.

Assuming no flux leakage, the entire flux Φ is trapped within the core of the iron ring, and is also the flux through the 7-turn coil.

Magnetic flux density (magnetic flux per unit area) is higher at the part of the core with smaller cross-sectional area; while magnetic flux linkage (number of turns multiplied by magnetic flux) is higher at the coil with greater number of turns.

14 A soft-iron ring of variable cross-section has four coils wound round it at the positions shown. The coils have 2, 3, 4 and 5 turns. The 3-turn coil is connected to an a.c. supply.
In which coil does the magnitude of the magnetic flux density have the greatest variation?



N10/I/32

Solution Answer: **C**

Magnetic flux density $B = \Phi/A$ where Φ is the magnetic flux in the core which is the same throughout, and A the cross-sectional area of the core. Since section C has the smallest cross-sectional area, it has the greatest magnetic flux density.

Short Questions

15 Two coils, A and B, are placed one on top of the other, as shown in Fig. 6.1.

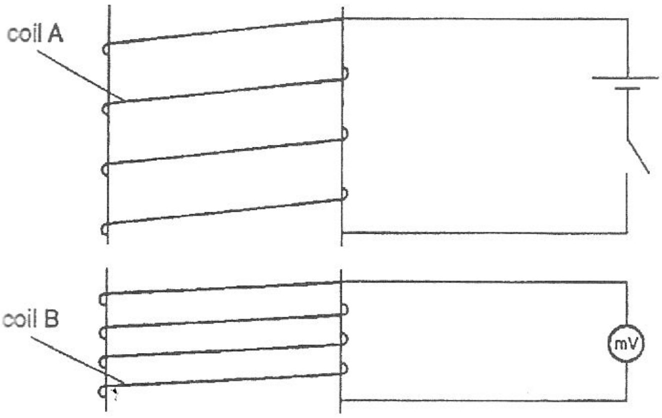


Fig. 6.1

Coil A is connected in series with a battery and a switch. A millivoltmeter is connected across the terminals of coil B.

(a) On Fig. 6.1, draw an arrow inside coil A to show the direction of the magnetic field in coil A when the current is switched on. [1]

(b) State Faraday's law of electromagnetic induction and hence explain why, when the current in coil A is switched on, the millivoltmeter indicates an induced e.m.f. for a short period of time. [4]

(c) (i) On Fig. 6.1, draw an arrow inside coil B to show the direction of the magnetic field due to the induced current in coil B. [1]

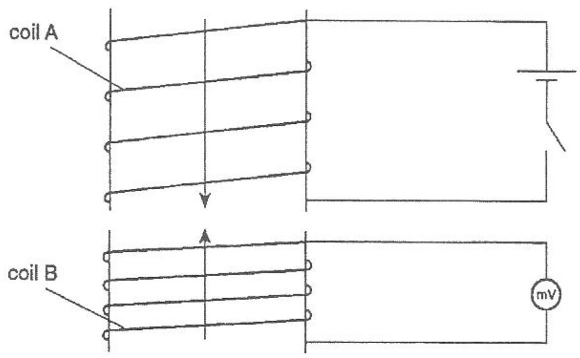
(ii) State Lenz's law and hence explain the direction chosen in (i). [2]

(d) The current in coil A remains constant. Explain why the millivoltmeter indicates an induced e.m.f. as a ferrous core is lowered into coil A. [2]

N04/II/6

Solution

(a) The direction of the magnetic field is shown in the figure below.



(b) Faraday's law of electromagnetic induction states that the electromotive force (e.m.f.) induced in a conductor is directly proportional to the rate of change of its magnetic flux linkage.

When the current in coil A is switched on, a magnetic field is produced by A. This results in a change in the magnetic flux linking coil B, and an e.m.f. is induced in B.

Once the magnetic flux produced by A has reached its steady maximum value, the rate of change of flux linkage in B is zero, and induced e.m.f. then falls to zero.

(c) (i) The direction of the magnetic field due to the induced current in B is shown in the figure above.

(ii) Lenz's law states that the e.m.f. induced in an electric circuit has a polarity such that the current it drives around a closed circuit produces a magnetic field which opposes the change in magnetic flux causing it.

The magnetic field produced by the induced current in coil B thus acts in a direction to oppose the increase in magnetic flux linkage generated by coil A.

Since the magnetic flux density produced by A is increasing in the downward direction, the magnetic field produced by the induced current in B has to act in the upward direction.

(d) Ferrous core has a higher permeability than air hence when lowered into coil A, it concentrates and increases the magnetic flux density produced by coil A.

This results in an increase (a change) in the magnetic flux linking coil B, and according to Faraday's law of electromagnetic induction, an e.m.f. is induced in B.

16 (a) A simple generator consists of a coil with a large number of turns that rotates at a constant rate in a uniform magnetic field, as shown in Fig. 2.1.

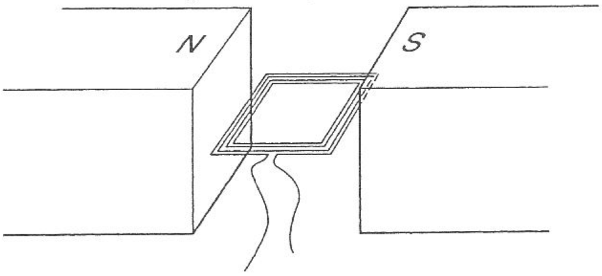


Fig. 2.1

(i) Explain why an e.m.f. is generated when the coil rotates. [2]

(ii) State two factors that affect the magnitude of the maximum e.m.f.
1. ....
2. ....
[2]

(iii) Explain briefly, in words, why the e.m.f. is sinusoidal. [2]

(b) The output from a similar generator is connected to the input of a transformer. The transformer has 30 turns on its primary coil and 600 turns on its secondary coil. The transformer may be considered to be ideal. The input e.m.f. is 72 V r.m.s. The output from the transformer is connected to a resistor of resistance 160 Ω, as shown in Fig. 2.2.

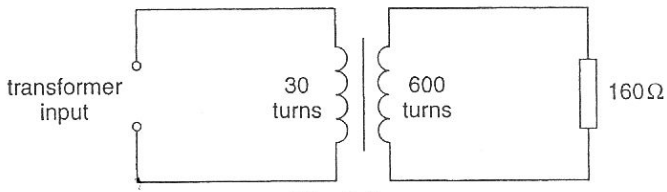


Fig. 2.2

Deduce
(i) the peak input e.m.f.,
peak input e.m.f. = ... V [1]
(ii) the r.m.s. value of the p.d. across the resistor,
r.m.s. p.d. = ... V [1]
(iii) the r.m.s. value of the current in the resistor,
r.m.s. current = ... A [1]
(iv) the mean power dissipated in the resistor,
power = ... W [1]
(v) the r.m.s. value of the current from the generator,
current = ... A [2]
N08/III/2

Solution

(a) (i) When the coil rotates, there is a rate of change of magnetic flux linking the circuit. According to Faraday's law of electromagnetic induction, an e.m.f. is induced in a circuit when there is a change in its magnetic flux linkage. The magnitude of the induced e.m.f. is directly proportional to the rate of change of magnetic flux linkage.

(ii) 1. The magnetic flux density within which the coil is located.

2. The frequency of rotation of the coil.

(iii) Magnetic flux is the product of magnetic flux density perpendicular to the surface, $B \sin \theta$, and the area of the coil, where θ is the angle between the plane of the coil and the field (Fig. 2.3).

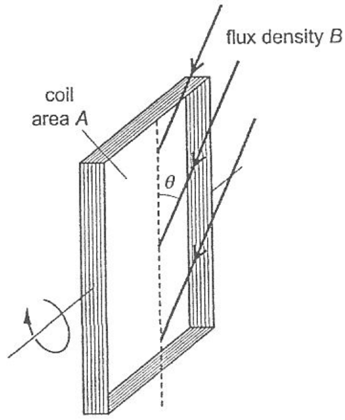


Fig. 2.3

As the coil rotates at a constant angular frequency ω, the angle $\theta = \omega t$ changes at constant rate. Hence the magnetic flux density perpendicular to the coil ($B \sin \theta$ or $B \sin \omega t$) varies sinusoidally, and the rate at which it is changing (the e.m.f.) is also sinusoidal.

Note: If the angle φ is defined as that between the magnetic flux density and the normal to the area of the coil, then the magnetic flux density perpendicular to the surface will be $B \cos \varphi$ or $B \cos \omega t$, still sinusoidal.

(b) (i) For sinusoidally varying waveforms, the peak e.m.f. $E_0 = \sqrt{2} \times E_{r.m.s.} = \sqrt{2} \times 72\ V = 102\ V$.

(ii) For a transformer, the voltage ratio between the secondary and primary is equal to its turns ratio.

$$V_S / V_P = N_S / N_P$$

$$V_S / 72\ V = 600 / 30$$

$$V_S = 1440\ V$$

(iii) $I_{r.m.s.} = V_{r.m.s.} / R = 1440\ V / 160\ \Omega = 9.0\ A$.

(iv) Mean Power

$$P_{mean} = I_{r.m.s.} V_{r.m.s.} = (9.0\ A)(1440\ V) = 1.3 \times 10^4\ W.$$

(v) Since transformer is ideal, input power = output power. Hence at the input,

$$I_{r.m.s.} = P_{mean} / V_{r.m.s.} = 1.3 \times 10^4\ W / 72\ V = 180\ A$$

17 Fig. 6.1 shows a rectangular coil.

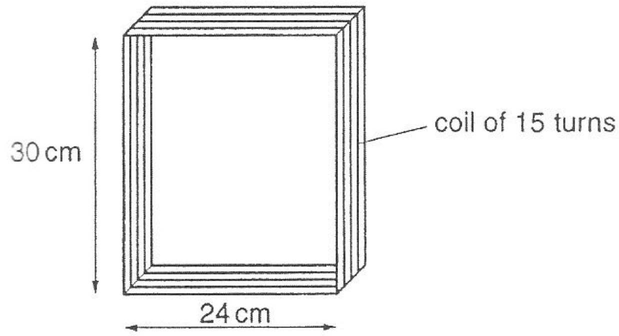


Fig. 6.1

(a) The coil has dimension 30 cm by 24 cm and has 15 turns.
A uniform magnetic field of flux density 0.018 T is at right-angles to the plane of the coil.
Show that the magnetic flux linkage of the coil is 0.019 Wb turns. [1]

(b) The uniform magnetic field in (a) is kept constant for 2.0 s and then reduced to zero over a time of 4.0 s, as shown in Fig. 6.2.

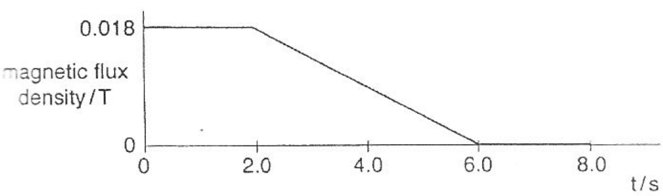


Fig. 6.2

(i) Explain why an e.m.f. is induced in the coil.
....
.... [1]

(ii) Calculate the magnitude of the induced e.m.f.
e.m.f. = ... V [2]

(iii) On Fig. 6.3, sketch a graph to show the variation with time of the e.m.f. E induced in the coil.

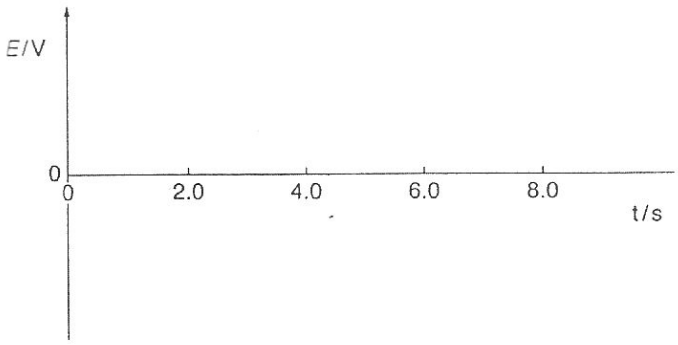


Fig. 6.2

N09/II/6

Solution

(a) Magnetic flux linkage of the coil = number of turns x magnetic flux density x area of the coil perpendicular to the magnetic field.

$= 15$ turns x (0.018 T) x (0.30 x 0.24) m^2

$= 0.01944 = 0.019$ Wb turns

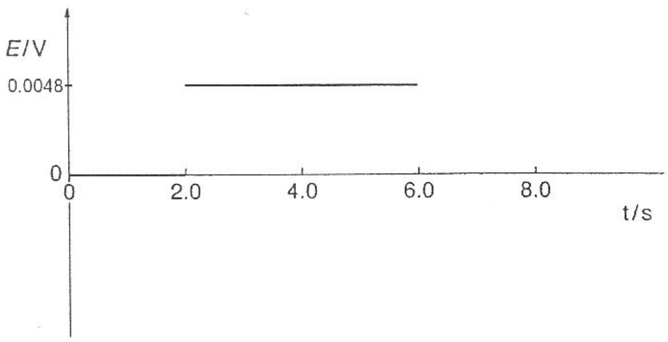
(b) (i) The change in magnetic flux density between 2.0 s and 6.0 s results in a rate of change of magnetic flux linkage. According to Faraday's law of electromagnetic induction, this will result in an induced e.m.f. which is directly proportional to the rate of change of magnetic flux linkage.

(ii) Based on the values calculated in (a),

$E = (0.019 \text{ Wb turns}) / 4.0 \text{ s} = 4.8 \times 10^{-3}$ V

e.m.f. $= 4.8 \times 10^{-3}$ V

(iii) In the first 2.0 s, there is no e.m.f. because there is no rate of change of magnetic flux linkage.



18 (a) Define magnetic flux. [1]

(b) A coil with 500 turns is placed in a uniform magnetic field of flux density 5.0×10^{-2} T. The area of the coil perpendicular to the field is 2.5×10^{-2} m^2, as shown in Fig. 4.1.

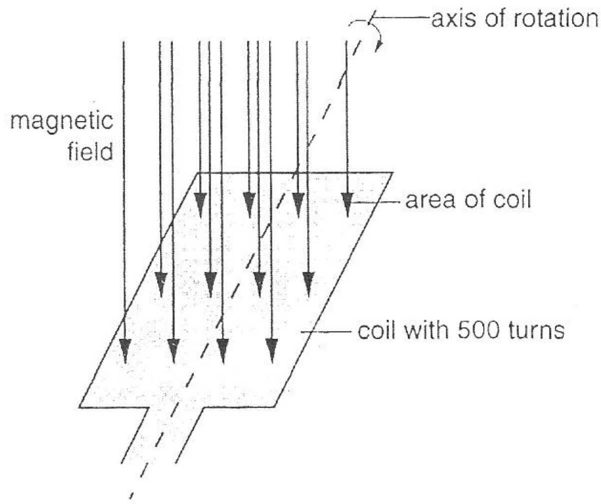


Fig. 4.1

Calculate the magnetic flux linkage of the coil. Give an appropriate unit.

magnetic flux linkage = ... [2]

(c) The coil in (b) is rotated around the axis shown in Fig. 4.1. The flux linkage Φ of the coil varies with time t, as shown in Fig. 4.2.

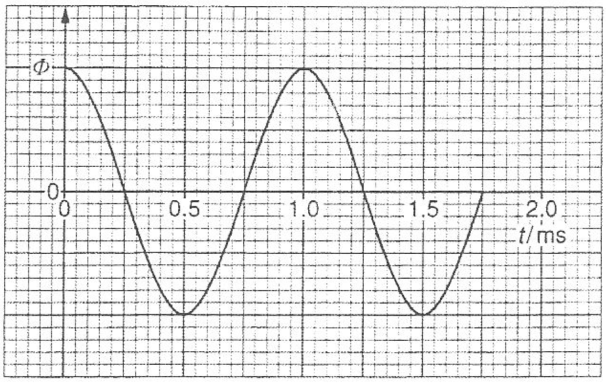


Fig. 4.2

(i) Explain why the flux linkage changes as the coil is rotated. [1]

(ii) Calculate the average e.m.f. induced across the coil when it rotates through the first quarter of a revolution.

e.m.f. = ... V [2]

(iii) Explain how the maximum e.m.f. induced across the coil is determined from Fig. 4.2. [1]

N10/II/4

Solution

(a) The magnetic flux through a surface is the product of the magnetic flux density perpendicular to the surface and the area of the surface.

(b) Magnetic flux linkage
$= NBA = (500)(5.0 \times 10^{-2}\ T)(2.5 \times 10^{-2}\ m^2)$
$= 0.625$ weber-turns (Wb-turns)

(c) (i) As the coil rotates, the magnetic flux density (B) is no longer perpendicular to the plane of the surface.

The component of the magnetic flux density perpendicular to the surface ($B\cos\theta$ where θ is the angle between the magnetic field and the normal to the plane of the surface) changes hence the magnetic flux linkage changes.

(ii) Average e.m.f.
= (change in magnetic flux linkage)/time
$= \Delta\Phi/t$
$= (0.625 - 0)/(0.25 \times 10^{-3})$
$= 2500$ V

(iii) The maximum e.m.f. is the gradient at the steepest parts of the graph (e.g. at t = 0.25 ms, 0.75 ms, 1.25 ms etc.). According to Faraday's law of electromagnetic induction, the induced e.m.f. (E) is directly proportional to the rate of change of magnetic flux linkage, i.e. $E \propto d\Phi/dt$. Hence when the rate of magnetic flux linkage is the greatest, the e.m.f. is the highest.

Long Questions

19 *(a)* Define the following quantities, and state the SI unit for each.

(i) *magnetic flux density*

(ii) *magnetic flux* [4]

(b) Fig. 5.1 shows the position of a wire in the radial magnetic field of an electric motor where the magnetic flux density is 0.20 T.

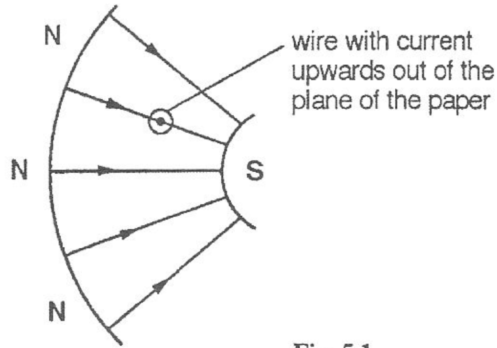


Fig. 5.1

The wire, which is of length 0.060 m, carries a current of 4.5 A out of the plane of the paper.

(i) Draw a diagram to show the shape of the combined magnetic field of the current in the wire and the motor's magnet. [3]

(ii) State the direction in which the force on the wire acts. [1]

(iii) Calculate the force on the wire. [3]

(c) When the current in the wire is switched off, the motor acts as an electrical generator when the wire is moved to cut magnetic flux.

(i) Name this process of electrical generation. [1]

(ii) Calculate the speed of the wire required to give an instantaneous e.m.f. of 0.12 V. [4]

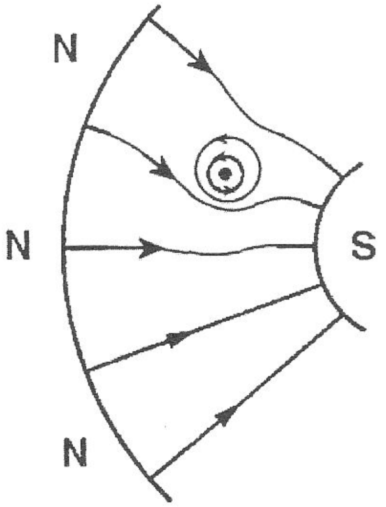
(d) Describe the energy changes that take place in *(b)* and *(c)*. [4] N03/III/5

Solution

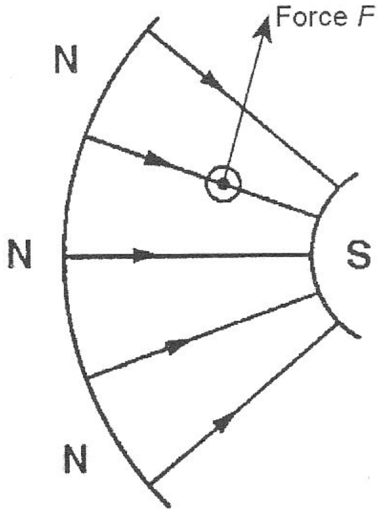
(a) (i) *Magnetic flux density B* is defined as the magnetic force per unit current per unit length on a wire at right angles to the magnetic field. Its unit is the tesla (T).

(ii) *Magnetic flux Φ* through a surface is the product of its area A and the magnetic flux density B perpendicular to the surface. Its unit is the weber (Wb).

(b) (i) A diagram of the combined magnetic field is shown in the figure below.



(ii) The force acts in a direction perpendicular to the plane containing both the current and the radial magnetic field, in the upward direction slightly tilted to the right, as shown in the figure below (Fleming's left-hand rule).



(iii) Force
$F = B I l \sin 90°$
$= (0.20\ T)(4.5\ A)(0.060\ m)$
$= 0.054\ N.$

(c) (i) Electromagnetic induction.

(ii) E.m.f
$E = B l v$ hence
$v = E / Bl$
$= (0.12\ V)/(0.20\ T \times 0.060\ m)$
$= 10\ m\ s^{-1}$.

(d) In (b), electrical energy is converted into mechanical energy.

In (c), mechanical energy is converted into electrical energy.

20 *(a)* Define *magnetic flux* and state the SI unit in which it is measured. [2]

(b) In a power station generator, a large rectangular coil is rotating at 50 revolutions per second in a magnetic field of magnetic flux density 0.29 T. The coil, as shown in Fig. 6.1, has 38 turns each 2.0 m long and 1.2 m wide.

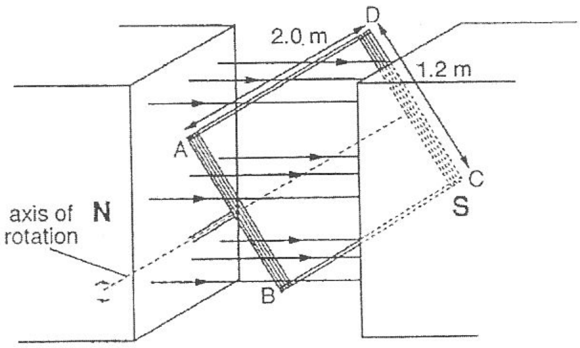


Fig. 6.1

The maximum output e.m.f. of the coil occurs when it moves near the plane of the magnetic field as shown in Fig. 6.2.

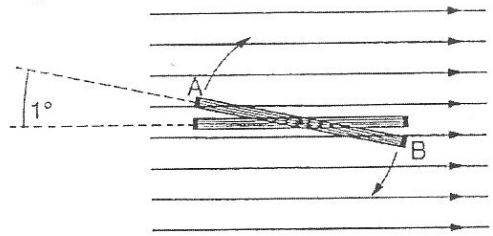


Fig. 6.2

For the coil moving through an angle of 1.0° near the plane of the magnetic field, calculate

(i) the time taken for it to rotate 1.0°, [2]

(ii) the flux cut by one turn of the coil in this time, [3]

(iii) the e.m.f. generated by one turn of the coil in this time, [2]

(iv) the e.m.f. generated by all 38 turns of the coil in this time. [1]

(c) (i) The value obtained in *(b)*(iv) is the peak value of the sinusoidal output of the coil. Calculate the r.m.s. value of the output of the coil. [1]

(ii) State the direction of the current induced in side AD as a result of this e.m.f. Explain how you deduced your answer. [2]

(d) Explain why, from both the scientific and economic points of view, the transmission of electrical energy is carried out using alternating currents and high voltages. [5]

(e) Draw a circuit diagram of a bridge rectifier being used to give full-wave rectification of an alternating current. [2]

N05/III/6

Solution

(a) (i) The magnetic flux Φ through a surface is defined as the product of its area A and the magnetic flux density B perpendicular to the surface.
The SI unit for magnetic flux is the weber (Wb).

(b) (i) The period of rotation
$T = 1/f = 1/50$
$= 0.020$ s (for one complete revolution of 360°).
The time to rotate 1° is
$t = (1/360) \times 0.020 \text{ s} = 5.556 \times 10^{-5} = 5.6 \times 10^{-5}$ s.

(ii) The flux $\Phi = B\,A \cos\theta$ where B is the magnetic flux density, A the area of the coil and θ the angle between the normal to the plane of the coil and the magnetic field.
Initially when the plane of the coil was parallel to the magnetic field, $\theta = 90°$ hence $\Phi = B\,A \cos 90° = 0$.
After rotating 1.0°, the angle $\theta = 89°$ hence
$\Phi = B\,A \cos 89° = (0.29 \text{ T})(1.2 \text{ m} \times 2.0 \text{ m}) \cos 89°$
$= 0.01215 = 0.012$ Wb.
Therefore the flux cut by the coil = 0.012 Wb.

(iii) According to Faraday's law of electromagnetic induction, the e.m.f. generated is equal to the rate of change of magnetic flux linkage.
For one turn, the rate of change is
$0.01215 \text{ Wb} / 5.556 \times 10^{-5} \text{ s} = 218.7 = 220$ V.

(iv) For 38 turns, the change in flux linkage ($N\Phi$) is 38 times the change in flux (Φ) hence the total e.m.f. is
$38 \times 218.7 \text{ V} = 8311 = 8300$ V.

(c) (i) For a sinusoidal varying e.m.f. generated by a rotating coil,
$V_{r.m.s.} = V_0 / \sqrt{2} = 8311 / \sqrt{2} = 5877 = 5900$ V.

(ii) The direction of the current is from A to D.

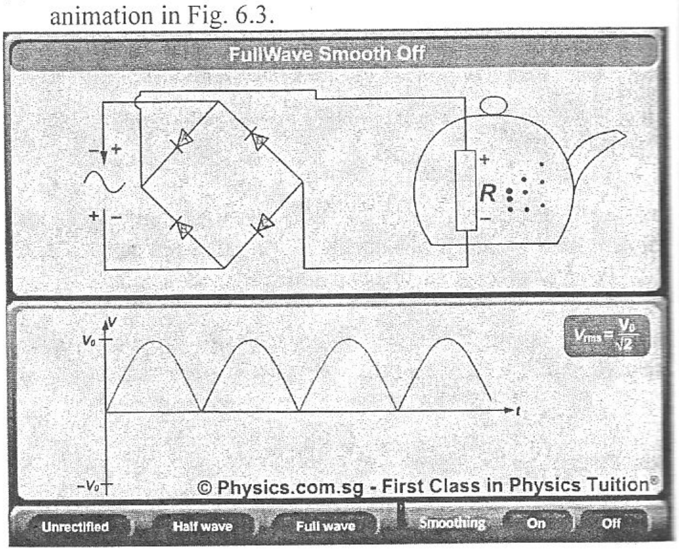
Detailed explanation (Fleming's left-hand rule)
The upward motion of the mobile free electrons contained in wire AD results in an equivalent current in the downward direction (conventional current flows in opposite direction to the movement of negative charges).
Using Fleming's left-hand rule, with the first finger in the direction of the magnetic field and the middle finger in the direction of this downward current, the force acting on these electrons will be from D to A.
As these electrons are accelerated from D to A, the conventional current flow in the wire will be from A to D (conventional current flows in opposite direction to the movement of negative charges).

Shorter explanation (Lenz's law)
As the coil rotates from its horizontal position, there is an increase in its magnetic flux linkage (with more magnetic field lines passing through the coil, towards the right). According to Lenz's law, the induced e.m.f. and resulting current will oppose this change by attempting to produce a magnetic field towards the left. Using the right-hand grip rule, the current has to be in the direction ADCB.

A simple rule-of-thumb would be to use "Fleming's right-hand rule" (strictly speaking not acceptable) with the first finger pointing to the right (in the direction of the magnetic field), the thumb upwards (in the direction of the motion of AD) and the middle finger points in the direction of the current (A to D).

(d) Alternating currents (a.c.) are used because they can be easily stepped up and down using simple transformers which are cost effective.
Transformers work based on the principle of electromagnetic induction, which requires an alternating magnetic field only an a.c. can produce.
With the ability of being stepped up and down, a.c. voltage supplies at households need to have only one value (e.g. 230 V) and transformers will make the necessary conversions for different electrical appliances.
Direct currents (d.c.), on the other hand, do not produce an alternating magnetic field hence they cannot be stepped up and down using transformers. The stepping up and down of d.c. requires switching circuits which are complex and expensive.
Power is transmitted at high voltages because at high voltages, current transmitted can be low, since current $I = P_{gen}/V$ where P_{gen} is the power output of the generator and V the transmitted voltage. The power loss in the transmission cable $P_{loss} = I^2R$ where R is the resistance of the transmission cable.
With a lower current, the power loss in the transmission cable can be lower, or alternatively, with the same power loss, a thinner cable (with higher resistance R) can be used thus saving on cabling costs.
Again, to reduce transmitted current by stepping up the voltage, the transmitted power has to be an a.c. so that transformers can be used to step them up and down economically.

(e) A full-wave bridge rectifier is shown in the Flash animation in Fig. 6.3.



http://www.physics.com.sg/H2/Alternating_Currents.swf

Fig. 6.3

The output could be smoothed by connecting a capacitor in parallel with the load resistor (not shown in figure).

TOPIC 17 Alternating Currents

1 There is a sinusoidal alternating current in a resistor.

What is the mean power dissipated in the resistor?

A $\frac{1}{\sqrt{2}}$ (maximum current)^2

B $\frac{1}{\sqrt{2}}$ (maximum power)

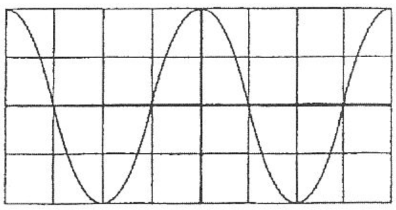
C $\frac{1}{2}$ (maximum current)^2

D $\frac{1}{2}$ (maximum power) N04/I/26

Solution Answer: **D**

For a sinusoidal current, $I_{r.m.s.} = I_0/\sqrt{2}$.
Mean power $P_{mean} = I_{r.m.s.}^2 R = (I_0/\sqrt{2})^2 R = I_0^2 R / 2 = P_0 / 2$ where P_0 is the maximum power.

2 A cathode-ray oscilloscope screen with a grid of 1 cm squares displays an alternating voltage waveform. The settings of the oscilloscope are: gain = 0.50 V cm^{-1}, time-base = 5.0 ms cm^{-1}.



The voltage V of this waveform is related to time t by the expression

A $V = 1.0 \cos 50\,t$.

B $V = 1.0 \cos 130\,t$.

C $V = 1.0 \cos 310\,t$.

D $V = 1.0 \cos 630\,t$. N06/I/25

Solution Answer: **C**

The period T is 4 cm x 5.0 ms cm^{-1} = 20 ms.
$V = V_0 \cos \omega t = 1.0 \cos (2\pi/T)t$
$= 1.0 \cos (2\pi/0.020)t = 1.0 \cos 310\,t$

3 A sinusoidal alternating current has a peak value I_0.
Which expression is correct for the root-mean-square current I_{rms}?

A $I_{rms} = \frac{I_0}{2}$

B $I_{rms} = \frac{I_0}{\sqrt{2}}$

C $I_{rms} = \sqrt{2 I_0}$

D $I_{rms} = 2I_0$

N07/I/32

Solution Answer: **B**

For a sinusoidally alternating current, the root-mean-square value

$$I_{rms} = [\int_0^{T/2} I_0^2 \sin^2\omega t \, dt \,/\, (T/2)]^{1/2} = I_0/\sqrt{2}$$

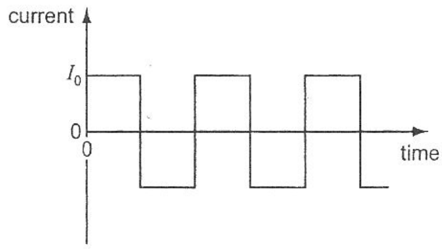
<u>Explanation of the above equation</u>
The root-mean-square value of a waveform is found by

1) Squaring the waveform ($I_0^2 \sin^2\omega t$).

2) Finding the mean of the square, which is the area under the squared graph in a time interval of one period, divided by the period (In this example, the period is $T/2$ since that is the time interval after which the sine-squared graph repeats itself). The area under the graph is found by integrating from limits 0 to $T/2$.
For sinusoidal waveforms, the mean-square current is simply half of the square of the peak current, i.e. $I_0^2/2$.

3) Square-rooting the result (which is $I_0/\sqrt{2}$).

4 The graph shows an alternating current with a square waveform.



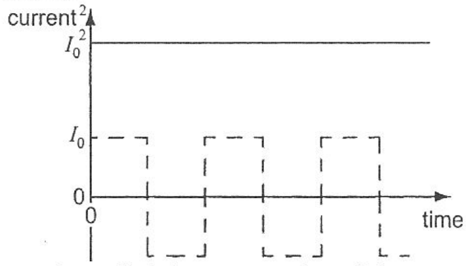
The peak value of the current is I_0.
What is its root-mean-square value?

A 0 B $\frac{I_0}{2}$ C $\frac{I_0}{\sqrt{2}}$ D I_0

N08/I/34

Solution Answer: **D**

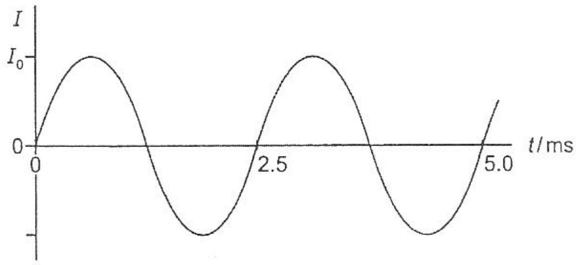
The first step to finding root-mean-square is to square the graph, as shown.



The next step is to find the mean value of the squared graph, which is I_0^2 (since it's a constant value).

The last step is to square-root the mean-squared value, which brings it back to I_0.

5 The graph shows how an alternating current I of peak value I_0 varies with time t.



Which expression gives the alternating current I?

A $I = I_0 \sin(5\pi t)$

B $I = I_0 \sin\left(\frac{2\pi t}{2.5}\right)$

C $I = I_0 \sin\left(\frac{\pi t}{0.0025}\right)$

D $I = I_0 \sin(800\pi t)$

N09/I/34

Solution Answer: **D**

The equation is $I = I_0 \sin(\omega t) = I_0 \sin(2\pi t / T)$

$= I_0 \sin(2\pi t / 0.0025\text{ s}) = I_0 \sin(800\pi t)$

6 A sinusoidally-alternating voltage supply at 50 Hz connected across a load resistor of 200 Ω delivers a peak current of 2.0 A. The frequency of the supply is doubled to 100 Hz.

What is the mean power dissipated in the load resistor at the higher frequency?

A 200 W B 400 W C 800 W D 1600 W

N09/I/35

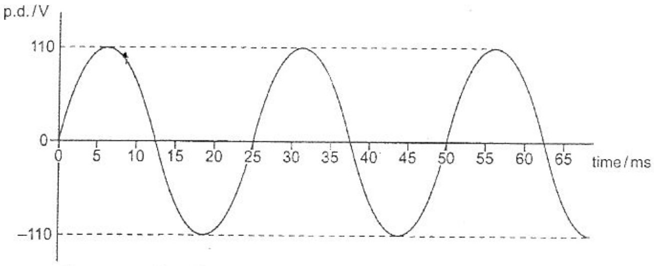
Solution Answer: **B**

The mean power dissipated in a load is independent of the frequency of the supply.

The mean power

$P_{mean} = I_{r.m.s.}^2 R = (I_0/\sqrt{2})^2 R = (2.0/\sqrt{2})^2(200\ \Omega) = 400$ W.

7 The graph shows how the potential difference across an alternating supply varies with time.



What are the frequency f and root-mean-square potential difference V_{rms} of the a.c. supply?

	f/Hz	V_{rms}/V
A	40	156
B	40	78
C	400	156
D	400	78

N10/I/33

Solution Answer: **B**

Frequency $f = 1/T = 1/(25 \times 10^{-3}\text{ s}) = 40$ Hz.

For sinusoidal waveforms, $V_{rms} = V_0/\sqrt{2} = 110/\sqrt{2} = 78$ V.

Long Questions

8 *(a)* Explain what is meant by the terms *peak voltage* and *root-mean-square voltage* when applied to a sinusoidal alternating voltage. [3]

(b) The equation $V = 564\,000 \sin 100\pi t$ represents a sinusoidal alternating voltage for an overhead cable on an electrical distribution system. State the frequency, peak voltage and root-mean-square voltage for this alternating voltage. [3]

(c) Explain why such a high voltage is advantageous for the transmission of electrical energy. [3]

(d) Describe briefly how such high voltages are obtained in practice. [3]

(e) A cable used for the transmission detailed in *(b)* has a circular cross-section of radius 0.012m. Fig. 4.1 is a full-scale drawing showing the electric field surrounding the cable together with lines of equal potential at an instant when the potential of the cable is +564 000 V.

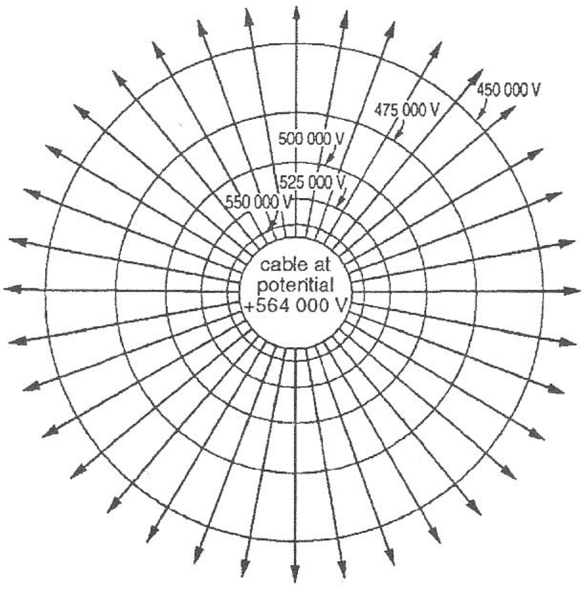


Fig. 4.1

(i) State the equation relating field strength with potential gradient.

(ii) Use Fig. 4.1 to estimate the potential gradient near the surface of the cable.

(iii) Explain why a cable of larger radius, but at the same potential, will have a smaller electric field at its surface.

[5]

(f) Sketch a diagram suggesting electric field lines near a cable of square cross-section. [3]

N02/III/4

Solution

(a) *Peak voltage* refers to the maximum instantaneous value of the alternating voltage that occurs at regular intervals, in either polarity direction.

Root-mean-square voltage refers to the equivalent value of the steady direct voltage that will dissipate heat at the same average rate as the alternating voltage, when applied across a given resistance.

(b) Comparing this equation with the general form $V = V_0 \sin(2\pi f t)$, it can be seen that:

Frequency $f = 100/2 = 50$ Hz.

Peak voltage $V_0 = 564000$ V.

Root-mean-square voltage $V_{r.m.s.} = V_0/\sqrt{2}$
$= 564000/\sqrt{2} = 398808 = 399000$ V.

(c) A high voltage is advantageous because when the transmitted voltage (V) is high, the transmitted current (I) is low since $I = P/V$ where P, the transmitted power, is a constant for a particular generating system.

The power loss in the transmitting cable $P_{loss} = I^2R$ where R is the resistance of the cable.

With a low current, the power loss is minimized or alternatively, a thinner cable with a higher resistance R can be used for the same amount of power loss, thus saving on cabling costs.

(d) Voltages are stepped up using step-up transformers.

In transformers, the voltage ratio V_s/V_p is equal to the turns ratio N_s/N_p. Thus if the transformer's secondary has more turns than the primary, then the secondary voltage will be higher than the primary.

By connecting the power generator to the primary coil of a transformer, the transmitted voltage can be stepped up to a high value for transmission.

(e) (i) Electric field strength E is the negative of the potential gradient ($E = -dV/dx$), where x is the distance from the centre of the cable.

(ii) On the diagram the radius of the cable is measured as 0.0075 m whereas the actual radius given in the question is 0.012 m.

The scale of the diagram is 7.5 : 12, or 15 : 24.

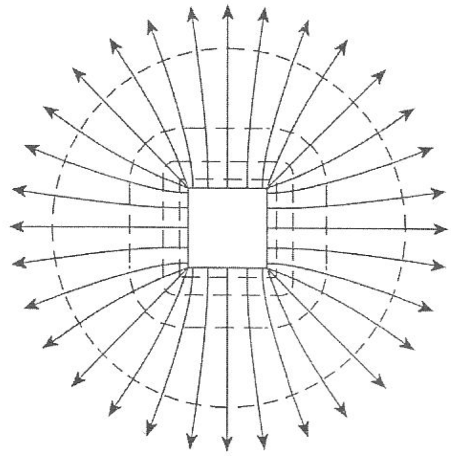
On the diagram, the measured value of the distance of separation between the 550000 V and 564000 V equipotential lines is 0.0015 m hence the actual distance is (24/15) x 0.0015 m = 0.0024 m.

Therefore, near the surface of the wire,

$|E| \approx \delta V/\delta x = (564000 - 550000)/(0.0024 \text{ m})$
$= 5.833 \times 10^6 = 5.8 \times 10^6 \text{ V m}^{-1}$.

(iii) The electric field E at the surface of the conductor is directly proportional to its surface charge density σ.
$E = \sigma/\varepsilon_0$ where ε_0 is the permittivity of free space.
A cable with a larger radius has a lower surface charge density σ, since the charges are distributed over a larger surface area.
Hence the electric field strength is weaker.

(f) The electric field lines near a cable of square cross-section are shown in the figure below.



Electric field lines (solid arrows) are always perpendicular to equipotential lines (dotted). The surface of the conductor is an equipotential line.
Electric field strength is greatest at the corners because charges tend to accumulate at locations where the radius of curvature of the surface is the smallest, i.e. at sharp points (Gauss's Law).
With increasing distance from the cable, the equipotential surfaces become increasingly circular hence the electric field lines, being perpendicular to the equipotential lines (dotted), become increasingly radial in direction.

9 (*a*) A transformer has a sinusoidal input of 7.3 μV r.m.s. and an output of 55 μV r.m.s.

(i) State the meaning of *root-mean-square* (r.m.s.). [1]

(ii) Calculate the peak output voltage. [2]

(iii) Assuming this to be an ideal transformer, calculate the turns ratio. [1]

N04/III/6 (part)

Solution

(a) (i) The *root-mean-square* value of an alternating current (a.c.) is defined as the equivalent value of the steady direct current (d.c.) which dissipates heat at the same average rate as the a.c., in a given resistance.

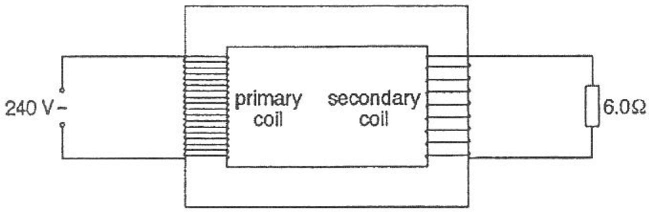
(ii) $V_0 = V_{r.m.s.}\sqrt{2} = (55\ \mu\text{V})\sqrt{2} = 77.78 = 78\ \mu\text{V}$.

(iii) $N_s/N_p = V_s/V_p = (55\ \mu\text{V})/(7.3\ \mu\text{V}) = 7.5$.

Transformer

10 The diagram shows an iron-cored transformer assumed to be 100% efficient. The ratio of the secondary turns to the primary turns is 1:20.

A 240 V a.c. supply is connected to the primary coil and a 6.0 Ω resistor is connected to the secondary coil.



What is the current in the primary coil?

- A 0.10 A
- B 0.14 A
- C 2.0 A
- D 40 A

N02/I/27

Solution Answer: **A**

$V_s/V_p = N_s/N_p = 1/20$.

$V_s = V_p / 20 = 240\text{ V} / 20 = 12\text{ V}$.

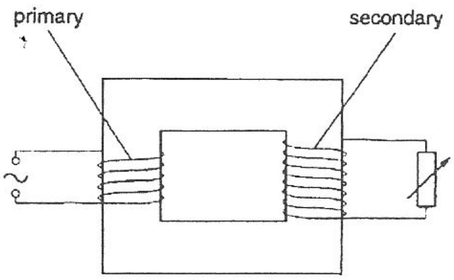
$I_s = 12\text{ V} / 6.0\ \Omega = 2.0\text{ A}$.

$I_p/I_s = N_s/N_p = 1/20$,

$I_p = (1/20)\, I_s = (1/20)(2.0\text{ A}) = 0.10\text{ A}$.

11 The primary coil of a transformer is connected to an alternating voltage supply. The secondary coil is connected across a variable resistor.

Which change will cause a decrease in the p.d. across the secondary coil?



- A increasing the cross-sectional area of the secondary coil
- B increasing the current in the primary coil
- C increasing the number of turns of the primary coil
- D increasing the resistance of the variable resistor

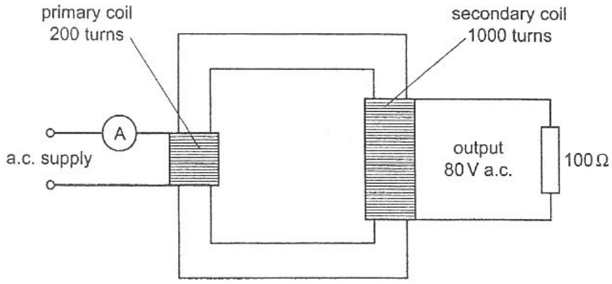
N03/I/24

Solution Answer: **C**

In a transformer, the voltage ratio V_s/V_p = turns ratio N_s/N_p.

With primary voltage V_p fixed, the secondary voltage can be decreased either by reducing the secondary turns N_s or increasing the primary turns N_p.

12 A 100% efficient transformer is connected as shown to a sinusoidal a.c. supply.



What is the reading on the ammeter?

A 0.16 A B 0.80 A C 2.8 A D 4.0 A

N07/I/33

Solution Answer: **D**

The secondary current $I_S = 80\text{ V} / 100\ \Omega = 0.80\text{ A}$.

$$I_P/I_S = N_S/N_P$$

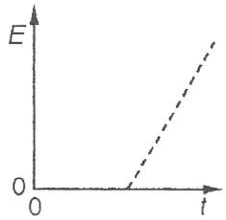
$$I_P/0.80\text{ A} = 1000 / 200$$

$$I_P = 4.0\text{ A}$$

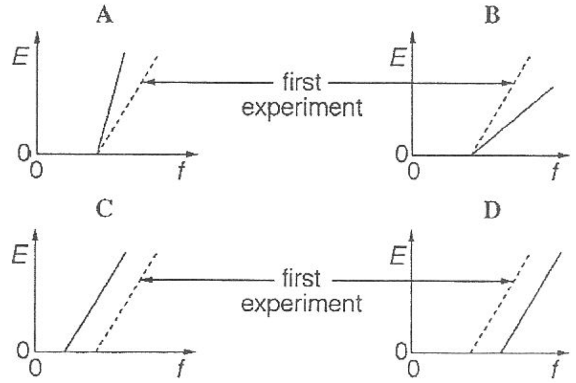
TOPIC 18 Quantum Physics

Photoelectric Effect

1 When electromagnetic radiation of frequency f falls on a particular metal surface, photoelectrons may be emitted. The graph shows the variation with f of the maximum kinetic energy E of these electrons.



Which graph is obtained when the experiment is repeated using another metal of greater work function energy?



N02/I/29

Solution Answer: **D**

$eV_s = K.E._{max} = hf - \Phi$.

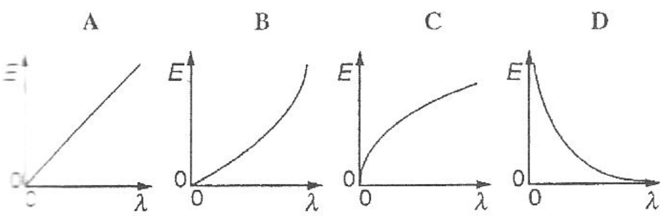
Dividing throughout by e, the equation becomes

$V_s = (h/e)f - \Phi/e$.

Hence a graph of stopping potential V_s versus frequency f is a straight line with a gradient of (h/e) and a negative y-intercept of $(-\Phi/e)$.

When work function energy Φ is greater, the y-intercept $-\Phi/e$ becomes more negative but the gradient h/e remains unchanged since both the Planck constant h and the elementary charge e are invariable.

2 Which graph shows how the energy E of a photon of light is related to its wavelength λ?



N03/I/28

Solution Answer: **D**

Photon energy $E = hc/\lambda$ where h is the Planck constant and λ the wavelength of the radiation.

A graph of E vs. λ is thus a rectangular hyperbola graph of the form $y \propto 1/x$.

3 In the photoelectric effect, light incident on a metal surface causes electrons to be ejected from the surface.

Which statement is correct?

A Electrons are ejected only if the wavelength of the incident light is greater than some minimum value.

B The de Broglie wavelength of the ejected electrons is the same as the wavelength of the incident light.

C The maximum energy of the electrons is independent of the intensity of the incident light.

D The maximum energy of the electrons is independent of the type of metal.

N06/I/28

Solution Answer: **C**

The maximum kinetic energy of the emitted photoelectrons is given by the equation: $K.E._{max} = hf - \Phi$ where h is the Planck constant, f the photon frequency and Φ the work function energy of the metal.

Increasing the intensity of the incident light merely increases the number of photons arriving per unit time, but not the photon energy or the work function energy of the metal.

Hence the maximum energy of the electrons is independent of light intensity.

4 What is a reasonable estimate, to one significant figure, of the energy of a photon of violet light?

A 4 eV B 6 eV C 3×10^{-19} J D 5×10^{-19} J

N07/I/34

Solution Answer: **D**

The visible spectrum ranges from 400 nm (violet) to 750 nm (red). The wavelength of violet light is approximately 400 nm or 4×10^{-7} m.

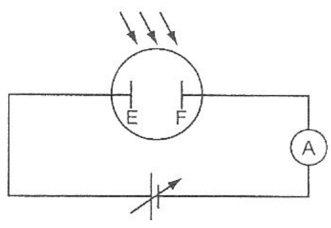
Energy of a violet light photon is

$$E = hc/\lambda$$

$$= (6.63 \times 10^{-34} \text{ J s} \times 3.00 \times 10^{8} \text{ m s}^{-1})/(4 \times 10^{-7} \text{ m})$$

$$= 5 \times 10^{-19} \text{ J}$$

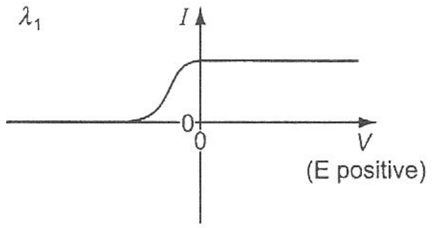
5 The diagram shows a circuit used for photoelectric emission experiments.



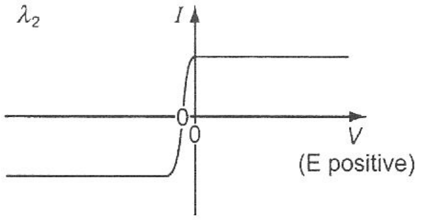
The two electrodes E and F are made of different metals. The work function of electrode E is Φ_E, and the work function of electrode F is Φ_F.

Current-voltage (I-V) characteristics are obtained when both electrodes are illuminated with monochromatic light.

When the wavelength of the light is λ_1 the *I-V* characteristic is as shown.



When the wavelength of the light is λ_2 the *I-V* characteristic is as shown.



Which line of the table relates the magnitudes of the wavelengths and the magnitudes of the work functions?

	wavelength	work function
A	λ_1 is less than λ_2	ϕ_E is less than ϕ_F
B	λ_1 is less than λ_2	ϕ_E is greater than ϕ_F
C	λ_1 is greater than λ_2	ϕ_E is less than ϕ_F
D	λ_1 is greater than λ_2	ϕ_E is greater than ϕ_F

N07/I/36

Solution Answer: **D**

The fact that light of wavelength λ_1 causes photoelectric emission first means that its photons have a lower energy and hence its wavelength λ_1 is longer.

The fact that the forward current flows first means that the electrons start emitting from plate F first (since current *I* flows in the opposite direction to electron flow) hence F has a lower work function.

6 The relation between the energy *E* of a photon and its wavelength λ is

$$E = K/\lambda$$

where *K* is a constant.
When *E* is measured in electronvolts and λ in nanometres, what is the numerical value of *K*?

A 3.18×10^{-53}
B 3.18×10^{-35}
C 1.24×10^{-15}
D 1.24×10^{3}

N08/I/21

Solution Answer: **D**

A photon's energy $E = hc/\lambda$, where *h* is the Planck constant and *c* the speed of light in vacuum. Hence $K = hc$.

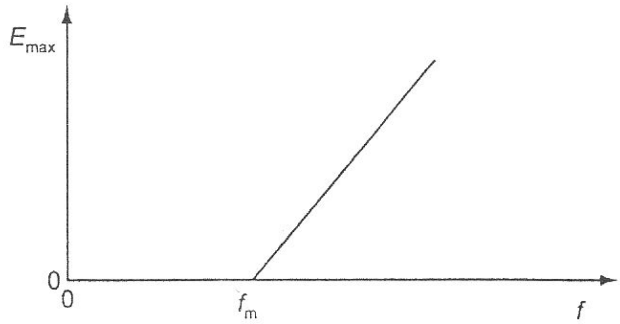
1 eV = 1.60×10^{-19} J; 1 nm = 10^{-9} m.

$h = 6.63 \times 10^{-34}$ J s = $(6.63 \times 10^{-34} / 1.60 \times 10^{-19})$ eV s
= 4.14×10^{-15} eV s

$c = 3.00 \times 10^{8}$ m s^{-1}
= $(3.00 \times 10^{8} \times 10^{9})$ nm s^{-1} = 3.00×10^{17} nm s^{-1}

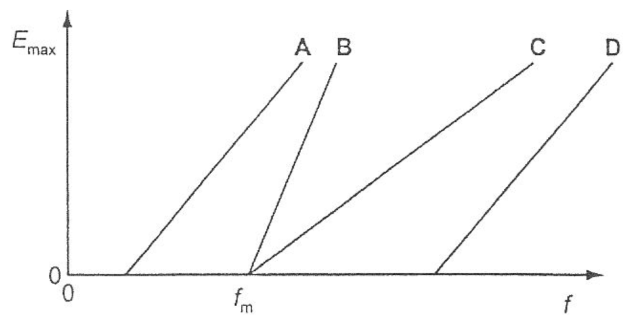
$K = hc = 4.14 \times 10^{-15}$ eV s $\times$ 3.00×10^{17} nm s^{-1}
= 1.24×10^{3} eV nm

7 The graph shows the variation of maximum kinetic energy E_{max} with frequency *f*, for photoelectrons emitted from the surface of a metal illuminated with electromagnetic radiation.



The metal is replaced with one that has a smaller work function.

Which line shows the variation of maximum kinetic energy with frequency for the new metal?



N09/I/36

Solution Answer: **A**

The equation is: $E_{max} = hf - \Phi$

where *h* is the Planck constant and Φ the work function of the metal.

The gradient of the graph, *h*, is a constant and should remain unchanged irrespective of the type of metal.

With a smaller work function, the graph will have a less negative *y*-intercept and hence shift up (and to the left), with a smaller threshold frequency f_m as well.

8 A metal surface is illuminated with a beam of monochromatic electromagnetic radiation.
What determines the maximum speed of the photoelectrons that are emitted from the surface?

A the intensity of the radiation only
B the frequency of the radiation only
C the intensity and the frequency of the radiation
D the frequency of the radiation and the work function of the metal

N10/I/35

Solution Answer: **D**

The maximum kinetic energy of emitted photoelectrons $KE_{max} = hf - \Phi$ where h is the Planck constant, f the frequency of the incident radiation and Φ the metal's work function. Thus radiation with higher frequency and/or metals with lower work functions will result in higher maximum photoelectron kinetic energy (KE_{max}) and speed (v_{max}) since $KE_{max} = \frac{1}{2}mv_{max}^2$.

Long Questions

9 (a) State what is meant by the *photoelectric effect* and describe how it provides evidence for the particulate nature of electromagnetic radiation when it is interacting with matter.

....
....
....
....
....
....
....
.... [6]

(b) State an equation to express the principle of energy conservation as applied to the photoelectric effect. Explain the meaning of all the terms that you use.

equation ....
terms used ....
....
.... [3]

(c) Light of wavelength 3.82×10^{-7} m is incident on a substance and electrons are emitted with a maximum speed of 6.87×10^{5} m s^{-1}. Calculate the work function energy of the substance.

Work function energy = ... J [3]

(e) Show, with the aid of a diagram, what is meant by a *potential barrier*. Discuss how the wave nature of particles allows particles to tunnel through such a barrier.

....
....
....
....
....
....
.... [3]

N07/III/7 (part)

Solution

(a) *Photoelectric effect* refers to the emission of electrons from a cold and clean metal surface when electromagnetic radiation of sufficiently high frequency is incident on it.

The fact that:

- Only radiation of frequency above a minimum threshold frequency can cause photoemission,
- The photoelectrons' maximum kinetic energy is dependent on the radiation's frequency but not its intensity, and
- There is no noticeable time lag between the arrival of the radiation and the emission of the photoelectrons,

show that electromagnetic radiation arrives as discrete packets of energy known as photons, which interact with the electrons on a one-to-one basis.

The individual photon's energy is proportional to the radiation frequency f but not its intensity ($E = hf$). If this energy exceeds the metal's work function, it will be totally acquired by the electron instantaneously, and the photoelectron will be emitted from the metal.

This provides evidence for the particulate nature of electromagnetic radiation.

(b) The equation is: $KE_{max} = hf - \Phi$

KE_{max} is the kinetic energy of the most energy photoelectron emitted from the metal surface.

h is the Planck constant (6.63×10^{-34} J s).

f is the frequency of the incident radiation.

hf represents the energy of the incident photon.

Φ is the work function of the metal, i.e. the minimum energy of the incident photon required for photoemission to take place.

(c) Radiation frequency $f = c/\lambda$

$= (3.00 \times 10^{8}\ \text{m s}^{-1})/(3.82 \times 10^{-7}\ \text{m}) = 7.8534 \times 10^{14}$ Hz.

Photon energy $E = hf$

$= (6.63 \times 10^{-34}\ \text{J s})(7.8534 \times 10^{14}\ \text{Hz}) = 5.2068 \times 10^{-19}$ J

Maximum kinetic energy $KE_{max} = \frac{1}{2}mv^2$

$= \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(6.87 \times 10^{5}\ \text{m s}^{-1})^2 = 2.1498 \times 10^{-19}$ J.

Work function energy $\Phi = hf - KE_{max}$

$= 5.2068 \times 10^{-19}\ \text{J} - 2.1498 \times 10^{-19}$ J

$= 3.057 \times 10^{-19} = 3.06 \times 10^{-19}$ J.

10 (a) By reference to the photoelectric effect, state what is meant by the *work function* of a surface. [1]

(b) Light of wavelength 540 nm is incident on a metal surface having a work function of 2.5 eV.
Determine whether electrons are emitted from the surface. [4]

(c) Explain whether your conclusion in (b) is affected by the intensity of light incident on the surface. [2]

N10/III/5

Solution

(a) The work function of a surface is the minimum energy of the individual photons in the incident radiation on the surface required to cause photoemission of electrons.

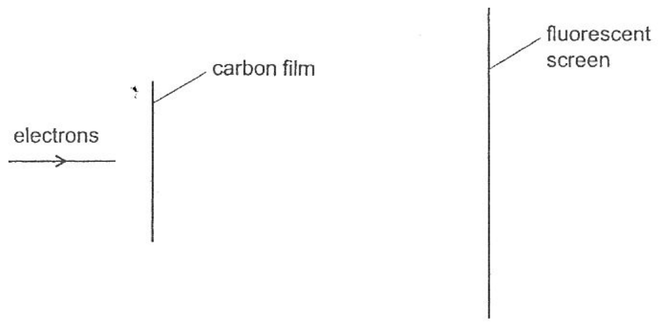
(b) The energy of the photon
$E = hc/\lambda = (6.63 \times 10^{-34})(3.00 \times 10^{8})/(540 \times 10^{-9})$
$= 3.683 \times 10^{-19}$ J = 2.3 eV

Since the energy of the incident photons (2.3 eV) is lower than the work function of the metal (2.5 eV), no electrons are emitted from the surface.

(c) The intensity does not affect the conclusion if the wavelength of the light remains unchanged.
Increasing the intensity merely increases the number of photons incident on the metal surface per unit time per unit area. If the wavelength remains unchanged and the individual photon's energy is still lower than the work function of the metal, no photoemission takes place.

Wave-Particle Duality

11 Electrons with velocity v travel through a vacuum and are incident on a thin carbon film, as shown.



The electrons produce a pattern of concentric circles on the fluorescent screen.

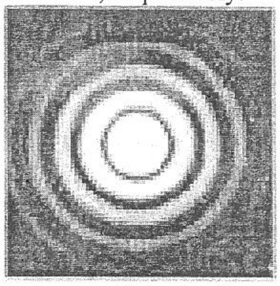
What causes the pattern and which change to the pattern occurs when the velocity v is increased?

	cause	change to pattern
A	diffraction	diameters of circles increase
B	diffraction	diameters of circles decrease
C	refraction	diameters of circles increase
D	refraction	diameters of circles decrease

N08/I/35

Solution Answer: **B**

The concentric bright and dark rings are due to the diffraction of the matter wave of the electron beam, due to constructive and destructive interference, respectively.



From de Broglie's wave-particle duality equation, $\lambda = h/mv$, when the velocity v of the electrons is increased, the wavelength of the matter wave decreases.
Using the diffraction grating formula $d \sin \theta = n\lambda$, the angular separation between successive fringes decreases.

Short Questions

12 A stationary nickel nucleus of mass 9.95×10^{-26} kg emits a γ-ray photon of energy 1.17 MeV.

(a) For the γ-ray photon,

(i) show that its wavelength is approximately 1.06×10^{-12} m, [3]

(ii) calculate its momentum.

momentum = ... N s [2]

(b) The direction in which the γ-ray photon is emitted is shown in Fig. 2.1.

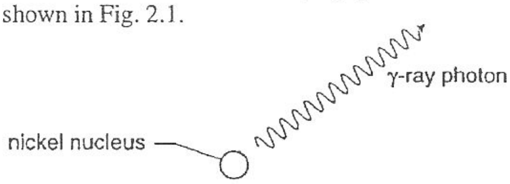


Fig. 2.1

(i) On Fig. 2.1, draw an arrow to show the direction of motion of the nickel nucleus after the emission of the photon. [1]

(ii) Calculate the speed of the nickel nucleus after the emission of the photon.

speed = ... ms^{-1} [2]

(c) A second nickel nucleus that is moving emits a γ-ray photon. Suggest, with a reason, whether the angle between the final direction of motion of the nucleus and that of the emitted photon will be the same as that in Fig. 2.1. [2]

N05/II/2

Solution

(a) (i) 1 eV = 1.6×10^{-19} J.

$$1.17 \text{ MeV} = 1.17 \times 10^6 \times 1.6\text{x}10^{-19} \text{ J} = 1.872 \times 10^{-13} \text{ J}.$$

$$E = hf = hc/\lambda$$

$$1.872 \times 10^{-13} \text{ J} = (6.63 \times 10^{-34} \text{ J s})(3 \times 10^8 \text{ m s}^{-1}) / \lambda$$

$$\lambda = 1.0625 \times 10^{-12} = 1.06 \times 10^{-12} \text{ m}.$$

(ii) Momentum
$p = h/\lambda$
$= 6.63 \times 10^{-34}$ J s / 1.0625×10^{-12} m
$= 6.24 \times 10^{-22}$ N s.

(b) (i) The direction of the motion of the nickel nucleus is indicated in Fig. 2.2.

The nickel nucleus moves in the opposite direction to that of the photon so that linear momentum remains constant before and after emission (i.e. zero).

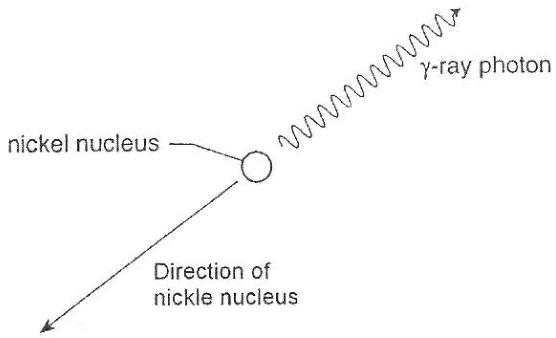


Fig. 2.2

(ii) The momentum of the nickel nucleus has to be equal but opposite to that of the photon in order that they cancel out vectorially.

$$mv = 6.24 \times 10^{-22} \text{ N s}$$

$$(9.95 \times 10^{-26} \text{ kg})\, v = 6.24 \times 10^{-22} \text{ N s}$$

$$v = 6.2724 \times 10^3 = 6.27 \times 10^3 \text{ m s}^{-1}$$

(c) The angle could be the same (i.e. 180°) only if the nucleus' initial momentum is collinear with the photon's momentum (either initially in the opposite direction, or in the same direction but with a momentum lower than the photon's momentum).

Otherwise, in accordance with the law of conservation of linear momentum, the nucleus will move in a different angle to the emitted photon (i.e. not 180°) such that momentum components perpendicular to the nucleus's initial momentum cancel out vectorially.

13 X-ray diffraction may be observed using a crystal as the diffraction grating. Electron diffraction may also be observed using a similar crystal if the de Broglie wavelength of the electron is appropriate.
The X-rays have wavelength 2.4×10^{-10} m. For an electron to have a de Broglie wavelength of 2.4×10^{-10} m, determine

(a) the momentum of the electron,
momentum = ... N s [2]

(b) the potential difference through which the electron must be accelerated from rest.
potential difference = ... V [4]

N09/III/3

Solution

(a) Using de Broglie's wave-particle duality equation, the momentum

$$p = h/\lambda = (6.63 \times 10^{-34})/(2.4 \times 10^{-10}) = 2.763 \times 10^{-24} \text{ N s}$$

momentum = <u>2.8×10^{-24} N s</u>

(b) The kinetic energy of a particle:

$$E_k = p^2/2m_e = (2.763 \times 10^{-24})^2/(2 \times 9.11 \times 10^{-31}) = 4.19 \times 10^{-18} \text{ J}$$

The gain in kinetic energy is also equal to the loss in electric potential energy:

$$eV = E_k = 4.19 \times 10^{-18} \text{ J}$$

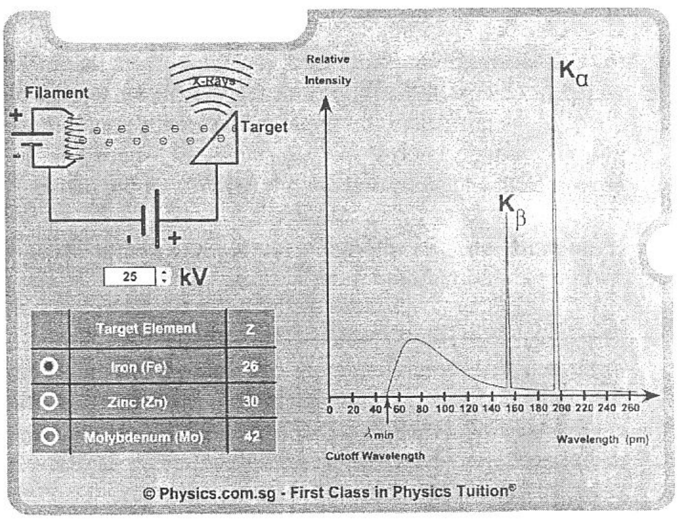
$$V = (4.19 \times 10^{-18} \text{ J})/(1.60 \times 10^{-19} \text{ C}) = 26.19 \text{ V}$$

potential difference = <u>26 V</u>

The effect of the accelerating potential difference on the cut-off wavelength (λ_{min}) in the X-ray spectrum is shown in the Flash animation below (not to be confused with the de Broglie wavelength of the electron).

Cut-off wavelength of X-ray spectrum $\lambda_{min} = hc/E_k$

de Broglie wavelength of electron $\lambda = h/\sqrt{(2m_eE_k)}$



http://www.physics.com.sg/H2/X_Rays.swf

Long Questions

14 Outline the experimental evidence that electromagnetic radiation is

(i) a wave, [4]

(ii) particulate. [4]

N03/III/6 (part)

Solution

(i) Young's double-slit experiment shows that electromagnetic radiation behaves like a wave.

When coherent light waves pass through the two slits, they diffract and their wavefronts spread out from both slits. The spreading wavefronts interfere with each other.

At locations where the wave trains are in phase, their displacements add up vectorially, according to the principle of superposition, to produce a resultant wave with double the amplitude and four times the intensity. These are the regions of constructive interference, also known as maxima.

At locations where the wave trains are 180° out-of-phase with each other, their displacements cancel vectorially to produce a resultant wave with zero amplitude and intensity. These are the regions of destructive interference, also known as minima.

At other locations where the wave trains are neither in phase nor completely out-of-phase, the amplitude and intensity are between the minimum and maximum.

The fact that electromagnetic radiation has phase information and are able to undergo diffraction and interference provide evidence for its wave nature.

(ii) The photoelectric effect experiment shows that electromagnetic radiation behaves like particles.

The *photoelectric effect* refers to the emission of photoelectrons from a cold and clean metal surface when electromagnetic radiation of frequency f above the threshold frequency f_0 is incident upon it.

When this condition is satisfied, the electrons in the metal which interact with the photons will acquire the energy of the photons and leave the metal surface with a maximum possible kinetic energy $K.E._{max} = hf - \Phi$ where f is the photon frequency and Φ the work function of the metal.

If the radiation intensity is increased, more photoelectrons will be emitted with the same $K.E._{max}$.

The fact that only radiation of frequency above a minimum threshold frequency can cause photoemission; the independence of the photons' maximum kinetic energy on radiation intensity; and the instantaneousness of the emission show that the radiation arrive as discrete particles known as photons.

15 *(b)* Electrons can sometimes be assumed to be particles and sometimes it is necessary to consider them having a wavelength.

Describe briefly an example of a situation in which an electron is considered as a particle and another example where it is considered as a wave. [4]

(d) Some of the energy levels for an electron in a hydrogen atom are –13.6 eV, –3.40eV and –1.51 eV. An electron moving from one level to a lower level emits electromagnetic radiation. For one of these lower levels, the radiation emitted is visible light.

(i) State the equation relating the change in energy ΔE to the wavelength λ of the emitted radiation. [1]

(ii) Deduce the wavelength of the line in the visible spectrum of hydrogen that results from electrons moving between two of the energy levels quoted. [4]

N06/III/4 (part)

Solution

(b) Electrons are considered as particles in Millikan's oil drop experiment whereby they are used to determine the magnitude of the elementary charge e. In this experiment, each oil drop will have a specific number N of excess electrons, resulting in a net charge Ne which could be determined.

Electrons are considered as waves in the electron microscope. The electrons, upon hitting the atomic lattice of a target, undergo diffraction and interference producing bright and dark patterns due to constructive and destructive interference respectively. The ability to undergo diffraction and interference is a property of waves.

(d) (i) $\Delta E = hc/\lambda$.

where h is the Planck constant and c the speed of light in free space.

(ii) For the hydrogen atom, visible lines can be seen only in the Balmer* series which involve transitions from a higher level to $n = 2$ (-3.40 eV).
For the three energy levels given, the only possibility is from $n = 3$ (-1.51 eV) to $n = 2$ (-3.40 eV).
The change in energy $\Delta E = hc/\lambda$ hence,

$$\lambda = hc / \Delta E$$

$$= hc / (-1.51 \text{ eV} - (-3.40 \text{ eV})) = hc / 1.89 \text{ eV}$$

$$= hc / (1.89 \text{ eV} \times 1.60 \times 10^{-19} \text{ J eV}^{-1})$$

$$= hc / (3.024 \times 10^{-19} \text{ J})$$

$$= (6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^{8} \text{ m s}^{-1})/(3.024 \times 10^{-19} \text{ J})$$

$$= 6.58 \times 10^{-7} \text{ m}$$

This wavelength 658 nm is in the red region of the visible spectrum.

* *The Lyman, Balmer, Paschen, Brackett, Pfund series etc. are not explicitly mentioned in the syllabus. A student who is not familiar with these series can still work out the above problem by trial and error, and select the wavelength in the visible range (410 nm to 750 nm), as follows:*

-1.51 eV to -3.40 eV : 658 nm (visible red)

-3.40 to -13.6 eV : 122 nm (ultra violet)

-1.51 eV to -13.6 eV : 103 nm (ultra violet)

Line Spectra

16 An electron in an atom makes a transition from a state of energy E_2 to one of lower energy E_1.

Which of the following gives the wavelength of the radiation emitted, in terms of the Planck constant h and the speed of light c?

A $\dfrac{E_2 - E_1}{hc}$ C $\dfrac{hc}{E_2 - E_1}$

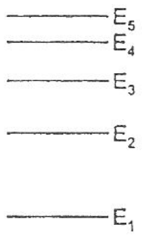
B $\dfrac{hc}{E_2} - \dfrac{hc}{E_1}$ D $\dfrac{c}{h(E_2 - E_1)}$

N04/I/29

Solution Answer: **C**

The photon's energy $E = hc/\lambda$ hence $\lambda = hc/E$.
The photon's energy is also equal to the difference between the higher and lower energy levels across which the transition takes place.
Hence $E = E_2 - E_1 = hc/\lambda$ and $\lambda = hc/(E_2 - E_1)$.

17 The diagram shows the first five energy levels of an atom.



Photons of electromagnetic radiation are emitted when an electron falls from one energy level to a lower level.

Which transition corresponds to a photon with the greatest wavelength?

A E_2 to E_1
B E_5 to E_1
C E_4 to E_3
D E_5 to E_4

N05/I/29

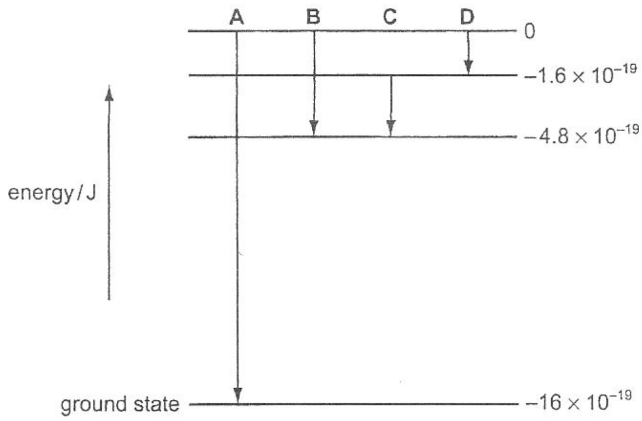
Solution Answer: **D**

The energy of the photon $E = hc/\lambda$ where h is the Planck constant, c the speed of light in free space and λ the wavelength of the photon. The longest wavelength belongs to the photon with the lowest energy.

Since the energy of an emitted photon is equal to the difference in the energy level across which the transition takes place, this also means that the difference in the energy levels must be the lowest. The lowest gap is between E_4 and E_5.

18 The diagram illustrates the electron energy levels, along with four possible electron transitions, in an atom.

Which transition will produce a photon of wavelength 6.2 x 10^{-7} m?



N07/I/35

Solution Answer: **C**

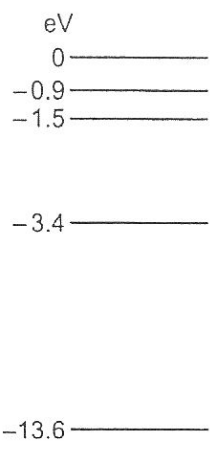
The energy of a photon of wavelength 6.2 x 10^{-7} m can be found from the equation

$$E = hc/\lambda = (6.63 \times 10^{-34} \text{ J s} \times 3.00 \times 10^{8} \text{ m s}^{-1})/(6.2 \times 10^{-7} \text{ m})$$

$$= 3.2 \times 10^{-19} \text{ J}$$

The energy of an emitted photon is equal to the difference in energy levels across which the transition takes place. In this case the energy difference between the -1.6 x 10^{-19} J level and the -4.8 x 10^{-19} J level is 3.2 x 10^{-19} J.

19 The diagram shows five energy levels of the hydrogen atom, labelled in the unit of electron-volt.



Which statement is correct?

A An atom in the level -3.4 eV can change levels by emitting photons of energy 1.9 eV, 2.5 eV, 3.4 eV and 10.2 eV.

B An atom in the level -3.4 eV can emit a photon of wavelength 650 nm to arrive in the level -1.5 eV.

C An electron with energy 10.2 eV colliding with an atom in level -13.6 eV can move it to the level -3.4 eV by losing all its kinetic energy.

D Most hydrogen atoms will be found in the level with zero energy.

N09/I/37

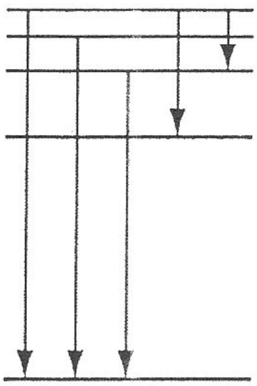
Solution Answer: **C**

The hydrogen atom can only move between these discrete levels. By absorbing all the kinetic energy of a 10.2 eV electron, the atom can be excited from -13.6 + 10.2 = -3.4 eV.

Statements A and B are incorrect because when an atom emits a photon, its energy decreases and not increases.

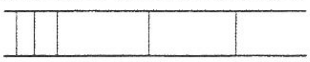
Statement D is incorrect because most atoms are found in the ground state (-13.6 eV), which is the lowest energy state.

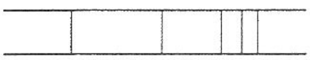
20 The diagram shows five energy levels of an atom. Five transitions between the levels are shown.

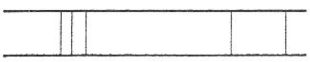


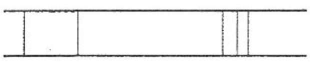
In the spectra below, the frequency scale is linear and increases to the right.

Which spectrum best corresponds to the five transitions between the levels shown above?

A


B


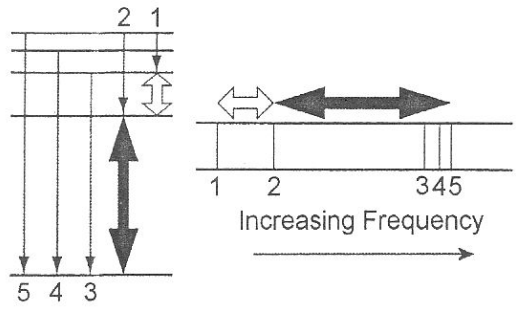
C


D


N10/I/36

Solution Answer: **D**

The convergence limit (where spectral lines converge) is towards the high frequency end since the higher energy levels are closer together. This leaves either answer B or D. Additionally, the energy of the 2[nd] lowest energy photon (labeled '2' below) is closer to that of the lowest energy photon (labeled '1' below) than the highest energy photon (labeled '5' below).



Short Questions

21 Fig. 4.2 shows some energy levels for the hydrogen atom.

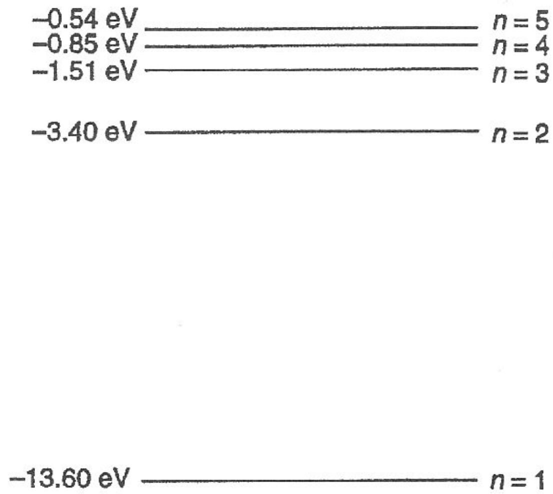


Fig. 4.2

A line emission spectrum is produced when electrons make transitions down to the $n = 1$ state.

Show, using a suitable calculation, that this line spectrum is **not** within the visible region of the electromagnetic spectrum. [5]

N04/II/4 (part)

Solution

The visible spectrum lies between wavelengths λ = 410 nm (violet end of spectrum) and λ = 750 nm (red end of spectrum).

The energy of a photon $E = hc/\lambda$ where h is the Planck constant and c the speed of light in free space. Hence $\lambda = hc/E$.

The energy of photons emitted is equal to the difference in energy levels across which the transition takes place.

The lowest possible energy transition is from n = 2 to n = 1, resulting in the emission of a photon with energy

E = -3.40 eV – (-13.60 eV) = 10.2 eV or 1.632 x 10^{-18} J

(since 1 eV = 1.6 x 10^{-19} J).

Hence $\lambda = hc/E$

= (6.63 x 10^{-34} J s)(3.00 x 10^{8} m s^{-1})/(1.632 x 10^{-18} J)

= 1.2188 x 10^{-7} m or 122 nm,

which is shorter than the shortest visible wavelength 410 nm at the violet end of the visible spectrum. In fact, 122 nm lies in the ultra violet range.

Photons emitted as a result of transitions from higher levels to n = 1 will result in even greater photon energies. Since wavelength λ is inversely proportional to photon energy E, these will result in even shorter wavelengths than ultra violet, which will definitely not be in the visible region.

Long Questions

22 *(b)* Explain how the existence of electron energy levels in atoms gives rise to line spectra, which may be emission or absorption spectra. [4]

(c) Some of the energy levels in atomic hydrogen are shown in Fig. 6.1.

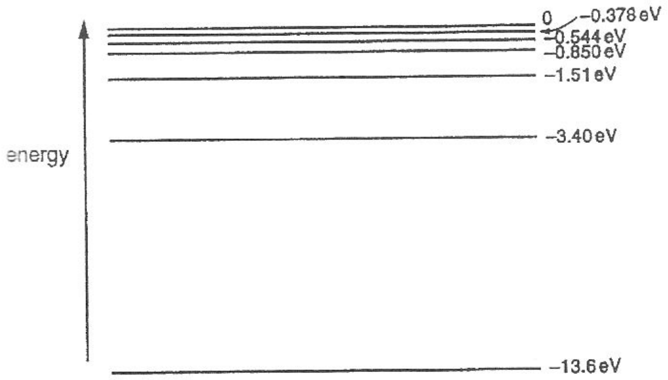


Fig. 6.1

(i) Calculate the minimum wavelength of the radiation that could be emitted from atomic hydrogen. [5]

(ii) Sketch the pattern of the visible line emission spectrum of hydrogen. This takes place when electrons fall to the –3.40 eV level. Mark the red and violet ends of the spectrum. [3]

N03/III/6 (part)

Solution

(b) Line spectra are produced when the atom is excited by an external source of energy such as collision with high speed electrons or photons.

The atom gains energy from the colliding electron or photon, and is excited to a higher energy state.

In the case of the emission line spectrum, the atom returns to the ground state, either directly or via intermediate energy levels, and emits a photon of energy equal to the difference between the higher and lower energy levels across which the transition takes place.

Due to the fact that the atom's energy levels are quantized, the energy and wavelength of the emitted photons can only take on certain discrete values.

When passed through a diffraction grating, the radiation will be split up according to the equation $d \sin \theta = n\lambda$ hence there are coloured lines only at certain angles, producing the emission line spectra.

In the case of the absorption line spectra, those photons which's energies are exactly equal to the difference between the atom's energy levels are absorbed and hence missing from the absorption spectra, accounting for the dark lines.

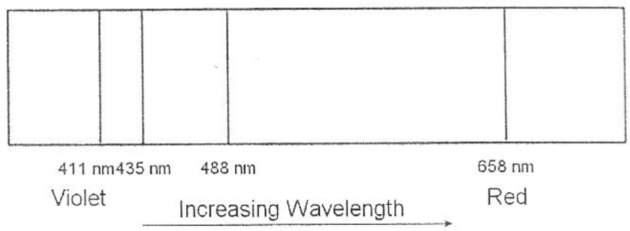
(c) (i) Minimum wavelength corresponds to the maximum energy ($E = hc/\lambda$), which is due to the transition from the highest excited state to the ground state.

hc/λ = 0 eV – (-13.6 eV)
= 13.6 eV
= 13.6 x 1.60 x 10^{-19} J
= 2.176 x 10^{-18} J.

$\lambda = hc/E$
= (6.63 x 10^{-34} Js)(3.00 x 10^{8} ms^{-1})/(2.176 x 10^{-18} J)
= 9.1406 x 10^{-8}
= 9.14 x 10^{-8} m.

(ii) The visible wavelengths are 411 nm, 435 nm, 488 nm, 658 nm.

A sketch of the emission line spectrum is shown in the figure below.



Uncertainty Principle

Long Questions

23 (d) A pulse of radio wave lasts for 1.0 x 10^{-5} s. A photon of the radio wave may be considered to be at a point anywhere within this pulse, although the location of the point is not known. Calculate

(i) the length of the pulse,

length of pulse = ... m [1]

(ii) the uncertainty in the position of the photon,

uncertainty in position = ... m [1]

(iii) the uncertainty in the momentum of the photon,

uncertainty in momentum = ... [3]

N07/III/7 (part)

Solution

(d) (i) Length = ct

= (3.00 x 10^{8} m s^{-1})(1.0 x 10^{-5} s) = 3.0 x 10^{3} m.

(ii) Uncertainty in position Δx = 3.0 x 10^{3} m.

(iii) Apply Heisenberg's uncertainty principle,

$$\Delta x\, \Delta p \geq \hbar/2$$

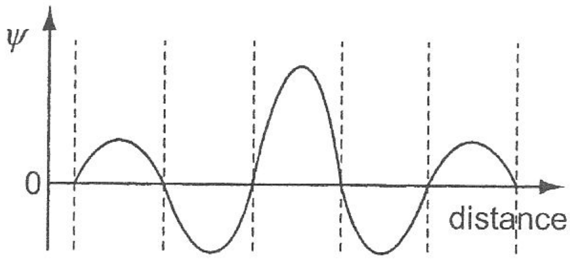
$$\Delta p \geq \hbar/(2\Delta x) = (h/2\pi)/(2\Delta x)$$

$$= 1.8 \times 10^{-38}\ \text{kg m s}^{-1}$$

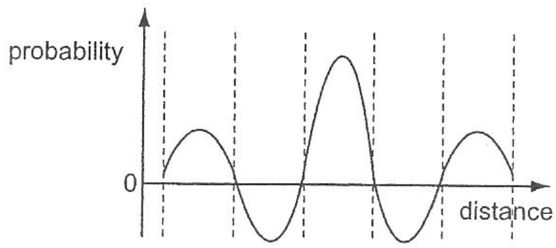
The uncertainly in momentum is at least 1.8×10^{-38} kg m s^{-1}.

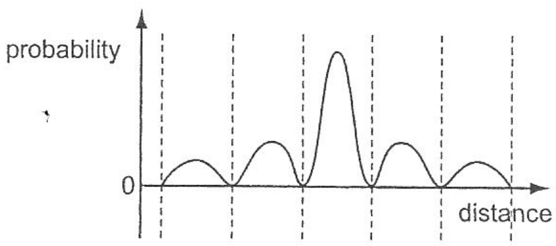
Schrödinger Model

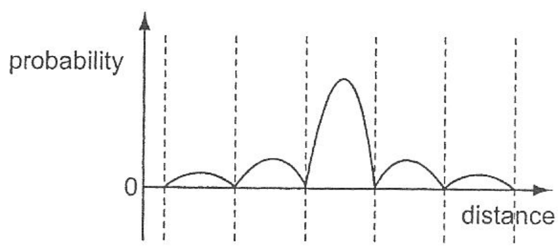
24 The sketch graph shows how the wave function ψ of an electron varies with position.

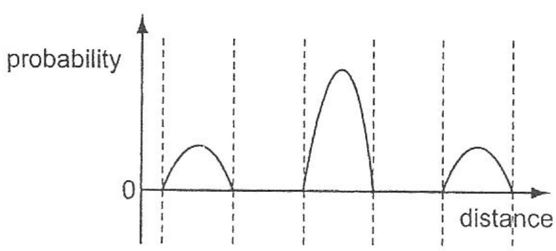


Which graph, drawn on the same horizontal scale, gives the probability of finding an electron at each position?

A


B


C


D


N08/I/36

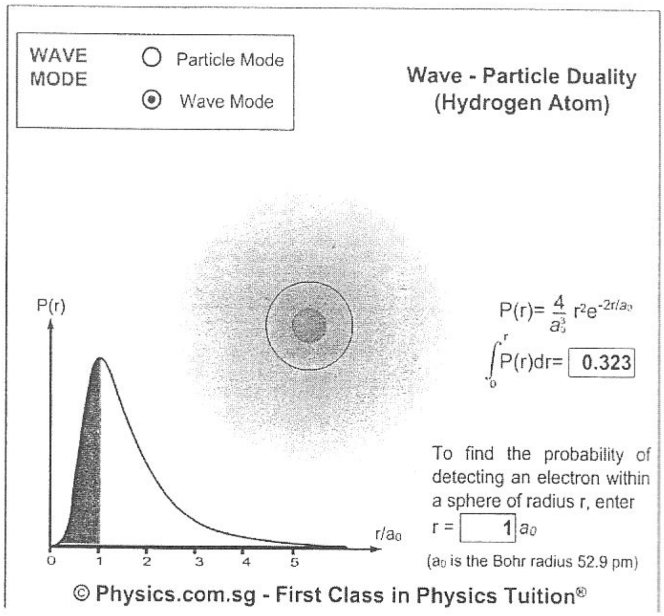
Solution Answer: **B**

The probability of finding an electron at a certain position can be deduced from the probability density function $|\psi|^2$, which is the square of the wave function ψ.

Both positive and negative parts of the wave function, when squared, becomes positive.

The answer is B and not C because the square of sinusoidal-shaped function is curved at the points it touches the x-axis, as in answer B.

Answer C has the shape of a $|\psi|$ function, rather than $|\psi|^2$.
The radial probability density function for a hydrogen atom in its ground state is shown in the Flash animation below.

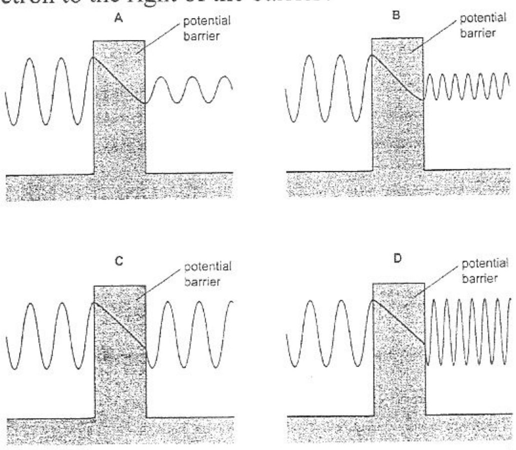


http://www.physics.com.sg/H2/Schrodinger_Model.swf

Interpretation of the above figure: the probability of finding an electron within one Bohr radius (52.9 pm) from the centre of the nucleus is 0.323.

Barrier Tunnelling

25 An electron of energy E is incident on the left-hand side of a potential barrier of energy U. The energy U is greater than E.
Which diagram represents the wave function of the electron to the right of the barrier?



N10/I/34

Solution Answer: **A**

When the electron tunnels through the barrier, only the amplitude of its wave function decreases due to there being a lower probability of detection to the right of the barrier (as probability density is proportional to the square of the probability amplitude). However, there is no change in the wavelength of the wave function since its kinetic energy ($h^2/2m\lambda^2$) remains unchanged if the potential energy on both sides are at the same level.

Long Question

26 (e) Show, with the aid of a diagram, what is meant by a *potential barrier*. Discuss how the wave nature of particles allows particles to tunnel through such a barrier.

....

....

....

....

....

....

.... [3]

N07/III/7(part)

Solution

(e) A potential barrier is a region where the potential energy U of a particle is higher than its total energy E, and where the force exerted on the particle is such as to oppose its passage through the region.

This is illustrated as the shaded region in Fig. 7.1.

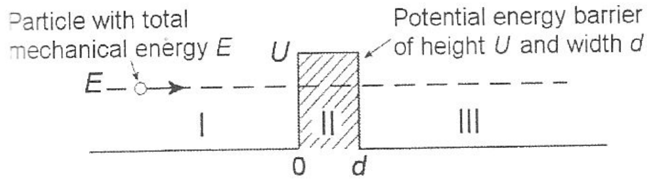
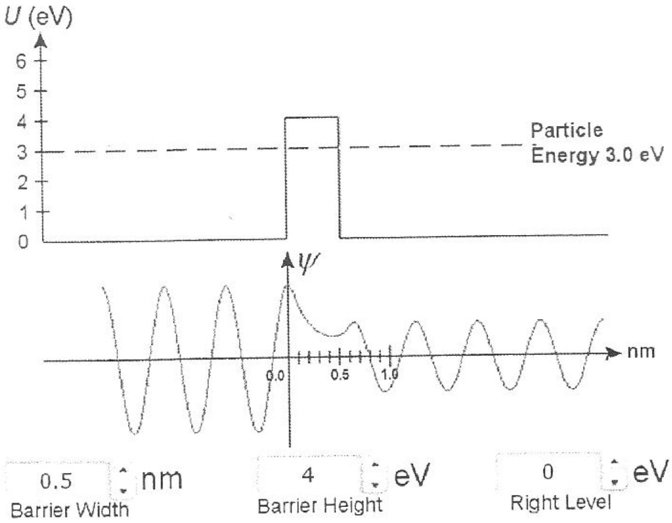


Fig. 7.1

Classically, a particle with total mechanical energy E which is lower than U will never be able to penetrate through the barrier.
In quantum mechanics, however, particles are treated as waves with a wave function ψ. The square of the wave function $|\psi^2|$, its probability density, provides information about the probability of locating the classical particle at various positions and times.
Treating the particle as a wave, the Schröedinger equation predicts that there is a nonzero probability that a particle can tunnel through a barrier – a region in which $E < U$ and the kinetic energy K is negative.

The probability that a particle of mass m penetrates through to the other side of the barrier is represented by the transmission coefficient $T = e^{-2kd}$ where d the width of the potential barrier and $k = \sqrt{(8\pi^2 m(U - E)/h^2)}$. Refer to the Flash Animation below.



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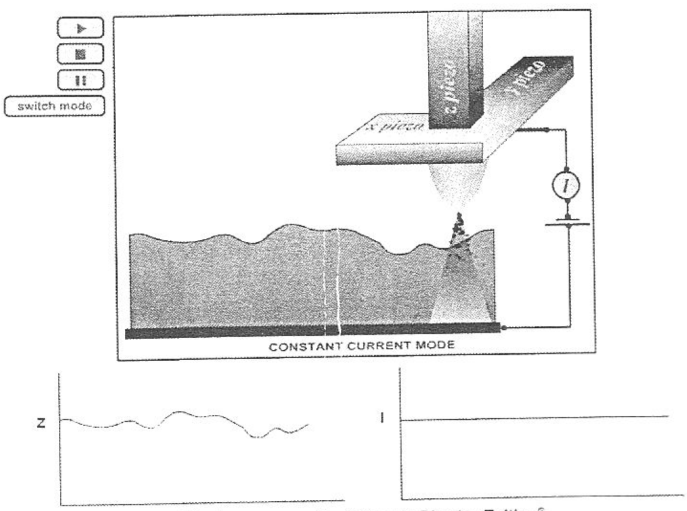
http://www.physics.com.sg/H2/Barrier_Tunnelling.swf

The application of barrier tunnelling in the Scanning Tunnelling Microscope (STM) is shown in the Flash animation below.



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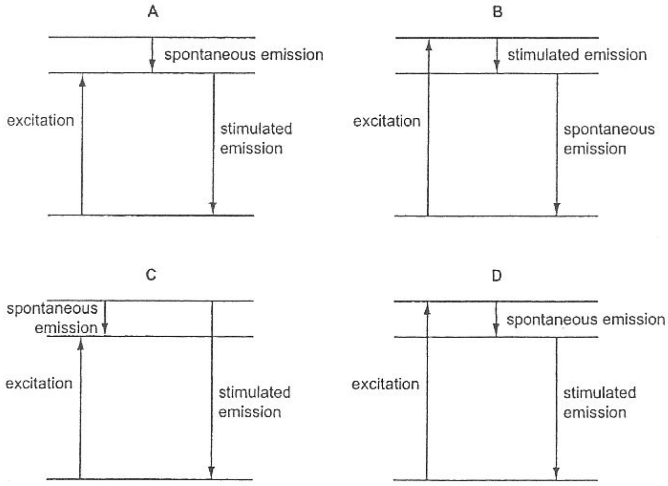
http://www.physics.com.sg/H2/Scanning_Tunnelling_Microscope.swf

TOPIC 19 Lasers and Semiconductors

Basic Principles of Lasers

1 In a helium-neon laser, helium atoms collide with neon atoms and excite them. This produces a population inversion which allows stimulated emission.
Which neon energy level diagram correctly shows the excitation of the neon atoms by the helium atoms, the spontaneous infra-red emission from the neon, and the stimulated emission of red light?



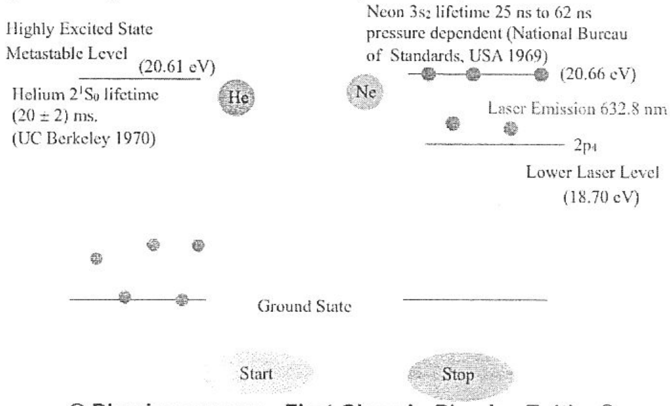
N08/I/38

Solution Answer: **D**

The excitation energy must bring the neon atom to the highest energy state, so that subsequent emissions (whether spontaneous or stimulated) can take place from there. This leaves only either answer B or D as the possible choices.

The infra-red radiation emitted from the spontaneous emission has a longer wavelength than that of the red light emitted from the stimulated emission. Therefore, the energy of the infra red photons is less than that of the red light photons, and the corresponding energy gap for the infra-red transition (spontaneous) is lower than that for the red light (stimulated). Therefore, only answer D is possible.

In the Flash animation below, the stimulated emission of red light (632.8 nm) takes place between the upper laser level (20.66 eV) and lower laser level (18.70 eV).



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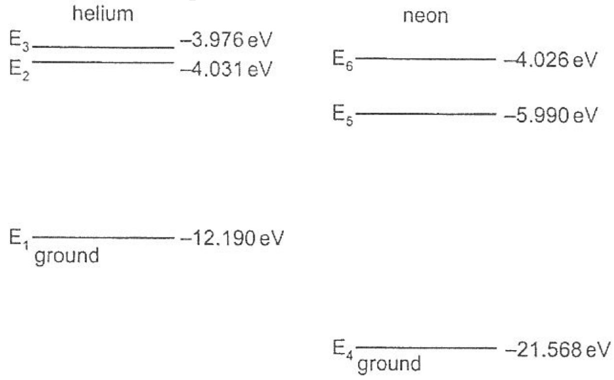
http://www.physics.com.sg/H2/Helium_Neon_Laser.swf

2 The diagram shows some of the energy levels of helium and neon. The energies of the levels are given in electron volts. The elements are the major constituents in a laser that emits red light.



Which transition between the labelled levels gives rise to the emission of the laser light?

A E_3 to E_2 **B** E_6 to E_5 **C** E_2 to E_1 **D** E_5 to E_4

N10/I/38

Solution Answer: **B**

The red laser light (632.8 nm) from the helium-neon laser is due to transition between the $3s_2$ (E_6) and $2p_4$ (E_5) levels in neon. To confirm this, a calculation can be performed as follows:

$$hc/\lambda = \Delta E = (E_6 - E_5) = (-4.026 - (-5.990)) \times 1.60 \times 10^{-19}\ \text{J}$$
$$\lambda = 633\ \text{nm (red)}$$

Energy Bands, Conductors and Insulators

3 Which statement about conduction of electricity in solids is correct?

A Free electrons are found both in the conduction band and in the valence band.
B In a metal, there is large energy gap between the conduction and valence bands.
C The presence of impurities in an intrinsic semiconductor is used to increase its resistance.
D In an intrinsic semiconductor, electrons travel in the opposite direction to holes.

N07/I/37

Solution Answer: **D**

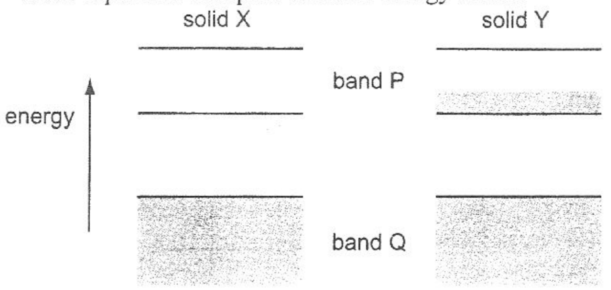
Holes behave like positive charges hence they travel in the opposite direction to electrons under an applied electric field.

Answer A is wrong because free electrons are found only in the conduction band. The valence band is completely filled hence the electrons within the valence band are not free to move to the higher occupied levels within the band.

Answer B is wrong because in a metal, the conduction and valence bands overlap. The conduction band is partially filled hence there is no band gap.

Answer C is wrong because impurities reduce, not increase, the resistance of semiconductors.

4 The diagram illustrates the upper energy bands in two different classes of solid at absolute zero. The shaded areas represent occupied electron energy levels.



What are bands P and Q, and what are the classes X and Y of the solids?

	band P	band Q	solid X	solid Y
A	conduction	valence	intrinsic semiconductor	metal
B	conduction	valence	metal	intrinsic semiconductor
C	valence	conduction	intrinsic semiconductor	metal
D	valence	conduction	metal	intrinsic semiconductor

N07/I/38

Solution Answer: **A**

The higher energy band is the conduction band and the lower energy band the valence band.

At absolute zero, the valence band in a semiconductor is completely filled while the conduction band is empty. There is a band gap between the two bands.

On the other hand, for a metal, the conduction band is partially filled hence there are plenty of empty energy levels for the electrons to be excited to. Hence electrons flow freely in a metal under the application of an electric field.

5 Which statement about the energy bands in an ideal intrinsic semiconductor is correct?

A The conduction band lies just below the valence band.

B The number of electrons in the conduction band equals the number of holes in the valence band.

C There is an energy gap of 5 eV to 10 eV between the valence and conduction bands.

D There is a small overlap between the valence and conduction band.

N09/I/38

Solution Answer: **B**

In an intrinsic semiconductor, each electron in the conduction band is excited from the valence band, leaving behind a hole in the valence band.

Statement A is a reversal of fact.

Statement C is for insulators as semiconductors have band gaps of around 1 eV.

Statement D is for conductors.

6 Energy levels in low-pressure gases are represented as lines whereas in solids, the levels are shown as bands. What is responsible for the formation of bands?

A Atoms in solids are much closer together than those in gases.

B Atoms in solids are much denser than those in gases.

C Solids are better electrical conductors than gases.

D Solids are not fluids but gases are fluids.

N10/I/37

Solution Answer: **A**

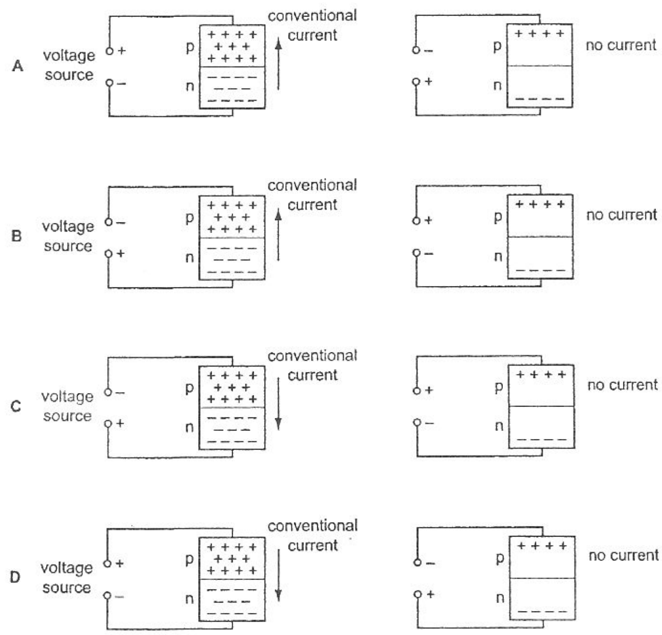
When the atoms are close together, their wave functions overlap and the effect of Pauli exclusion principle (whereby no two electrons in the solid can have the same energy state) becomes pronounced.

Answer B is wrong because although solids are much denser than gases, it is due to the atoms in solids being much closer together than those in gases, rather than the atoms of solids being much denser than those in gases.

Semiconductors

7 In the diagram below, the symbols + + + and − − − represent the majority carriers in the p-type and n-type sides of a p-n junction.

Which pair of diagrams illustrates how a p-n junction acts as a rectifier?



N08/I/37

Solution Answer: **D**

Under forward bias, the p-side is at a higher potential than the n-side, and conventional current flows from p-side to n-side due to the movement of majority carriers (holes from p-side to n-side and electrons from n-side to p-side).

Under reverse bias, the p-side is at a lower potential than the n-side, and the p-n diode does not conduct. There are movements of minority carriers (holes from the n-side and electrons from the p-side), but these are negligibly small.

Short Questions

8 A junction is formed between slices of p-type and n-type semiconductor material, as shown in Fig. 5.1.

Fig. 5.1

(a) On Fig. 5.1, draw an arrow to show the direction of movement of electrons as the two slices are brought into contact. [1]

(b) Describe the origin of the depletion region at the junction. [4]

(c) On Fig. 5.1, draw the symbol for a battery, connected so as to increase the width of the depletion region. [1]

N07/II/5

Solution

(a) The arrow indicating the direction of movement of electrons is shown in Fig. 5.2 .

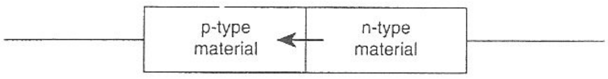


Fig. 5.2

Electrons, the majority charge carrier in the n-type material, diffuse over to the p-type material.

(b) When the p-type and n-type materials come into contact, electrons in the conduction band diffuse from the n-type to the p-type material. At the same time, holes in the valence band diffuse from the p-type to the n-type material.

The recombination of electrons with holes on both sides give rise to a region depleted of electrons and holes, resulting in a depletion region about 1 μm thick.

The absence of charge carriers in the depletion region exposes fixed positively charged donor ions on the n-side of the depletion region, and fixed negatively charged acceptor ions on the p-side.

These region of immobile fixed charges set up an electric field directed from the positively charged n-region to the negatively charged p-region, thus preventing further diffusion of charges in either direction.

(c) To increase the width of the depletion region, the p-n junction has to be in reverse bias. This means the positive terminal of the battery applied to the n-type material, and the negative terminal of the battery applied to the p-type material. Refer to Fig. 5.3.

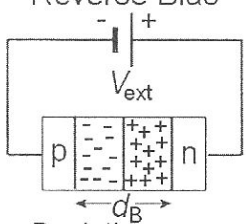
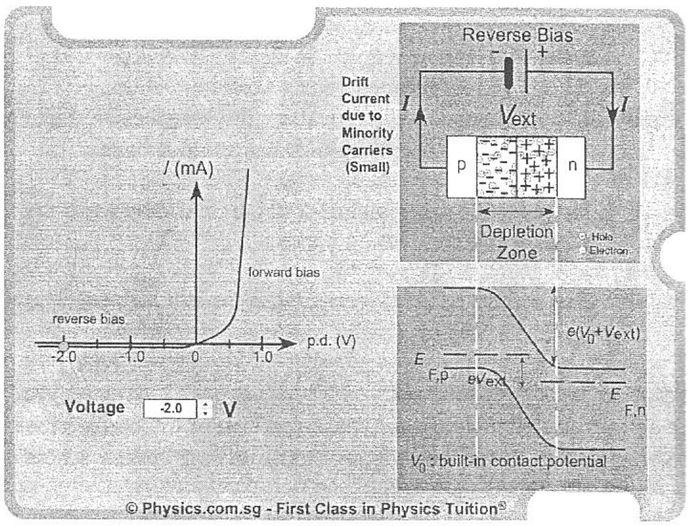


Fig. 5.3

This change in width of the depletion zone can be seen in the following Flash animation.



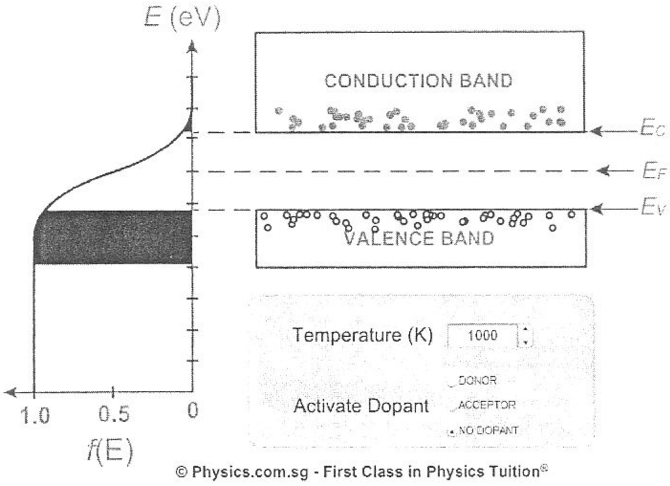
http://www.physics.com.sg/H2/pn_Junction_Diode.swf

9 By reference to the band theory of conduction, explain why the electrical resistance of an intrinsic semiconductor material decreases as its temperature rises.
You may draw a diagram if you wish.

....
....
....
....
....
.... [4]

N08/II/5

Solution
Refer to the Flash animation in Fig. 5.1.



http://www.physics.com.sg/H2/Band_Theory.swf

Fig. 5.1

An intrinsic semiconductor has a small energy band gap (E_g) (approximately 1 eV) between the bottom of the conduction band (E_C) and top of the valence band (E_V) band.

When temperature rises, more electrons in the valence band are thermally excited, across the small band gap, into the conduction band, leaving behind holes in the valence band.

The numerous empty energy states above the occupied states in the conduction band enable electrons in the conduction band to gain energy, when under the application of an external electric field, and contribute to the conduction of electric current.

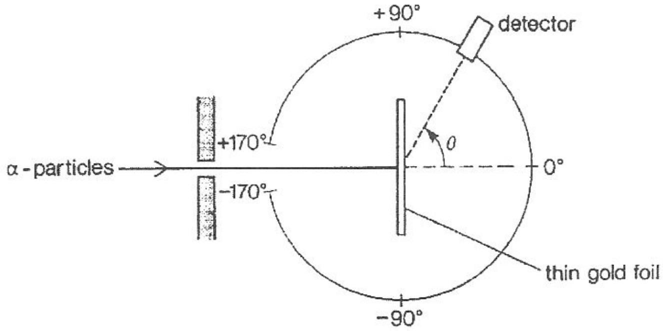
Similarly, vacancies left behind (holes), which behave like positive charges amongst the occupied states in the valence band, also contribute to the conduction of electric current.

The increase in charge carrier density (both electrons and holes) enables a greater current with the same potential difference, hence reducing the resistivity of semiconductor.

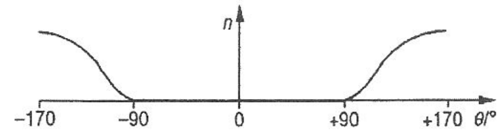
When temperature rises, the Fermi-Dirac distribution function (left side of Fig. 5.1) spreads out, resulting in an increase in the probability of energy states near the bottom of the conduction band being occupied by electrons; and a corresponding vacancy near the top of the valence band.

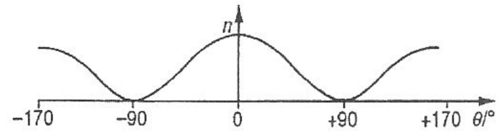
TOPIC 20 Nuclear Physics

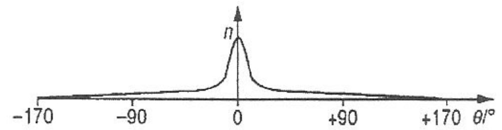
1 In repeating Rutherford's α-particle scattering experiment, a student used the apparatus shown, in a vacuum, to determine n the number of α-particles incident per unit time on a detector held at various angular positions θ.

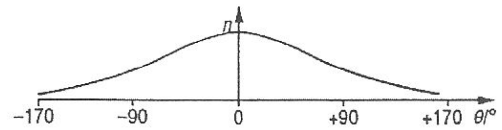


Which graph best represents the variation of n with θ?

A


B


C


D


N02/I/30

Solution Answer: C

Rutherford's alpha-scattering experiment demonstrates that the atom is mostly empty space with a small but massive nucleus at its centre.

Hence most of the α-particles will pass straight through undeflected ($\theta = 0°$).

The count rate falls sharply to both sides of the straight-through position (Answer C, and not gradually as depicted in the normal distribution graph in Answer D).

2 Which equation correctly shows an α-particle causing a nuclear reaction?

A ${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}n$

B ${}^{17}_{8}O + {}^{4}_{2}He \rightarrow {}^{20}_{9}F + {}^{1}_{1}p$

C ${}^{17}_{8}O + {}^{0}_{-1}e \rightarrow {}^{13}_{5}B + {}^{4}_{2}He$

D ${}^{14}_{7}N + {}^{1}_{1}p \rightarrow {}^{11}_{6}C + {}^{4}_{2}He$

N05/I/30

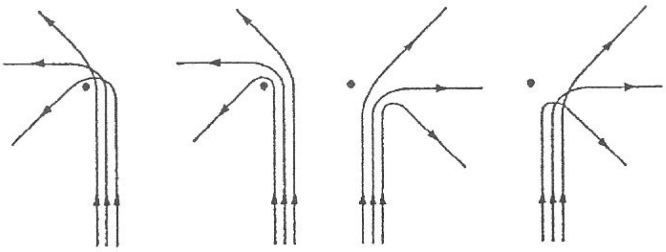
Solution Answer: B

An α-particle is a helium nucleus ${}^{4}_{2}He$ hence the answer can only be either A or B where ${}^{4}_{2}He$ is a reactant.

However, the symbol of a neutron is ${}^{1}_{0}n$ and not ${}^{1}_{1}n$ as depicted in answer A, hence answer A is wrong.

3 Which diagram best represents the paths of α-particles scattered by a gold nucleus?

A B C D



N06/I/29

Solution Answer: D

Both α-particles and the gold nucleus are positive charges, and like charges repel.

The closer an α-particle is to the nucleus, the greater the repulsive force it experiences (Coulomb's law) and the greater the degree of deflection.

4 Nuclei of atoms can exist in excited states. When an excited nucleus returns to its state of lowest energy (the ground state), a γ-ray photon may be emitted.

The mass of a nucleus in its ground state is 59.9308 u. The energy of the photon emitted when this nucleus returns from an excited state to the ground state is 2.13×10^{-13} J.

What is the mass of the nucleus in the excited state?

A 59.9280 u
B 59.9294 u
C 59.9322 u
D 59.9337 u

N07/I/40

Solution Answer: C

Energy of photon $E = \Delta mc^2$ where Δm is the loss in mass and c the speed of light in free space.

$$\Delta m = E/c^2 = 2.13 \times 10^{-13}\text{ J} / (3.00 \times 10^8\text{ m s}^{-1})^2$$

$$= 2.36667 \times 10^{-30}\text{ kg} = 1.4257 \times 10^{-3}\ u$$

The mass of the nucleus in the excited state is thus

$$59.9308\ u + 1.4257 \times 10^{-3}\ u = 59.9322\ u$$

5 The nucleus Z has the notation ${}^{y}_{x}Z$.

The mass defect of this nucleus is Δm.

What is the binding energy per nucleon of the nucleus?

A $\dfrac{\Delta m}{x}$ B $\dfrac{\Delta m}{y}$ C $\dfrac{c^2\Delta m}{x}$ D $\dfrac{c^2\Delta m}{y}$

N09/I/39

Solution Answer: **D**

The binding energy of the whole nucleus can be found using Einstein's mass-energy equivalence equation $E = \Delta mc^2$.

The number of nucleons in the nucleus is its nucleon number y.

6 Which pair of nuclides has nuclei containing the same number of neutrons?

A ${}^{107}_{47}Ag$ and ${}^{104}_{45}Rh$

B ${}^{109}_{47}Ag$ and ${}^{109}_{46}Pd$

C ${}^{108}_{46}Pd$ and ${}^{109}_{47}Ag$

D ${}^{105}_{45}Rh$ and ${}^{106}_{45}Rh$

N10/I/39

Solution Answer: **C**

In the nuclide notation ${}^{A}_{Z}X$, Z is the atomic or proton number while A the mass or nucleon number (proton plus neutron). The number of neutrons = A – Z. For Pd it is 108 – 46 = 62 while for Ag it is 109 – 47 = 62.

Short Questions

7 When an α-particle bombards a stationary nitrogen-14 nucleus, the following nuclear reaction may occur.

$$ {}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + X $$

The rest masses of the nuclei are

${}^{14}_{7}N$, 14.007525 u
${}^{4}_{2}He$, 4.003860 u
${}^{17}_{8}O$, 17.004507 u
X, 1.008142 u

(a) Identify the nucleus represented by the symbol X. [1]

(b) (i) Deduce that the change in rest-mass energy in this reaction is approximately 1.9 x 10^{-13} J. [2]

(ii) By reference to energy, suggest how it is possible for this reaction to occur. [2]

(iii) The oxygen-17 nucleus and the nucleus X move apart after the reaction.
Describe the effect of this movement on your answer in (ii). [2]

N07/II/6

Solution

(a) ${}^{1}_{1}H$ (proton).

(b) (i) The change in rest-mass in this reaction is

$$(17.004507\ u + 1.008142\ u) - (14.007525\ u + 4.003860\ u)$$

$$\Delta m = 1.264 \times 10^{-3}\ u$$

$$= 1.264 \times 10^{-3}\ (1.66 \times 10^{-27}\ \text{kg}) = 2.098 \times 10^{-30}\ \text{kg}$$

The change in rest-mass energy

$$E = \Delta mc^2 = (2.098 \times 10^{-30}\ \text{kg})(3.00 \times 10^{8}\ \text{m s}^{-1})^2 = 1.9 \times 10^{-13}\ \text{J}$$

(ii) The combined rest-mass of the products is greater than that of the reactants. This means that energy has to be supplied to enable the reaction to occur.

The energy is supplied from the kinetic energy of the α-particle, which is converted into mass through mass-energy equivalence $E = \Delta mc^2$.

(iii) The fact that the products possess kinetic energy means that the kinetic energy of the α-particle has to be greater than the increase in rest-mass energy (1.9 x 10^{-13} J).
The kinetic energy of the α-particle has to be equal to 1.9 x 10^{-13} J + total kinetic energy of the oxygen and proton after the reaction.

Long Questions

8 (a) An unstable nucleus of nucleon number (mass number) A undergoes α-decay, as illustrated in Fig. 8.1.

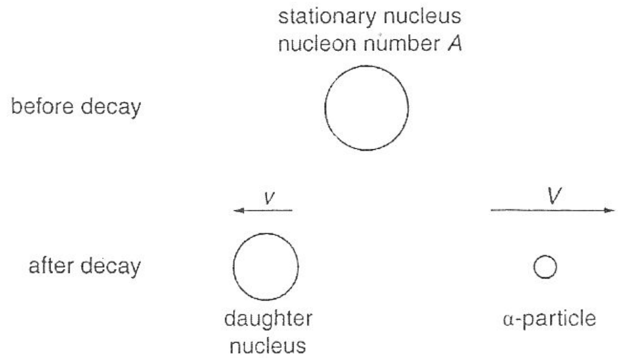


Fig. 8.1

The nucleus is stationary before the decay.
After the decay, the initial speed of the α-particle is V and that of the daughter nucleus is v.

(i) State an equation, in terms of A, v and V, to represent conservation of linear momentum for this decay. [2]

(ii) Show that the ratio

$$\frac{\text{initial kinetic energy of α-particle}}{\text{initial kinetic energy of daughter nucleus}}$$

is equal to ($\frac{1}{4}A$ – 1). [3]

(b) Data for the α-decay of bismuth-212 ($^{212}_{83}Bi$) to form thallium-208 ($^{208}_{81}Tl$) are given in Fig. 8.2.

nucleus	mass of nucleus / u
bismuth-212	211.9459
thallium-208	207.9374
helium-4	4.0015

Fig. 8.2

(i) Use the data of Fig. 8.2 to calculate, to two places of decimals, the energy released during the decay.
energy = ... MeV [4]

(ii) Use your answer in (i) to show that, based on the expression in (a)(ii), the energy of the α-particle is 6.42 MeV. [2]

(c) In practice, the α-particle is found to have an energy of 6.10 MeV, rather than 6.42 MeV as calculated in (b)(ii).
Suggest
(i) an explanation for the difference in energy, [1]
(ii) why it is likely that the thallium nucleus and the α-particle do not move off in opposite directions. [3]

N10/III/8 (part)

Solution

(a) (i) $0 = -(A-4)v + 4V$.

(ii) From (i), $V = \frac{1}{4}(A-4)v$.

Hence $KE_\alpha / KE_{\text{daughter}}$
$= \frac{1}{2}(4u)V^2 / \frac{1}{2}(A-4)uv^2$
$= 4V^2 / (A-4)v^2$
$= 4(\frac{1}{4}(A-4)v)^2 / (A-4)v^2$
$= \frac{1}{4}(A-4)$
$= (\frac{1}{4}A - 1)$.
where u is the unified atomic mass constant (which is cancelled out on both sides of the equation).

(b) (i) Change in mass Δm
$= (211.9459\ u) - (207.9374 + 4.0015)\ u$
$= 0.007$ u
$= 0.007 \times 1.66 \times 10^{-27}$ kg
$= 1.162 \times 10^{-29}$ kg

$E = \Delta mc^2$
$= (1.162 \times 10^{-29})(3.00 \times 10^8)^2$
$= 1.0458 \times 10^{-12}$ J
$= (1.0458 \times 10^{-12})/(1.60 \times 10^{-13})$ MeV
$= 6.53625$ MeV
$= 6.54$ MeV (2 d.p.)

Checking: 1 u ≡ 931.4 Mev
Hence 0.007 u = 0.007 x 931.4 = 6.52 MeV.
Slight discrepancy due to the different number of significant figures used for the constants.

(ii) $KE_\alpha / KE_{\text{thallium}} = (\frac{1}{4}A - 1)$.
$= (\frac{1}{4}(212) - 1) = 52$
Hence $KE_\alpha = (52/53)(6.54\text{ MeV}) = 6.42$ MeV.

(c) (i) The energy released is not purely in the form of kinetic energy of the daughter nucleus and α-particle. Some energy is released as γ-radiation.

Gamma decay accompanies either alpha or beta-decay whereby the product-nucleus' energy is too high. The nucleus relaxes from the excited state in which it is first formed to a lower energy state, and gives off the excess energy as gamma radiation.

$$^{212}_{83}Bi \rightarrow {}^{208}_{81}Tl^* + {}^{4}_{2}He + \text{energy}$$
$$^{208}_{81}Tl^* \rightarrow {}^{208}_{81}Tl + \gamma$$

*where the asterisk * refers to the excited state.*

(ii) The γ-photon has a small but non-zero momentum ($p = h/\lambda$ where h is the Planck constant and λ the wavelength of the radiation).

The γ-photon will likely move off in a direction that is not collinear with that of the thallium nucleus and α-particle, carrying with it a component of momentum perpendicular to the velocities indicated in this question.

Since momentum must be conserved in the perpendicular direction as well, the thallium nucleus and α-particle cannot move off in opposite directions.

Radioactive Decay

9 Which line in the table describes the nature of α-radiation, β-radiation and γ-radiation?

	α-radiation	β-radiation	γ-radiation
A	electrons	helium nuclei	uncharged particles
B	helium nuclei	electrons	uncharged particles
C	electrons	helium nuclei	packets of wave energy
D	helium nuclei	electrons	packets of wave energy

N03/I/29

Solution Answer: **D**

α-particles are helium nuclei with the nuclide notation ^4_2He.
β-particles are high speed electrons with nuclide notation $^0_{-1}e$.
Gamma-radiation is electromagnetic radiation which can be considered either as a wave or as being made up of photons, which are packets of wave energy (answer D).
Photons can also be considered as particles with no mass and no charge (answer C), but answer D is a more exact description of the photon.

10 Samples of two radioactive nuclides, **X** and **Y**, each have equal activity A_0 at time $t = 0$. **X** has a half-life of 24 years and **Y** a half-life of 16 years.

The samples are mixed together.

What will be the total activity of the mixture at $t = 48$ years?

A $\frac{1}{12}A_0$

B $\frac{3}{16}A_0$

C $\frac{1}{4}A_0$

D $\frac{3}{8}A_0$

N03/I/30

Solution Answer: **D**

48 years is two half-lives of nuclide **X** hence **X**'s activity has decreased to $(\frac{1}{2})^2$ or $\frac{1}{4}A_0$.

48 years is 3 half-lives of nuclide **Y** hence **Y**'s activity has decreased to $(\frac{1}{2})^3$ or $\frac{1}{8}A_0$.

The total activity of the mixture is $\frac{1}{4}A_0 + \frac{1}{8}A_0 = \frac{3}{8}A_0$.

11 The grid shows a number of nuclides arranged according to the number of protons and the number of neutrons in each.

A nucleus of the nuclide $^{8}_{3}$Li decays by emitting a β-particle.

What is the resulting nuclide?

number of protons							
4					A	B	
3				$^{6}_{3}$Li	$^{7}_{3}$Li	$^{8}_{3}$Li	
2		$^{3}_{2}$He	$^{4}_{2}$He			C	D
1	$^{1}_{1}$H	$^{2}_{1}$H					
number of neutrons	0	1	2	3	4	5	6

N04/I/30

Solution Answer: **A**

$^{8}_{3}Li \rightarrow {}^{8}_{4}X + {}^{0}_{-1}e$ + energy.

The resulting nuclide X has 4 protons and 4 neutrons.

One neutron in the nucleus has transformed into one proton and one electron $^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e$.

12 The initial activity of a sample of a radioactive isotope containing N_0 nuclei is A_0.

What is the number of unchanged nuclei when the activity has declined to $\frac{A_0}{2}$?

A $0.69\,N_0$ **C** $\frac{N_0}{2}$

B $\frac{0.69\,N_0}{2}$ **D** $\frac{N_0}{1.38}$

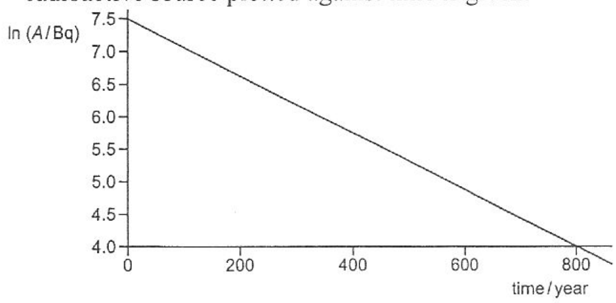
N06/I/30

Solution Answer: **C**

Activity A is directly proportional to the number of unchanged nuclei N ($A = \lambda N$) where λ is the decay constant.

When the activity has declined to $A_0/2$, the number of unchanged nuclei has correspondingly decreased to $N_0/2$.

13 A graph of the natural logarithm of the activity A of a radioactive source plotted against time is given.



What is the half-life of the source in years?

A 0.0044 **B** 160 **C** 400 **D** 860

N07/I/39

Solution Answer: **B**

Activity $A = A_0 \exp(-\lambda t) = A_0 \exp((-\ln 2 / T_{½})t)$

Natural log both sides of the equation

$$\ln A = \ln A_0 - (\ln 2 / T_{½})t$$

The gradient of the graph is

$$-(\ln 2 / T_{½}) = -(7.5 - 4.0) / 800 = -3.5 / (800 \text{ yrs})$$

$$T_{½} = 160 \text{ years}$$

14 A detector of ionising radiation gives a background count rate of 24 per minute.

A radioactive source is placed close to the detector and the reading is 532 counts per minute.

What will be the reading after two half-lives of the source?

A 127 **B** 133 **C** 151 **D** 157

N08/I/39

Solution Answer: **C**

The original true count rate is 532 – 24 = 508 counts per minute. After two half lives, it falls to $(\frac{1}{2})^2 \times 508 = 127$ counts per minute.

Adding back the background count of 24 counts per minute, the reading is 127 + 24 = 151 counts per minute.

15 A nucleus of bohrium $^{x}_{y}$Bh decays to mendelevium $^{255}_{101}$Md by a sequence of three α-particle emissions.

bohrium $^{x}_{y}$Bh → dubnium + α
→ lawrencium + α
→ mendelevium $^{255}_{101}$Md + α

How many neutrons are there in a nucleus of $^{x}_{y}$Bh?

A 267 **B** 261 **C** 160 **D** 154

N08/I/40

Solution Answer: **C**

$^{x}_{y}Bh \rightarrow {}^{255}_{101}Md + 3\,{}^{4}_{2}He$

x = 255 + 3(4) = 267.

y = 101 + 3(2) = 107.

Number of neutrons in bohrium = 267 – 107 = 160.

16 Which sample of nuclide has the greatest initial activity?

	nuclide	amount/mole	half-life/day
A	$^{225}_{89}Ac$	0.003	10
B	$^{228}_{90}Th$	0.1	400
C	$^{228}_{88}Ra$	0.6	2100
D	$^{241}_{94}Pu$	1.0	4800

N09/I/40

Solution Answer: A

Activity $A = \lambda N$ where λ is the decay constant ($\lambda = \ln 2 / T_{½}$) and N the number of nuclei.

Therefore, the nuclide with the greatest ratio of $N/T_{½}$ will have the highest activity, which are, respectively for the four answers: 0.00030; 0.00025; 0.00029; 0.00021.

17 Antimony-124 undergoes radioactive decay, with a half-life of 60 days, to become tin-124. Tin-124 is stable. Initially, a sample of antimony-124 contains no tin-124. For this sample, after what period of time will the ratio $\frac{\text{number of tin-124 nuclei}}{\text{number of antimony-124 nuclei}}$ equal 6?

A between 60 days and 120 days
B 120 days
C between 120 days and 180 days
D 180 days

N10/I/40

Solution Answer: C

After one half-life (60 days), half the antimony has decayed into tin hence the ratio is 1:1. After two half-lives (120 days), there is only ¼ of the initial antimony left hence the ratio is (3/4):(1/4) = 3:1. After three half-lives (180 days), there is only 1/8 of the initial antimony left hence the ratio is (7/8):(1/8) = 7:1. Therefore the ratio 6:1 is achieved between 120 and 180 days.

<u>Exact Calculation</u>

For the ratio of tin:antimony to be 6:1, there will only be 1/7 of the initial antimony left:

$$N = N_0 \exp(-\ln 2/T_{½})t$$
$$1/7 = \exp(-\ln 2/60)t$$
$$t = 168 \text{ days}$$

Short Questions

18 The nuclide Potassium-42 ($^{42}_{19}K$) undergoes radioactive decay to become Calcium-42 ($^{42}_{20}Ca$). A radioactive sample contains N_0 atoms of Potassium-42 and no atoms of Calcium-42 at time $t = 0$. Fig. 7.1 shows the variation with time t of the number N of atoms of Potassium-42.

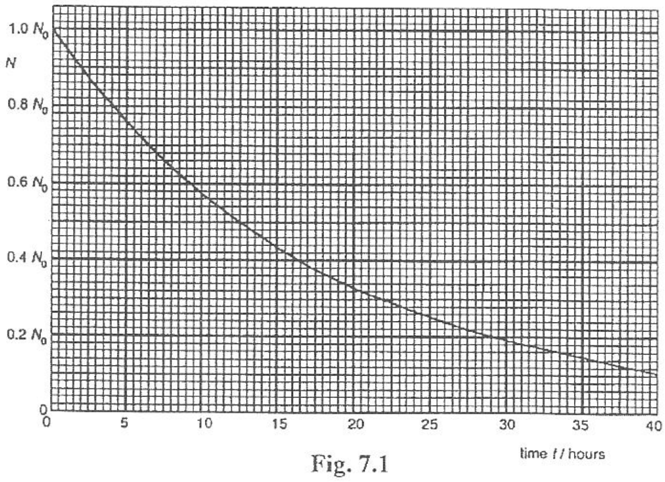


Fig. 7.1

(a) State the particles emitted in the radioactive decay of Potassium-42. [1]

(b) Use Fig. 7.1 to estimate the half-life of Potassium-42. [1]

(c) Calcium-42 is stable. Using the axes of Fig. 7.1, sketch a graph to show the variation with time t of the number of Calcium-42 atoms in the sample. [1]

(d) By reference to Fig. 7.1, or otherwise, determine the age of the radioactive sample when the ratio

$$\frac{\text{number of Calcium-42 atoms}}{\text{number of Potassium-42 atoms}}$$

is equal to 4.0. [2]

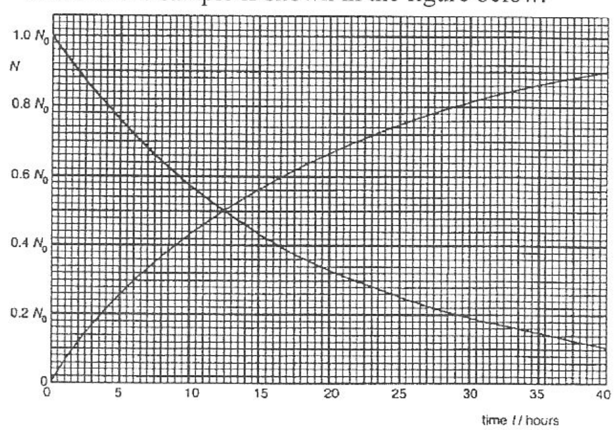
N04/II/7

Solution

(a) β particles ($^{0}_{-1}e$).

(b) 12.5 hours.

(c) The variation with time t of the number of Calsium-42 atoms in the sample is shown in the figure below.



(d) Let t be the time when the ratio of Calcium-42 to Potassium-42 atoms is 4.0.

The decay constant
$\lambda = \ln 2 / T_{½} = \ln 2 / (12.5 \text{ hrs}) = 0.05545 \text{ hr}^{-1}$.

$$(N_0 - N_0 e^{-\lambda t})/(N_0 e^{-\lambda t}) = 4.0$$

$$1 - e^{-0.05545\, t} = 4.0\, e^{-0.05545\, t}$$

$$5.0\, e^{-0.05545\, t} = 1$$

$$t = 29 \text{ hours}$$

Alternatively, from the graph, it can be seen that at $t = 29$ hours, the number of Calcium-42 atoms is $0.80\, N_0$ while that of Potassium-42 atoms is $0.20\, N_0$, a ratio of 4.0.

19 The fraction of atoms of the isotope Uranium-235 in samples of natural uranium ore taken from most parts of the Earth at the present time is very similar. This fraction is approximately $\frac{1}{140}$

For the fission of Uranium-235 to occur continuously, the fraction of Uranium-235 in natural uranium ore must be approximately $\frac{1}{20}$

The half-life of Uranium-235 is 7.0×10^8 years.

(a) Explain what is meant by
- (i) *isotopes*, [2]
- (ii) *fission*, [2]
- (iii) *half-life*. [2]

(b) Determine the time that has elapsed since the fraction of Uranium-235 in natural uranium ore was $\frac{1}{20}$

time = ... years [3]

(c) At one location in Gabon, West Africa, the fraction of Uranium-235 in uranium ore is now only $\frac{1}{2300}$.
Suggest an explanation for this value. [1]

N06/II/7

Solution

(a) (i) *Isotopes* are nuclides of the same element with the same atomic number but different mass numbers.

(ii) *Fission* is a type of nuclear reaction in which a heavy nucleus splits into fragments of comparable mass, losing mass and releasing energy in the process.

(iii) *Half-life* is the expected time for half the remaining unstable nuclei in a radioactive sample to decay.

(b) The decay constant $\lambda = \ln 2 / T_{½} = 9.902 \times 10^{-10} \text{ yr}^{-1}$.

$$N = N_0 \exp(-\lambda t)$$

$$\exp(-9.902 \times 10^{-10} \text{ yr}^{-1}\, t) = N / N_0 = (1/140)/(1/20)$$

$$-9.902 \times 10^{-10} \text{ yr}^{-1}\, t = \ln(20/140) = \ln(1/7)$$

$$t = 1.965 \times 10^9 = 2.0 \times 10^9 \text{ years}$$

(c) A large fraction of the Uranium-235 might have been used up in naturally-occurring underground fission reactions.

This could have occurred during the early formative years of the Earth, when the fraction of Uranium-235 in natural uranium ore was greater than 1/20 and hence sufficient to sustain nuclear fission.

20 Strontium-90 is a radioactive nuclide.

(a) Explain what is meant by *radioactive.* [2]

(b) A sample of Strontium-90 has a mass of 2.40×10^{-8} g. The average activity of this sample during a period of 1 hour is found to be 1.26×10^5 Bq.

(i) Define *decay constant.* [2]

(ii) Calculate the decay constant of Strontium-90.

decay constant = ... s^{-1} [3]

(c) Suggest why the determination of decay constant by measuring the mass and activity of a sample can be used only for nuclides that have relatively small decay constants. [1]

N08/II/6

Solution

(a) A nuclide is radioactive if it is unstable and may undergo random and spontaneous disintegration, and structural changes within its nucleus, to transform into another nuclide. In the process, radiation may be emitted in the form of either α-particles, β-particles, γ radiation or a combination of these.

(b) (i) *Decay constant* is defined as the probability of a radioactive nucleus disintegrating per unit time interval.

(ii) 90 g of Strontium-90 contains approximately one mole of Strontium atoms. Hence the number of atoms in 2.40×10^{-8} g of Strontium-90 is:

$$N = (2.40 \times 10^{-8} \text{ g} / 90 \text{ g}) \times 6.02 \times 10^{23} = 1.6053 \times 10^{14}$$

Activity A is related to the number of radioactive nuclides by the equation:

$$A = \lambda N$$

where λ is the decay constant of Strontium-90.

$$\lambda = A/N$$
$$= (1.26 \times 10^5 \text{ Bq})/(1.6053 \times 10^{14}) = 7.85 \times 10^{-10} \text{ s}^{-1}$$

(c) The activity in radioactive decay is not a constant value, but decreases with time as the number of radioactive nuclide remaining falls.

However, if the decay constant λ is small, then the half life is long (since half life $T_{½} = \ln 2 / \lambda$). Hence over a relatively short time interval over which the activity is measured (e.g. 1 hour as in this question), the activity A remains fairly stable, and can be assumed to be constant.

Long Questions

21 *(a)* State the atomic composition of a single neutral atom of the nuclide whose symbol is

$$^{12}_{6}\text{C}.$$ [3]

(b) Define the terms

(i) decay constant,

(ii) half-life [3]

(c) Explain why the random nature of radioactive decay makes it difficult to measure the values of the terms in *(b)* to a high degree of accuracy. [2]

(d) Measurements are made of the activity of a specimen of carbon from pieces of wood found in a fireplace at an archaeological site. The specimen is found to contain one Carbon-14 atom per 8.6×10^{10} Carbon-12 atoms. In similar carbon from a modern fire the concentration of Carbon-14 atoms is greater at one Carbon-14 atom per 3.3×10^{10} Carbon-12 atoms. The difference between these two figures is because Carbon-14 is radioactive and some atoms have decayed over the years.

Calculate the age of the wood from the ancient fire. The half-life of Carbon-14 is 1.8×10^{11} s. Carbon-12 is stable. [6]

(e) The technique of dating described in *(d)* is difficult to do accurately. This difficulty can be minimised by using all the Carbon-14 atoms rather than just those which happen to undergo radioactive decay when the dating is being carried out. Carbon atoms from the wood can be ionised by removing one electron from each atom. They are then formed into a beam which is passed through a magnetic field as shown in Fig. 5.1

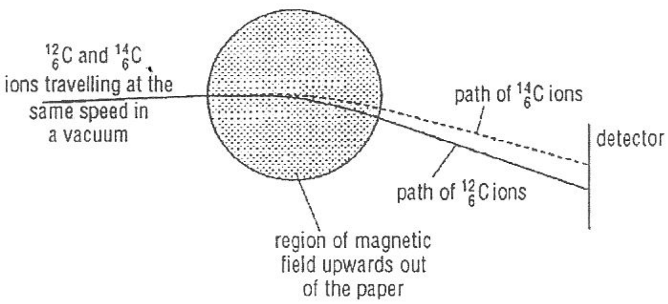


Fig. 5.1

(i) Explain why the paths of the two types of ion are different.

(ii) Explain why the three sections of each path followed by the ions are straight, circular, straight, and why there is no change in the speed of an ion throughout.

(iii) Suggest why this method of measuring the ratio of Carbon-14 to Carbon-12 atoms is more reliable. [6]

N02/III/5

Solution

(a) A single neutral atom of $^{12}{}_{6}$C has 6 protons, 6 neutrons and 6 orbital electrons.

(b) (i) Decay constant is defined as the probability of a radioactive nucleus disintegrating per unit time interval.

(iii) Half-life is the expected time for half the remaining radioactive nuclei to decay.

Its unit is the second (s).

(c) Since radioactive decay is random, it is not possible to predict exactly which radioactive nucleus in a given sample will disintegrate in unit time interval.

The result may fluctuate from measurement to measurement.

For the same reason, the time taken for half the remaining radioactive nuclei to disintegrate also fluctuates due to the random nature of radioactive decay.

Sometimes it may take longer and sometimes shorter than the expected time, making accurate measurements of half-life difficult to achieve, especially for low count rates.

(d) $N = N_0 \exp(-\lambda t) = N_0 \exp(-(\ln 2 / T_{½})t)$

$(1/8.6 \times 10^{10}) = (1/3.3 \times 10^{10}) \exp(-(\ln 2 / 1.8 \times 10^{11} \text{ s})t)$

$t = 2.487 \times 10^{11} = 2.5 \times 10^{11}$ s.

(e) (i) The magnetic force provides the centripetal force hence $Bqv = mv^2/r$, where B is the magnetic flux density, q the charge, m the mass, v the velocity and r the radius of curvature of the carbon ions.

Rearranging the equation, $r = mv/Bq$. Since v, B and q are the same for both ions, $r \propto m$.

Carbon-14 has a greater mass than carbon-12 hence it has a larger radius of curvature than that of carbon-12.

(ii) Before entering the magnetic field and also after leaving the magnetic field, there is no resultant force acting on the ions.

There is no electric or magnetic field, and the vacuum offers no resistance to the motion. According to Newton's first law of motion, a body in motion will continue moving in a straight line with uniform speed, provided no net external force acts on it.

Inside the magnetic field, a moving charge experiences a magnetic force at right angles to the plane containing the field and the charge's velocity. The direction can be found using Fleming's left-hand-rule. This magnetic force provides a centripetal force causing the ions to move in a circle.

Since this force is perpendicular to the ions' velocity, the resulting acceleration known as centripetal acceleration is also perpendicular to the velocity. The centripetal acceleration changes only the direction but not the magnitude of the velocity.

(iii) This method measures the ratio directly, and the random error is extremely small since the half-life of Carbon-14 is very long.

On the other hand, the other method of measuring the activity is subject to high random errors due to the random nature of radioactive decay.

Furthermore, there is background radiation which needs to be taken into account and eliminated.

22 *(a)* Some elements that are normally stable, such as lead (Pb), have isotopes which are radioactive. The nucleus $^{214}_{82}$Pb is one such isotope of lead.

(i) State what is meant by *isotopes*. [1]

(ii) Determine the constituents of a nucleus of $^{214}_{82}$Pb from its nucleon number and proton number. [2]

(b) A nucleus of $^{214}_{82}$Pb decays by β emission into $^{214}_{83}$Bi with a half-life of 27 minutes. This bismuth nuclide is itself radioactive with an unusual decay pattern. Sometimes it decays by α emission into tellurium (Tl) and sometimes by β emission into polonium (Po). Give nuclear equations for these two decays of $^{214}_{83}$Bi. [4]

(c) The two decay patterns of the $^{214}_{83}$Bi each give rise to γ ray photons. Suggest why each of these photons have different energies. [2]

(d) (i) Without numerical values, sketch a labelled graph to show how, starting with only $^{214}_{82}$Pb nuclei, the activity of the $^{214}_{82}$Pb changes with time. Do not use graph paper. [1]

(ii) On the same axes, sketch another graph to suggest how the activity of the $^{214}_{83}$Bi nuclei formed from the $^{214}_{82}$Pb nuclei changes with time. [2]

(e) A sample of $^{214}_{82}$Pb has mass 2.6 μg at time t = 0. Calculate

(i) approximately how many atoms it contains, [2]

(ii) its decay constant, [1]

(iii) its activity at time t = 0, [2]

(iv) the time at which its activity has fallen to 8.3×10^9 Bq. [3]

N05/III/5

Solution

(a) (i) Isotopes refer to two or more nuclides that have the same proton number but different nucleon numbers.

(ii) $^{214}_{82}$Pb has 82 protons and 132 neutrons.
The number of neutrons is obtained by subtracting the proton number (82) from the nucleon number (214).

(b) $^{214}_{83}\text{Bi} \rightarrow {}^{210}_{81}\text{Tl} + {}^{4}_{2}\text{He}$ + energy

$^{214}_{83}\text{Bi} \rightarrow {}^{214}_{84}\text{Po} + {}^{0}_{-1}\text{e}$ + energy

(c) Gamma decay accompanies either alpha or beta-decay whereby the product-nucleus' energy is too high.

The nucleus relaxes from the excited state in which it is first formed to a lower energy state, and gives off the excess energy as gamma radiation.

The energy of the gamma ray photon given off is equal to the difference in the quantum energy levels within the nucleus across which the transition takes place.

Since thallium and polonium have different nuclear structures, their nuclear energy levels are different. Hence the energy of the gamma ray photons given off is also different

(d) (i) The labelled graph is in Fig. 5.1.
The activity of $^{214}_{82}$Pb decreases exponentially with time following the radioactive exponential decay curve.
The activity A is proportional to the number of radioactive nuclides N remaining ($A = \lambda N$ where λ is the decay constant).
As the number of radioactive nuclides of $^{214}_{82}$Pb decreases exponentially, so does its activity.

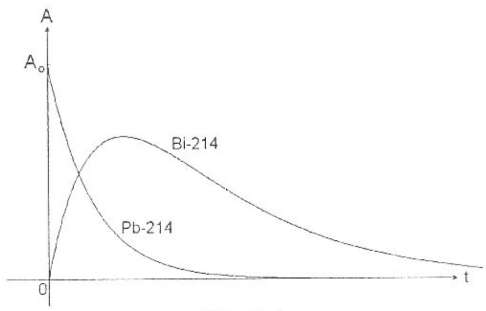


Fig. 5.1

(ii) The graph for the activity of $^{214}_{83}$Bi is also in Fig. 5.1.
Initially there was no $^{214}_{83}$Bi hence its activity starts from zero.
As more $^{214}_{83}$Bi are formed from $^{214}_{82}$Pb, its activity increases.
However, over time, as the $^{214}_{82}$Pb is exhausted and the number of $^{214}_{83}$Bi starts decreasing as well, its activity also decreases exponentially.

(e) (i) The mass of one $^{214}_{82}$Pb is approximately 214 u = 214 x 1.66×10^{-27} kg = 3.5524×10^{-25} kg. Therefore, in 2.6 μg, there are:

$$2.6 \times 10^{-6} \times 10^{-3} \text{ kg} / 3.5524 \times 10^{-25} \text{ kg}$$

$$= 7.3190 \times 10^{15} = 7.3 \times 10^{15} \text{ atoms}$$

(ii) Decay constant

$$\lambda = \ln 2 / T_{½} = \ln 2 / (27 \times 60 \text{ s}) = 4.279 \times 10^{-4}$$
$$= 4.3 \times 10^{-4} \text{ s}^{-1}$$

(iii) Activity

$A = \lambda N = 4.279 \times 10^{-4}\ s^{-1} \times 7.3190 \times 10^{15}$
$= 3.132 \times 10^{12} = 3.1 \times 10^{12}$ Bq

(iv) $A = A_0\ e^{-\lambda t}$

8.3×10^9 Bq = $(3.1 \times 10^{12}$ Bq) $\exp(-4.279 \times 10^{-4}\ t)$
t = 13,840 s = 230 minutes

23 *(a)* Define *electric field strength.* [2]

(b) Two parallel plates are separated by a distance of 0.0080 m and have a potential difference between them of 170V. Calculate the electrical force on a β-particle travelling in the space between the plates. [3]

**(c)* A β-particle is travelling with velocity 7.3×10^7 m s^{-1} at right angles to a magnetic field of field strength B. Calculate the value of B that causes the same force on the β-particle in *(b)*. [3]

**(d)* Draw a sketch showing how the two fields of *(b)* and *(c)* may be arranged in order to give zero resultant force on the beta particle. [2]

**(e)* To illustrate the effect of an electric field on the paths of α, β and γ radiations when travelling in a vacuum, a book contains a diagram similar to Fig. 5.1.

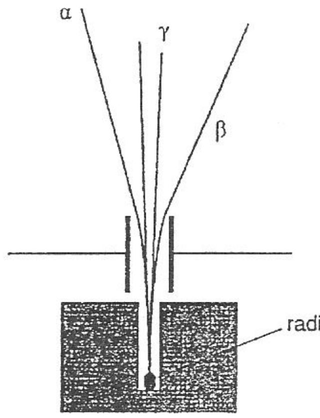


Fig. 5.1

Explain why

(i) no curvature of the gamma radiation takes place, [1]

(ii) the paths of the α and β curve when in the electric field, [2]

(iii) α and β radiations curve in opposite directions, [1]

(iv) there is no need to consider the gravitational field within which the particles are travelling. [2]

**(f)* For the paths shown in (e), suggest

(i) two reasons why a pattern of tracks, such as that shown in Fig. 5.1, is impossible to observe in practice, [2]

(ii) why it is unlikely that, from a given source of radiation, both alpha and beta particles have the same speed. [2]

N06/III/5

Solution

(a) The *electric field strength* at a point in an electric field is defined as the electric force per unit positive charge on a small test charge placed at that point.

The direction of the electric field is the direction of the electric force.

(b) The electric field strength between parallel plates $E = V/d$ where V is the potential difference between the plates and d the separation between the two plates.

$E = V/d$ = 170 V / 0.0080 m = 21250 V m^{-1} or N C^{-1}

The electric force F_E acting on a β-particle (which is an electron with a charge $q = -e = -1.60 \times 10^{-19}$ C), is given by:

$$F = qE = eE$$

$= (1.60 \times 10^{-19}$ C)(21250 N C^{-1}) $= 3.4 \times 10^{-15}$ N

(c) The magnetic force F_B acting on a charge q moving with velocity v at right angles to a magnetic field of flux density B is given by:

$F_B = Bqv = Bev = 3.4 \times 10^{-15}$ N

$B(1.60 \times 10^{-19}$ C)$(7.3 \times 10^7$ m s$^{-1}) = 3.4 \times 10^{-15}$ N

$B = 2.911 \times 10^{-4} = 2.9 \times 10^{-4}$ T

(d) The two fields can be arranged at right angles so that the electric force F_E and magnetic force F_B are equal in magnitude but opposite in direction, as shown in Fig. 5.2 below.

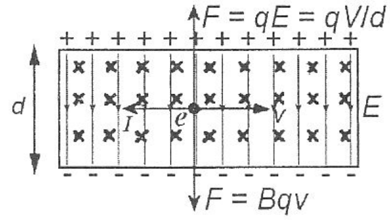


Fig. 5.2

(e) (i) Gamma radiation has no charge hence the magnetic force acting on it $F_B = Bqv = 0$. With no net force acting on the radiation, the photons will move in a straight line with uniform speed (Newton's first law of motion).

(ii) α-particles are Helium nuclei with a net positive charge of $+2e$. β-particles are electrons with a net negative charge of –e.

Charged particles in an electric field experience an electric force $F_E = qE$ causing them to curve in a parabolic path.

(iii) α-particles have a positive charge of +2e hence they experience a force in the same direction as the electric field; β-particles have a negative charge of –e hence they experience a force in the opposite direction to the electric field. Thus they curve in opposite directions.

(iv) Due to the small masses of the particles involved, the gravitational force is many orders of magnitude smaller than the electric force and can be neglected without much effect.

(f) (i) 1. Gamma radiation has very weak ionizing power hence even when placed in a diffusion cloud chamber, the tracks they create are hardly visible.

2. The electric force acting on the α to β-particle may be of the ratio 2:1 but the mass of the β to α-particle is 1:7300.

Hence the β-particle's acceleration is 3650 times that of the α-particle. The curvature of the two particles will be so different that it would be impossible to observe their curvature in the same photographic frame like in Fig. 5.1.

A similar question appeared in G.C.E. 'A' Level Physics June 1991 Paper 3 Question 6(e).

(ii) For a given source of radiation, the energy of α-particles has a small variance, whereas that of β-particles varies over a wide range.

Even though an α-particle typically has energy one order of magnitude higher than that of a β-particle, the mass of an α-particle is 7300 times that of a β-particle. Therefore, the speed of α-particles is less than that of β-particles, on average.

24 (a) Cobalt-60 is a beta-emitter.
A student has a sample of cobalt-60 inside a box. The student defines the half-life of cobalt-60 as the time taken for the number of nuclei inside the box to decay to one half of its initial value.

State and explain one reason why this definition is inappropriate.

....

....

.... [2]

(b) A mass of 4.3 x 10^{-9} g of cobalt-60 is found to have an activity of 1.8 x 10^{5} Bq.
Calculate the half-life, in years, of cobalt-60.
half-life = ... years [4]

N09/III/4

Solution

(a) Radioactive decay is a random process. It is not possible to predict exactly how many nuclei will decay per unit time interval.

Half-life should be defined, based on the average of a large number of samples, as the time when the expected number of decayed nuclei is equal to half its initial value.

(b) The mass of one cobalt-60 atom is approximately 60 u (where u is the unified atomic mass constant). The number of cobalt-60 atoms in 4.3 x 10^{-9} g:

$$N = (4.3 \times 10^{-9} \times 10^{-3}\text{ kg}) / (60\ u)$$

$$= (4.3 \times 10^{-9} \times 10^{-3}\text{ kg}) / (60 \times 1.66 \times 10^{-27}\text{ kg})$$

$$= 4.317 \times 10^{13}$$

Activity $A = \lambda N$ where λ is the decay constant.

$\lambda = \ln 2 / T_{½}$ where $T_{½}$ is the half-life of cobalt-60.

$$A = (\ln 2 / T_{½})\, N$$

$$T_{½} = N \ln 2 / A = (4.317 \times 10^{13}) \ln 2 / (1.8 \times 10^{5})$$

$$= 1.662 \times 10^{8}\text{ s} = 5.270\text{ years}$$

half-life = <u>5.3 years</u>

25 (d) Some data for the half-lives and decay constants of bismuth-212 and thallium-208 are given in Fig. 8.3.

nucleus	half-life / s	decay constant / s^{-1}
bismuth-212	....	1.9 x 10^{-4}
thallium-208	190	3.7 x 10^{-3}

Fig. 8.3

(i) Complete Fig. 8.3 by calculating the half-life of bismuth-212.

(ii) Initially, a radioactive source contains N nuclei of bismuth-212.
After two hours, it is found that the number of bismuth-212 nuclei has reduced to approximately ¼N. However, although bismuth-212 decays to form thallium-208, the number of thallium nuclei is much less than ¾N.
Suggest an explanation for these observations. [4]

N10/III/8 (part)

Solution

(d) (i)

nucleus	half-life / s	Decay constant / s^{-1}
bismuth-212	3600	1.9 x 10^{-4}
thallium-208	190	3.7 x 10^{-3}

Half-life $T_{½} = \ln 2 / \lambda$ where λ is the decay constant.

$T_{½} = \ln 2 / (1.9 \times 10^{-4}) = 3648 = 3600$ s (2 s.f.)

(ii) This is because thallium-208 is not stable but will decay to form yet another daughter nucleus (lead-208 through β-decay).

$${}^{208}_{81}\text{Tl} \rightarrow {}^{208}_{82}\text{Pb} + {}^{0}_{-1}\text{e} + \text{energy}$$

Since the half-life of thallium-208 is much shorter than that of bismuth-212, the rate of conversion of thallium into lead is much greater than the rate of conversion of bismuth into thallium.

This accounts for the very small amount of thallium present after 2 hours.

TOPIC 21 Data Analysis

1 Wind power can be used for the generation of electric power. Fig. 8.1 illustrates one particular type of wind turbine.

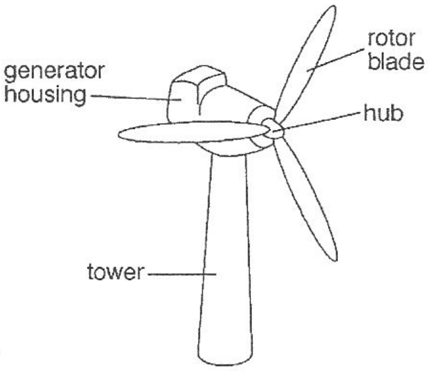


Fig. 8.1

The wind causes the rotor blades to turn and these drive an electric generator. The electric generator is situated in the housing at the top of the tower, as illustrated in Fig. 8.2.

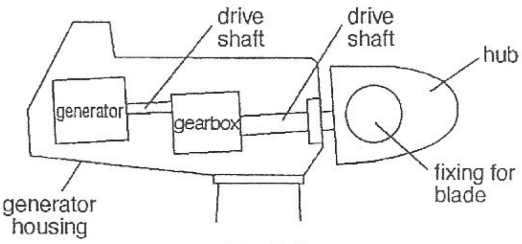


Fig. 8.2

Some information provided by a manufacturer of wind turbines is given in Fig. 8.3.

height of tower from ground to hub	56 m
rotor diameter	44 m
number of blades	3
nominal output	500 kW
voltage	690 V
frequency	50 Hz

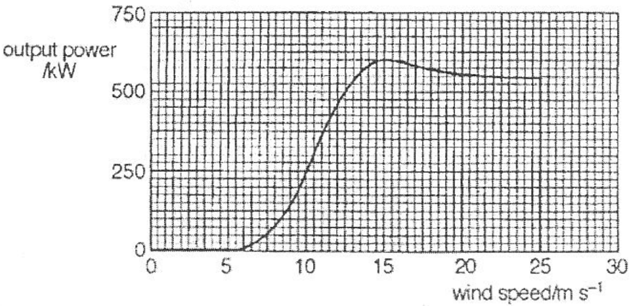


Fig. 8.3

One such wind turbine is built near a town. The local newspaper reported that the wind turbine 'could serve 200 homes'.

(a) Suggest reasons, one in each case,

(i) why the manufacturer specifies a *nominal* output power for the wind turbine,

(ii) why the report that the wind turbine can serve 200 homes is misleading. [2]

(b) Determine the minimum height of the tip of a rotor blade above ground level.

height = ... m [1]

(c) (i) Use the manufacturer's data to give values of

1. the maximum power output,

maximum power = ... kW

2. the wind speed for this maximum power.

wind speed = ... m s^{-1} [1]

(ii) Air of density ρ and speed v is incident normally on a rotor of radius r. The kinetic energy E of the air incident on the rotor in unit time is given by

$$E = -\pi r^2 v^3 \rho.$$

The air has density 1.25 kg m^{-3}.

Calculate, for the wind turbine operating at maximum output power,

1. the kinetic energy of air incident per second on the rotor (the incident wind power),

incident wind power = ... W

2. the overall efficiency of generation of electric power.

efficiency = ... % [4]

(iii) In addition to the power usefully transformed in the wind turbine, 10% of the incident wind power is lost. Calculate the power of the wind after it has passed through the rotor.

power = ... W [2]

(iv) At high wind speeds, the turbine is 'cut out', that is, the generator is no longer turned by the blades.

1. Use Fig. 8.3 to determine this cut-out speed.

cut-out speed = ... m s^{-1}

2. Suggest one reason why it is necessary to have a cut-out speed. [2]

(d) (i) State whether the generator produces direct current or alternating current, explaining how you came to your conclusion.

(ii) Calculate the nominal current from the generator.

current = ... A [4]

(e) The wind turbine must be protected from lightning strike.

(i) Suggest, with reasons, which part of the wind turbine is most likely to be struck by lightning.

(ii) Suggest how the risk of damage by lightning may be minimised. [4]

N02/II/8

Solution

(a) (i) The output power varies with wind speed.
The *nominal* value indicates the minimum power the generator is capable of generating under favourable wind conditions.

(ii) The generator output fluctuates with wind speed. Although it might be able to provide sufficient power under favourable wind conditions (2,500 W per household), it may not be able to serve even one household if there is no wind.

(b) The length of a blade = radius of the rotor = 22 m.
The minimum height = 56 m – 22 m = 34 m.

(c) (i) 1. 600 kW (peak of the graph).

2. 15 m s^{-1}.

(ii) 1. $E = \frac{1}{2}\pi r^2 v^3 \rho$
$= \frac{1}{2}\pi(22\ m)^2(15\ m\ s^{-1})^3(1.25\ kg\ m^{-3})$
$= 3.207 \times 10^6$
$= 3.2 \times 10^6$ W.

2. Efficiency
$= (600 \times 10^3\ W)/(3.207 \times 10^6\ W) \times 100\%$
= 18.71
= 19 %.

(iii) 10% of the incident wind power
$= 10\% \times 3.207 \times 10^6$ W
$= 3.207 \times 10^5$ W.

The power of the wind after passing through the rotor
$= 3.207 \times 10^6\ W - 3.207 \times 10^5\ W - 600 \times 10^3$ W
$= 2.286 \times 10^6$
$= 2.3 \times 10^6$ W.

(iv) 1. 25 m s^{-1} (when the output power drops to zero).

2. After peaking at 600 kW with a wind speed of 15 m s^{-1}, the power output does not increase any further with wind speed.
The cut-out reduces wear and tear of the generator and gears as they may not be able handle such high speeds of rotation.

(d) (i) The generator produces alternating current.
It is stated in the question that the output has a frequency of 50 Hz.

(ii) Nominal current = Nominal power / voltage
= 500 kW / 690 V
= 725 A.

(e) (i) The tip of the rotor blade when it is at the highest position, above the generator housing.
Lightning usually strikes the highest point of a structure and particularly sharp objects, such as the rotor blade tip, due to high electric field around them.

(ii) Install a lightning rod on the generator housing with an even higher and sharper tip than the rotor blade.
The sharp tip creates a high electric field which ionizes the air and relieves the potential build-up by diverting charges to the ground via the earth wire connected to the lightning rod

2 The large amount of energy released in a nuclear fission reaction, together with the emission of more than one neutron, has made it possible for neutron-induced fission to be used as a source of useful energy.

When a neutron is captured by a Uranium-235 nucleus, it causes the nucleus to fission. On average, 2.5 neutrons are emitted in these fission reactions. This is illustrated in Fig. 8.1.

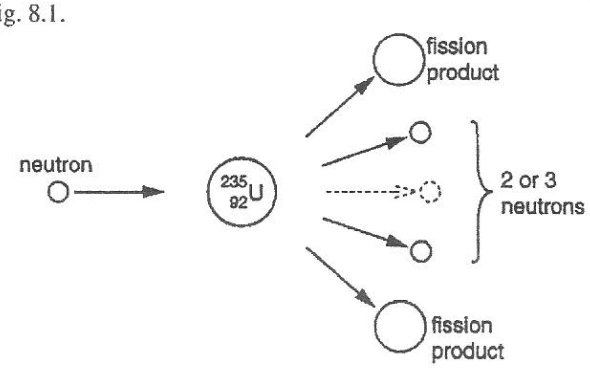


Fig. 8.1

When the conditions are suitable, a chain reaction can occur. If this chain reaction is not controlled, an explosion is likely. However, if the chain reaction is controlled, as in a nuclear reactor, a source of continuous power may be created.

(a) (i) Explain, with the aid of a diagram, what is meant by a *chain reaction*.

(ii) Suggest why, in an uncontrolled chain reaction where all neutrons are captured by Uranium-235 nuclei, the majority of the energy is released during the final stages of the fission of a sample of the uranium. [3]

(b) The induced fission reaction of Uranium-235 may be represented by a nuclear equation of the form

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{a}_{b}P + ^{c}_{d}Q + (2 \text{ or } 3)^{1}_{0}n + 198 \text{ MeV}.$$

The fission products P and Q have approximately equal masses. However, when any two nuclei are fissioned, the fission products may not be the same. If a large sample of Uranium-235 is fissioned, many different fission products will be produced. The percentage amount of each fission product in the fissioned material is referred to as the *percentage yield*. The variation with nucleon number of the percentage yield of different fission products is referred to as a 'fission yield curve' and is illustrated in Fig. 8.2.

(i) Suggest why the percentage yield is shown on a logarithmic scale. [1]

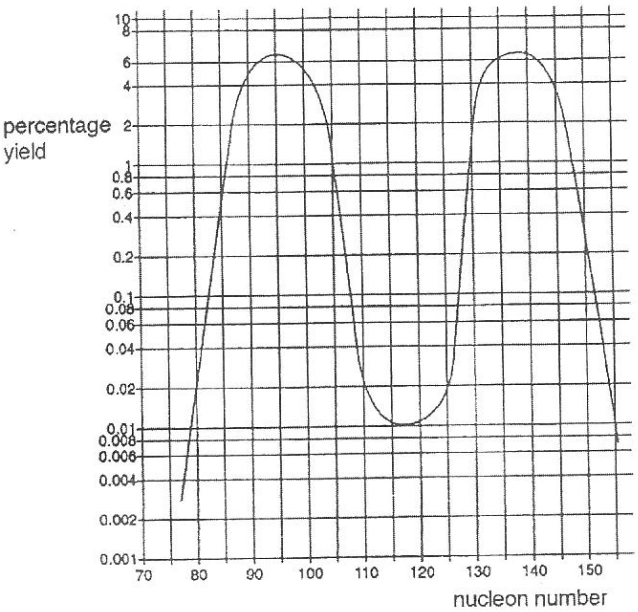


Fig. 8.2

(ii) Use Fig. 8.2 to determine the nucleon numbers of those fission products that have the same percentage yield as the nuclide with a nucleon number of 82. [2]

(iii) By reference to the nuclear equation, and your answer in (ii), suggest the nucleon number of the nuclide that would be produced in the same fission reaction as the nuclide with nucleon number 82. [1]

(iv) Use Fig. 8.2 to determine the percentage yield of fission products having nucleon numbers of 95 and 139. [1]

(v) Hence show that the fission products in (iv) are about 600 times more likely to be produced than those having masses equal to each other. [1]

(c) The energy released in the reaction in *(b)* occurs partly as kinetic energy of the fission products (167 MeV) and of the neutrons (5 MeV).

(i) Suggest **one** other mechanism by which energy is released in the fission reaction. [1]

(ii) In a nuclear power station, 25% of the energy of the fission products is converted into electrical energy. Calculate, for the fission of a mass of 1.0 kg of Uranium-235,

1. the number of nuclei in 1.0 kg of Uranium-235, [2]
2. the electrical energy generated, [3]
3. the average power output, in megawatts, of the power station if the uranium is fissioned in a time of 24 hours. [2]

(d) The fission products are usually radioactive and give rise to a series of radioactive decay products. Each decay product has its own half-life, but eventually a stable nuclide is reached.

Two such fission products with their decay products and half-lives are shown below.

$$^{99}_{42}\text{Mo} \xrightarrow{67\text{ hour}} {}^{99}_{43}\text{Tc} \xrightarrow{2\times10^{5}\text{ year}} {}^{99}_{44}\text{Ru (stable solid)}$$

$$^{140}_{54}\text{Xe} \xrightarrow{16\text{ second}} {}^{140}_{55}\text{Cs} \xrightarrow{1.1\text{ minute}} {}^{140}_{56}\text{Ba} \xrightarrow{13\text{ day}} {}^{140}_{57}\text{La} \xrightarrow{40\text{ hour}} {}^{140}_{58}\text{Ce (stable solid)}$$

Consider equal amounts of these two fission products.

Suggest why there are very different problems for the storage of this nuclear waste. [3]

N03/II/8

Solution

(a) (i) A *chain reaction* refers to a series of reactions in which one causes another to happen, and this goes on continuously.
In the case of fission, the neutrons produced by each reaction causes further neutron capture by other uranium atoms. This in turn produces more neutrons to cause further reactions.

(ii) In an uncontrolled chain reaction, the number of neutrons produced increases with each reaction according to the factor $(2.5)^n$ where n is the number of stages since the first reaction.
At the beginning, the number of neutrons available for capture is small but this escalates to a tremendous value towards the final stages, when there are a large number of simultaneous reactions taking place at the same time.
This causes a huge amount of energy to be released during the final stages.

(b) (i) A logarithmic scale is used because the percentage yields vary over a very wide range of many orders of magnitude.
The logarithmic scale compresses the numbers to a magnitude such that comparison within the same vertical scale of the graph is possible.

(ii) 108, 127, 152
All having a percentage yield of 0.08%.

(iii) 152.
The total nucleon number on both sides of the nuclear equation must balance.
The total nucleon number of the reactants is 236 hence if one of the products has a nucleon number of 82, the other must have a nucleon number slightly lower than 236 – 82 = 154.
The difference between 154 and 152 is due to the two neutrons produced.

(iv) The logarithmic yield is 6.4%.

(v) For fission products to have the same masses, the nucleon number must be around (236 – 2)/2 = 117 (the subtraction of '2' takes into account the estimated number of neutrons produced).
For the nucleus with nucleon number 117, the percentage yield is 0.01%.
The ratio is 6.4% / 0.01% = 640 times.

(c) (i) Electromagnetic radiation across the entire spectrum. One example is gamma radiation.

(ii) 1. $1.00 \text{ kg} / 235\,u$
$= 1.00 \text{ kg} / (235 \times 1.66 \times 10^{-27} \text{ kg})$
$= 2.5634 \times 10^{24}$
$= 2.56 \times 10^{24}$.

2. Electrical energy generated by the fission products (167 MeV per reaction)
$= 25\% \times 2.5634 \times 10^{24} \times (167 \times 1.6 \times 10^{-13} \text{ J})$
$= 1.7124 \times 10^{13}$
$= 1.71 \times 10^{13}$ J.

3. Power = energy / time
$= 1.7124 \times 10^{13} \text{ J} / (24 \times 60 \times 60 \text{ s})$
$= 1.9819 \times 10^{8}$
$= 1.98 \times 10^{8}$ W.

(d) Molybdenum ${}^{99}_{42}Mo$ decays fairly rapidly to Technetium ${}^{99}_{43}Tc$ but ${}^{99}_{43}Tc$ takes a very long time to decay hence it will be radioactive for many years.
Although the reaction shows only beta emission (${}^{99}_{42}Mo \rightarrow {}^{99}_{43}Tc + {}^{0}_{-1}e$), beta emissions are usually accompanied by gamma radiation which is highly penetrative.
Therefore, ${}^{99}_{42}Mo$ has to be sealed up in lead containers and buried for a very long time.

The Xenon ${}^{140}_{54}Xe$ series, on the other hand, has a much shorter half-life. Even the longest, ${}^{140}_{57}La$, has a half-life of only 40 hours.
By storing ${}^{140}_{54}Xe$ in a lead container for around two months, the activity will fall to less than a billionth of its initial activity. The final stable product Cerium ${}^{140}_{58}Ce$ can be discarded without container.

3 Solar cells are used in some appliances for the generation of electrical energy. When light energy is incident on the surface of such a cell, an e.m.f. is generated between the terminals of the cell. Connection of a resistor between these terminals will result in a current and electrical power dissipation in the resistor.

The variation with output potential difference V of the current I from a solar cell may be investigated using the circuit of Fig. 8.1.

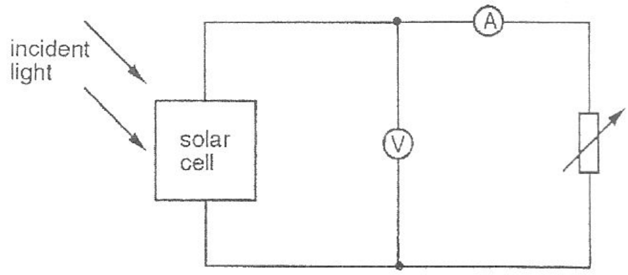


Fig. 8.1

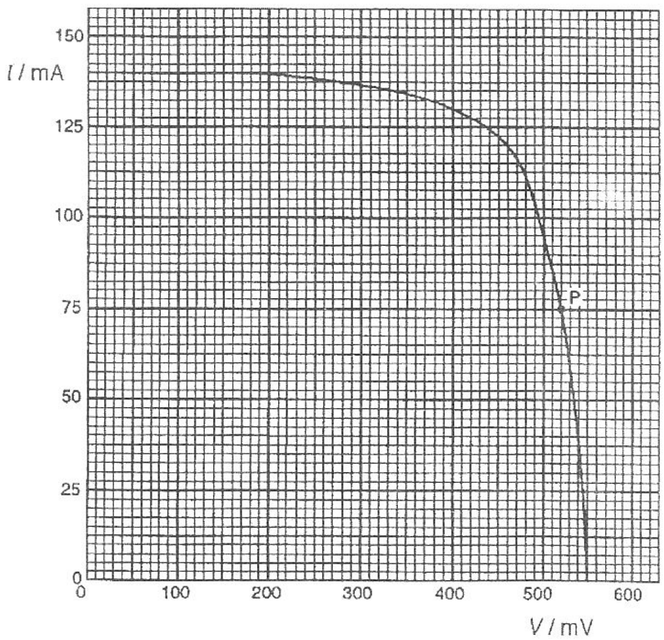


Fig. 8.2

The ammeter has negligible internal resistance and the voltmeter has a very high resistance. Light of constant intensity is incident on the solar cell. The I/V characteristic of one type of solar cell, when it is illuminated with a certain intensity of light, is shown in Fig. 8.2.

(a) (i) Determine the current from the solar cell for an output potential difference of 400 mV. [1]

(ii) Explain how the graph shows that the e.m.f. of the solar cell is 550 mV. [1]

(iii) Use your answers to (i) and (ii) to determine the internal resistance of the cell at an output potential difference of 400 mV. [2]

(b) (i) Determine the power dissipation in the load resistor for point P on Fig. 8.2. [3]

(ii) On Fig. 8.2, shade an area that represents the power dissipation calculated in (i). [2]

(c) (i) By reference to your answer in *(b)*(ii), state whether the power dissipated in the load resistor increases, stays the same, or decreases when the current

1. increases from 30 mA to 100 mA,
2. increases from 125 mA to 135 mA. [2]

(ii) Hence, or otherwise, mark with the letter M the approximate point on Fig. 8.2 at which the power dissipation is a maximum. [1]

(d) The solar cell is connected to a load resistor of resistance 4.2 Ω.

(i) Fig. 8.3 is the same graph as Fig. 8.2.

On Fig. 8.3, draw a line to show the variation with current I of the potential difference V across the resistor. [2]

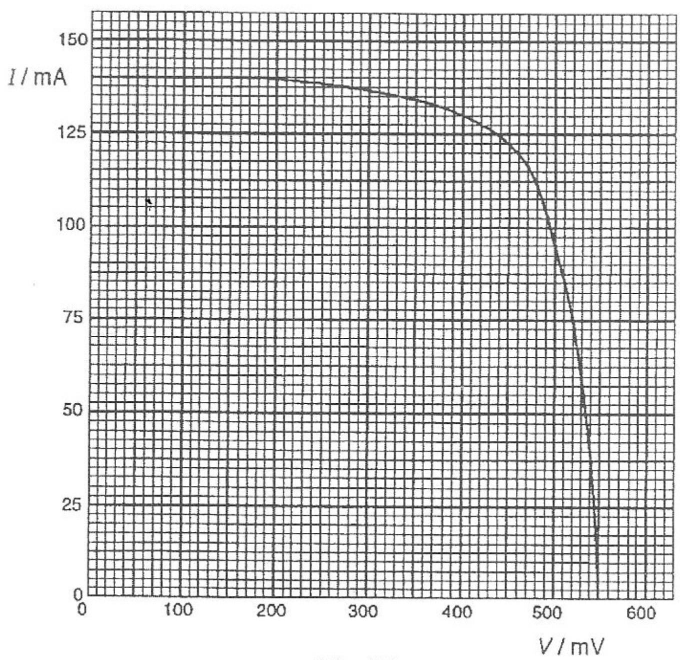


Fig. 8.3

(ii) The cell is illuminated with the same intensity of light as before. The printed curve of Fig. 8.3 is the I/V characteristic of the solar cell. Determine the power dissipation in the load resistor. [2]

(e) A number of identical solar cells of a different type each produce an output power of 75 mW at an output potential difference of 0.50 V.

Each cell may be represented by the symbol shown in Fig. 8.4.

Fig. 8.4

Draw suitable arrangements of solar cells so that the cells may be used to provide

(i) a power of 150 mW at a potential difference of 1.0 V,

(ii) a power of 150 mW at a potential difference of 0.50 V,

(iii) a power of 300 mW at a potential difference of 1.0 V. [4]

N04/II/8

Solution

(a) (i) 130 mA.

(ii) The terminal potential difference across the solar cell $V = E - Ir$ where E is its e.m.f. and r its internal resistance.

When the current I is zero, the potential drop across the internal resistance (Ir) is zero hence $V = E$.

It can be seen from the graph that when I is zero, V = 550 mV.

Hence E = 550 mV.

(iii) At an output potential difference of 400 mV, the current is 130 mA.

$$V = E - Ir$$

$$0.400 \text{ V} = 0.550 \text{ V} - (0.130 \text{ A})\, r$$

$$r = 1.1538\ \Omega = 1.15\ \Omega$$

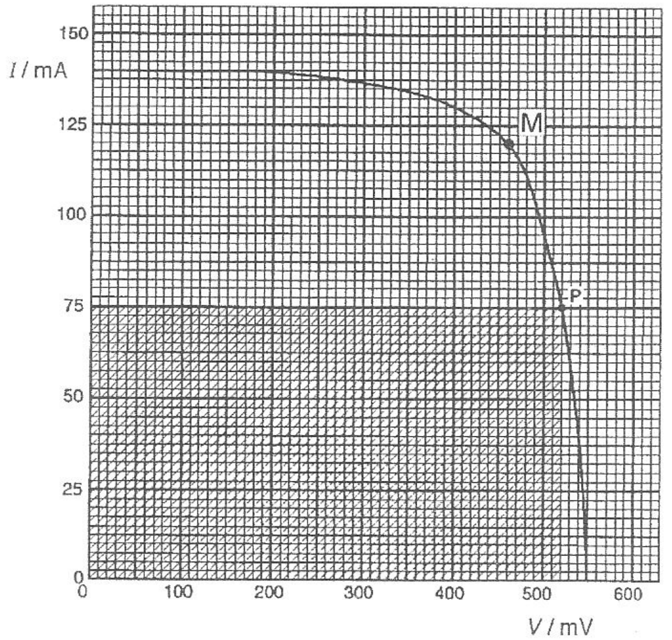
(b) (i) Power

$P = IV$

$= (75 \times 10^{-3} \text{ A})(520 \times 10^{-3} \text{ V})$

$= 3.9 \times 10^{-2}$ W.

(ii) The shaded area is shown below.

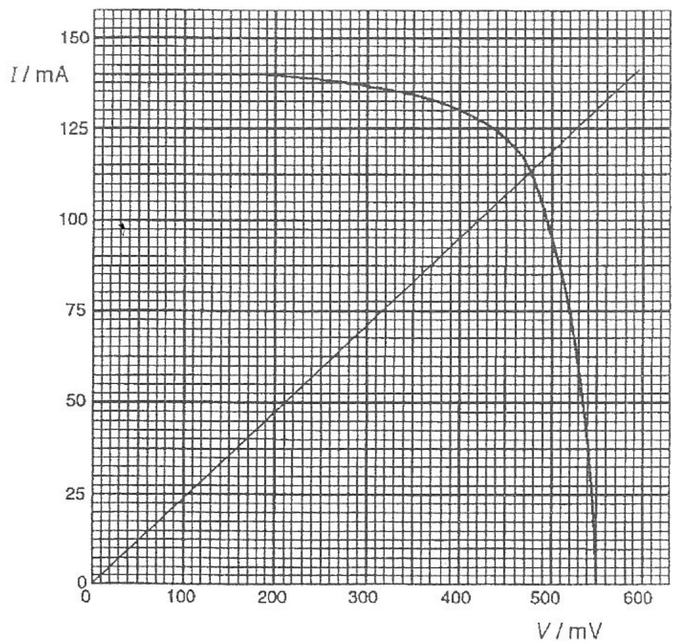


(c) (i) 1. Power increases (as area becomes larger).

2. Power decreases (as area becomes smaller).

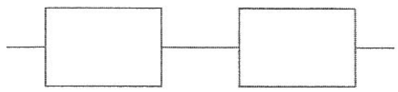
(ii) The letter M is marked in the figure above. The potential difference is 460 mV and the current 120 mA, producing a power of 55.2 mW.

(d) (i) The line is drawn in the graph below.

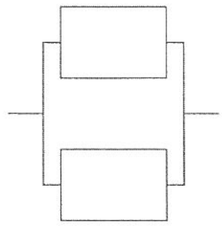


The two lines intersect at 475 mV, 113 mA. The power is 53.7 mW.

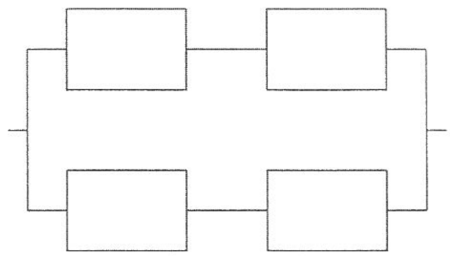
(e) (i) The arrangement for 150 mW, 1.0 V is shown in the figure below.



(ii) The arrangement for 150 mW, 0.5 V is shown in the figure below.



(ii) The arrangement for 300 mW, 1.5 V is shown in the figure below.



4 An object that is at a higher temperature than its surroundings loses thermal energy by emitting electromagnetic radiation.

For loss of thermal energy as electromagnetic radiation, the intensity I_λ of the emitted radiation of wavelength λ varies with wavelength λ as shown in Fig. 8.1.

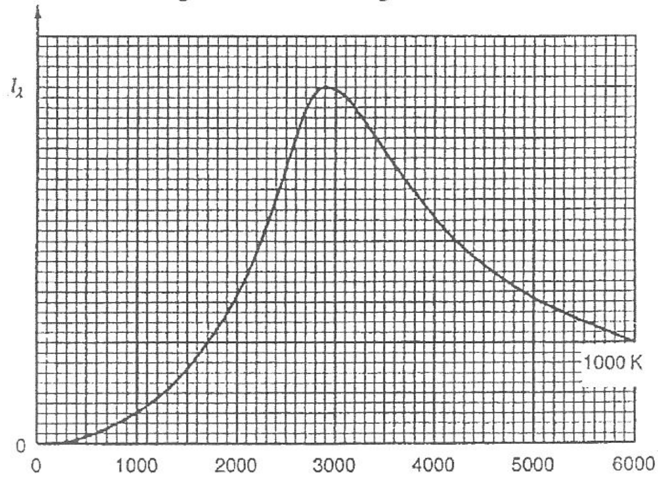


Fig. 8.1

Fig. 8.1 shows the variation of I_λ with λ for the body when it is at 1000 K. The distribution of intensity is different at different temperatures. This is illustrated in Fig. 8.2.

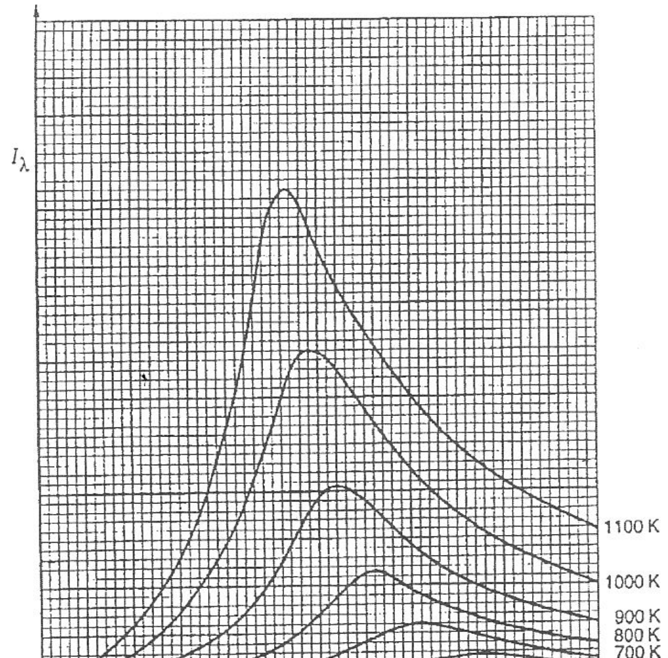


Fig. 8.2

(a) (i) On the horizontal axis of Fig. 8.2, indicate with the letter V a wavelength that is in the visible region of the electromagnetic spectrum.

(ii) Hence suggest why, at a temperature of 1100 K, the object would glow with a red colour. [2]

(b) At any temperature T, the graph of Fig. 8.2 shows a peak corresponding to a wavelength λ_{max} and an intensity I_{max}. Data for T and λ_{max} are shown in Fig. 8.3.

T/K	λ_{max}/nm
600	4830
700	4140
800	3610
900	3210
1000	2900
1100	2630

Fig. 8.3

(i) Without drawing a graph, show that

$$T \times \lambda_{max} = \text{constant},$$

and determine the constant.

constant= .... [3]

(ii) Hence determine the wavelength for maximum intensity at a temperature T of 1 200 K.

wavelength = ... m [2]

(c) The total intensity of emitted radiation from a particular body at temperature T is I_{tot}. Fig. 8.4 shows the values of lg(T/K) plotted against the corresponding values of lg(I_{tot} /W m^{-2}).

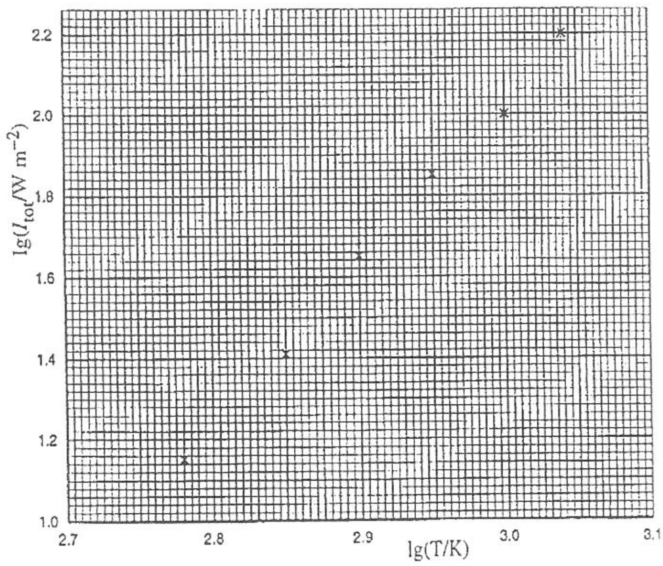


Fig. 8.4

It is known that I_{tot} varies with T according to the relation

$$I_{tot} = cT^n,$$

where c and n are constants.

(i) Use Fig. 8.4 to determine a value for n.

n = ... [3]

(ii) For this body at T = 900 K, I_{tot} is found to be 71 W m^{-2}.

Use these data and your answer to (i) to determine I_{tot} for the body at a temperature of 1200K.

I_{tot} = ... W m^{-2} [3]

(d) Using your answer to *(b)*(ii) sketch, on Fig. 8.2, the variation with wavelength λ of intensity I_λ for a temperature of 1200 K. [3]

(e) The radiation emitted by a hot body may be used as a means of determining the temperature of the body.

(i) Suggest and explain a property of the radiation that could be used for this purpose. [2]

(ii) Suggest one advantage and one disadvantage of this method for measuring temperature. [2]

N05/II/8

Solution

(a) (i) The wavelength of visible light ranges from 410 nm (violet) to 750 nm (red). One possible wavelength in the visible spectrum will be 700 nm (red), indicated with the letter V in Fig. 8.5 below.

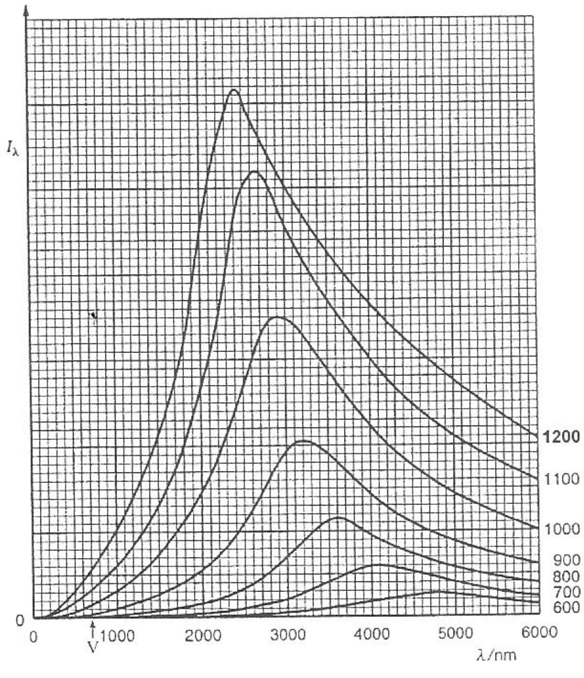


Fig. 8.5

(ii) Referring to the curve for 1100 K, the intensity of the longer wavelengths at the red end of the visible spectrum (> 600 nm) exceeds those at the violet end of the visible spectrum (<500 nm).
The longer wavelengths thus dominate, resulting in a reddish glow.

(b) (i) The products of $T \times \lambda_{max}$ for the six sets of data are respectively:
2.898 x 10^{-3} metre kelvin (m K),
2.898 x 10^{-3} m K,
2.888 x 10^{-3} m K,
2.889 x 10^{-3} m K,
2.900 x 10^{-3} m K,
2.893 x 10^{-3} m K.
The values are the same up to two significant figures (2.9 x 10^{-3} m K).
The constant can be obtained by averaging the six sets of values, and is equal to: 2.89 x 10^{-3} m K.

(ii) For T = 1200 K,

$$T \times \lambda_{max} = 2.89 \times 10^{-3} \text{ m K}$$

$$\lambda_{max} = 2410 \text{ nm}$$

(c) (i) If $I_{tot} = cT^n$, then,

$$\log(I_{tot}) = \log(c) + n \log(T)$$

Comparing this with the equation of a straight line graph $y = mx + c$ where $y = \log(I_{tot})$ and $x = \log(T)$, the gradient is n and the y-intercept is $\log(c)$.

Drawing a best-fit line through the points, the gradient of the straight line graph
n = (2.22 – 1.00) / (3.05 – 2.74)
= 1.22 / 0.31
= 3.94 (Fig. 8.6).

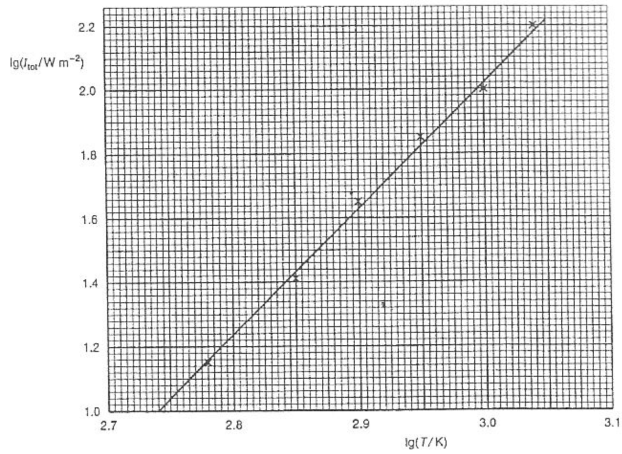


Fig. 8.6

(ii) Substituting T = 900 K and I_{tot} = 71 W m^{-2} into the equation $I_{tot} = cT^n$, with n = 3.94, c = 1.63 x 10^{-10}.

Using the same equation for T = 1200 K,
$I_{tot} = cT^n$ = (1.63 x 10^{-10})$(1200)^{3.94}$ = 220 W m^{-2}.

(d) For a temperature of $T = 1200$ K, the peak of the graph is at $\lambda_{max} = 2410$ nm (as worked out in (b)(ii)).

The total area under the graph is also greater than that at a temperature of $T = 900$ K.

Hence the graph will have a maximum to the left of 900 K. The peak of the graph will be approximately three times the height of the 900 K graph.

This is shown in Fig. 8.5 above.

(e) (i) The thermometric property is the peak wavelength λ_{max} of the electromagnetic radiation emitted.

The higher the temperature of the body emitting the radiation, the shorter the peak wavelength λ_{max}.

(ii) Advantage: No physical contact is necessary between the body and the thermometer, so that temperatures can be taken from a distance.

Disadvantage: At low temperatures of below 600 K, the radiation is very weak making the peak very difficult to detect and the thermometer highly inaccurate.

5 In the first half of the last century, numerous experiments were conducted to investigate the absorption and the scattering of X-rays by matter.

It was discovered that when a monochromatic beam of X-rays is incident on a light element such as carbon, the scattered X-rays have wavelengths dependent on the angle of scattering.

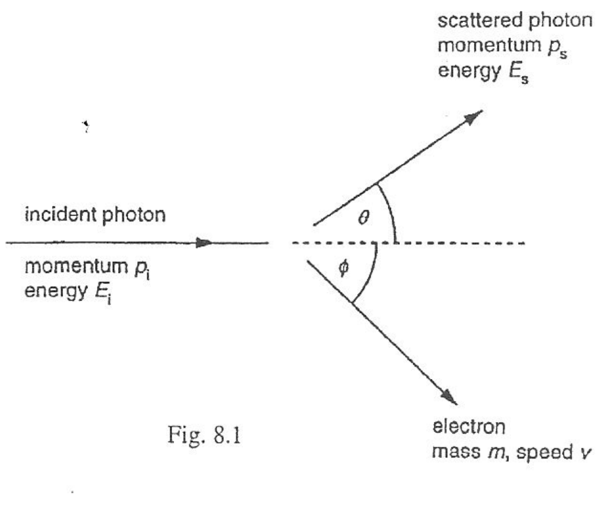


Fig. 8.1

Compton (1923) assumed that the scattering process could be treated as an elastic collision between an X-ray photon and a 'free' electron, and that energy and momentum would be conserved.

(a) Explain what is meant by a *photon*. [2]

(b) The elastic collision between a photon and a stationary electron may be represented as in Fig. 8.1.

The incident photon has momentum p_i and energy E_i. The photon is scattered through an angle θ and, after scattering, has momentum p_s and energy E_s. The electron of mass m, which was originally stationary, moves off with speed v at an angle ϕ to the original direction of the incident photon.

(i) Write down equations, in terms of p_i, p_s, E_i, E_s, m, v, θ and ϕ, that represent, for this interaction,

1. conservation of energy, [1]

2. conservation of momentum along the direction of the incident photon. [1]

(ii) Suggest, with a reason, whether the scattered photon will have a wavelength that is greater or less than that of the incident photon. [3]

(c) In an experiment to provide evidence to justify Compton's theory, measurements were made of the wavelength λ_i of the incident photon, the wavelength λ_s of the scattered photon and the angle θ of scattering. Some data from this experiment are given in Fig. 8.2.

λ_i /10^{-12} m	λ_s /10^{-12} m	θ /°
191.92	193.27	57
153.30	154.65	57
965.04	966.84	75

Fig. 8.2

(d) In this experiment, the uncertainty in the measurement of θ is ±5°.

Determine the value of cos θ, with its uncertainty, for the angle $\theta = 75° \pm 5°$.

cos θ = ... ± .... [3]

(e) Compton's theory suggests that the change in wavelength $\Delta\lambda$ is related to the angle θ of scattering by the expression

$$\Delta\lambda = k(1 - \cos\theta)$$

where k is a constant.

Experimental data for the variation with cos θ of $\Delta\lambda$ are shown in Fig. 8.3.

(i) On Fig. 8.3, draw the best-fit line for the points. [1]

(ii) State and explain two different ways by which the constant k may be determined from the graph of Fig. 8.3. [4]

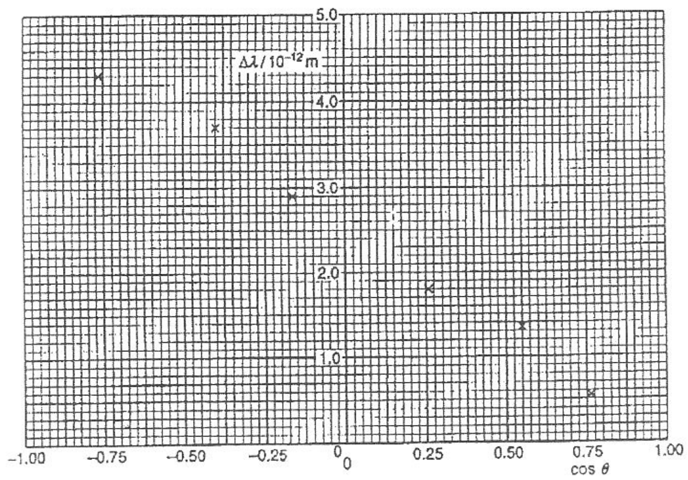


Fig. 8.3

(iii) Determine the constant k, with its unit, using **one** of the ways described in (ii). [2]

(*f*) For a carbon atom, the binding energy of an electron is of the order of a few electronvolts.

Compton's theory assumes that the electrons are not bound in the atoms but are free.

Suggest whether, for 30 keV photons, this assumption is justified. [1]

N06/II/8

Solution

(a) A *photon* is a quantum of electromagnetic radiation.

Electromagnetic radiation has both wave and particulate properties, and a *photon* is the particulate representation of the radiation, existing as a packet of wave energy.

Each *photon* carries with it an amount of energy $E = hc/\lambda$ where h is the Planck constant and λ the radiation's wavelength.

(b) (i) 1. $E_i = E_s + \frac{1}{2}mv^2$.

2. $p_i = p_s \cos\theta + mv \cos\Phi$.

(ii) Greater.

The energy of a photon $E = hc/\lambda$.

After the collision, the photon loses some of its energy to the recoiling electron. The photon's energy thus decreases and its wavelength increases.

(c) In the first two measurements when the angle θ is 57°, the change in wavelength is 1.35 x 10^{-12} m even though the incident wavelength λ_i is different.

In the third measurement, the change in wavelength is 1.80 x 10^{-12} m but that is due to the angle of scattering θ having changed to 75°.

(d) Angle $\theta = 75° \pm 5°$ means that $\theta_{min} = 70°$ and $\theta_{max} = 80°$.

For $\theta_{min} = 70°$, $\cos\theta_{min} = \cos 70° = 0.34202$.

For $\theta_{max} = 80°$, $\cos\theta_{max} = \cos 80° = 0.17365$.

$\cos\theta = (\cos\theta_{max} + \cos\theta_{min})/2 = 0.25784$

$\Delta\cos\theta = (\cos\theta_{max} - \cos\theta_{min})/2 = 0.08419 = 0.08$ (1 s.f.)

$\cos\theta = 0.26 \pm 0.08$

Note: For trigonometric functions, the uncertainty has to be derived from first principles and not by using standardized relations like $\Delta\cos\theta / \cos\theta = \Delta\theta / \theta$.

(e) (i) The best-fit line is shown in Fig.8.4 below.

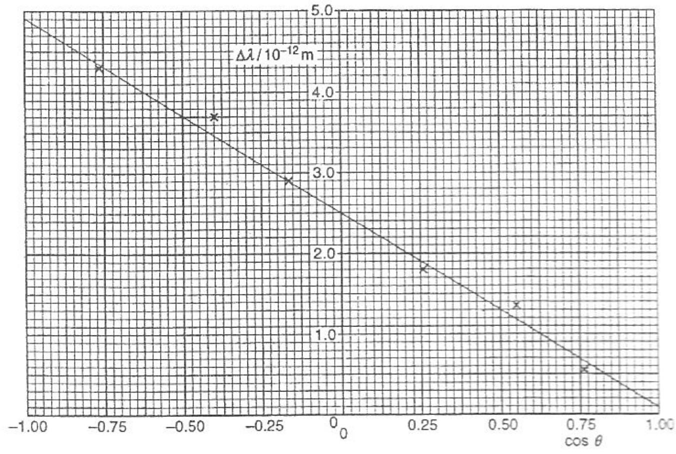


Fig. 8.4

(ii) 1. One way to find k is from the gradient of the graph.

The equation can be rearranged as

$\Delta\lambda = -k\cos\theta + k$ which is a straight line graph of the form $y = mx + c$ where the gradient $m = -k$.

2. The other way to find k is from the y-intercept where $c = k$.

(iii) From the y-intercept, $k = 2.5 \times 10^{-12}$ m.

The unit for k is the metre because the intercept is on the y-axis, which has units of metres.

(f) Taking into account the binding energy (E_B) of electrons, equation (b)(i)1. can be re-written as:

$$E_i - E_B = E_s + \frac{1}{2}mv^2$$

If E_i = 30 keV and E_B only a few eV, then $E_i - E_B$ is approximately equal to E_i, hence E_B can be ignored.

The assumption that the electrons are free is thus justified.

6 When some substances are in the solid state, they exist as positively-charged and negatively-charged ions arranged in a cubic lattice, as illustrated in Fig. 7.1.

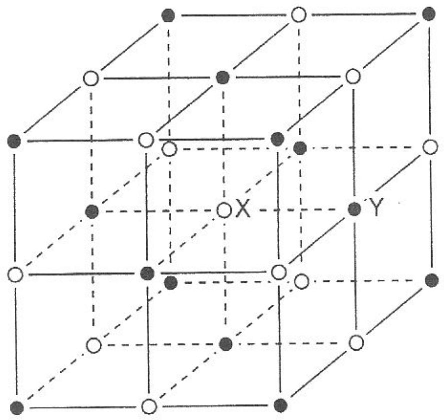


Fig. 7.1

A starting point for the understanding of lattice energies is to consider the potential energy E_P between two ions X and Y.

Fig. 7.2 shows the variation with distance r between X and Y of E_P.

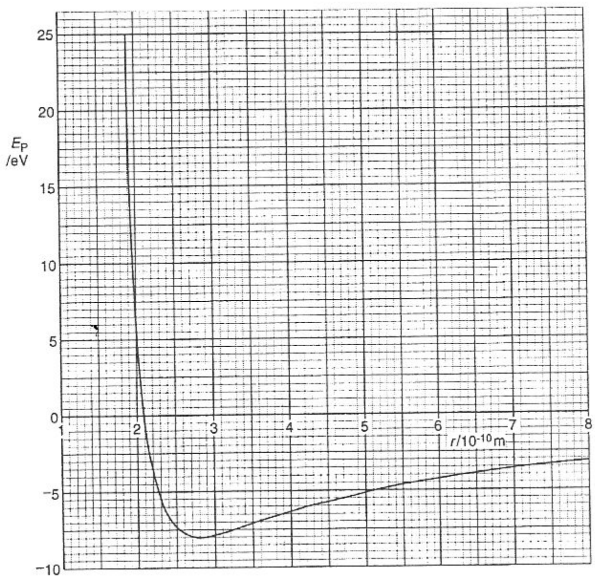


Fig. 7.2

(a) (i) The gradient G of the graph varies with the distance r. Show that, starting from the definition of *work done*, for any value of r the magnitude of the force F between X and Y is given by the expression

$$F = G.$$

....

....

.... [2]

(ii) Suggest how Fig. 7.2 indicates that, for some values of r, the force between X and Y is attractive and, for other distances, the force is repulsive.

....

....

.... [2]

(iii) Use Fig. 7.2 and the expression in (i) to determine the magnitude of the force, in newton, for values of the distance r equal to

1. 2.8×10^{-10} m,
force = ... N [1]
2. 5.0×10^{-10} m.
force = ... N [3]

(b) The variation with distance r of the potential energy E_P may be represented by the expression

$$E_P = -\frac{A}{r} + \frac{B}{r^8},$$

where A and B are constants.

By reference to Fig. 7.2, state two features of the force represented by the term B/r^8 in this expression.

1. ....

....

2. ....

.... [2]

(c) Fig. 7.3 shows part of Fig. 7.2, drawn on a larger scale.

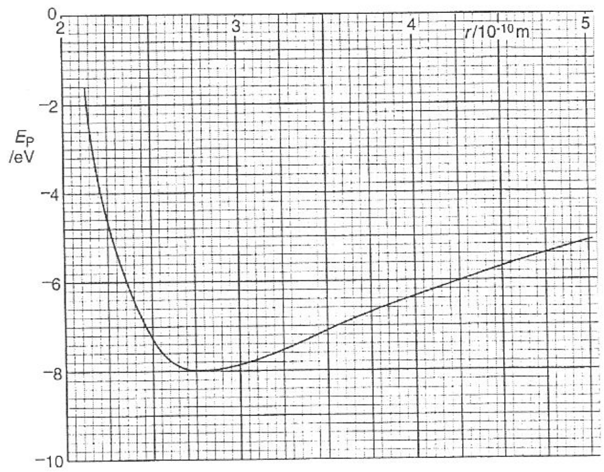


Fig. 7.3

Thermal energy of the ions causes them to vibrate.

The ions have a total energy of -6.0 eV.

(i) Use Fig. 7.3 to determine, for these ions,

1. the values of r between which they vibrate.
minimum value of r = ... m
maximum value of r = ... m [2]

2. the kinetic energy of the ions at distance r = 3.5 x 10^{-10} m.
   kinetic energy = ... eV [2]

(ii) State why, although the ions are oscillating, their motion is **not** simple harmonic.

....

.... [1]

(d) By reference to Fig. 7.3, suggest why the dimensions of the whole lattice increase as it is heated.

....

....

....

.... [3]

N07/II/7

Solution

(a) (i) The incremental work done δW by a force F is the product of the force and the incremental distance δr moved in the direction of the force.

$$\delta W = F\,\delta r$$

The total work done by a variable force will thus be

$$\int dW = \int F\,dr$$

$$W = \int F\,dr$$

The potential energy E_P is equal to the work done by an external agent in bringing the two ions together from infinity. The force exerted by the external agent is opposite to that of the electric force.

Hence, $$E_P = -\int F\,dr$$

$$F = -dE_P/dr = -G$$

$$|F| = G$$

(ii) For $r < 2.8 \times 10^{-10}$ m, the gradient dE_P/dr is negative. The force $F = -dE_P/dr$ is positive, indicating a repulsive force.

For $r > 2.8 \times 10^{-10}$ m, the gradient dE_P/dr is positive. The force $F = -dE_P/dr$ is negative, indicating an attractive force.

(iii) 1. At $r = 2.8 \times 10^{-10}$ m, the gradient of the graph is zero. Hence the force F = 0 N.

2. At $r = 5.0 \times 10^{-10}$ m, the gradient of the graph is

$$(6.0 \text{ eV} \times 1.60 \times 10^{-19} \text{ J eV}^{-1})/((8.0 - 3.0) \times 10^{-10} \text{ m})$$

$$F = 1.9 \times 10^{-9} \text{ N}$$

Refer to Fig. 7.4.

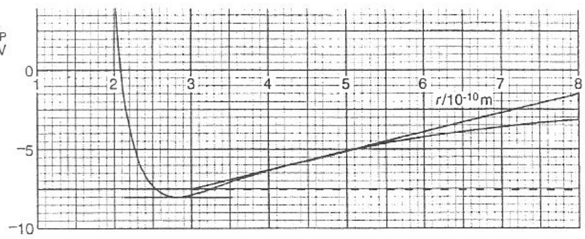


Fig. 7.4

(b) 1. The force causes repulsion between the ions.
This is because the gradient of the graph of the equation $E = B/r^8$ is negative, hence $F = -dE/dr$ is positive.

2. The force is significant when the ions are very close to each other, but decreases rapidly with distance.
This can be seen from the very steep gradient of the graph of the equation $E = B/r^8$ when the ions are close to each other.

The two components of the E_P vs r graph are segregated in Fig. 7.5. It can be seen that B/r^8 is the Pauli Repulsion.

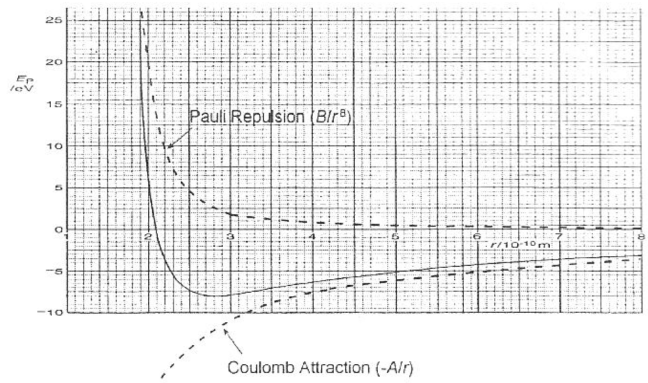


Fig. 7.5

(c) (i) Refer to Fig. 7.6.

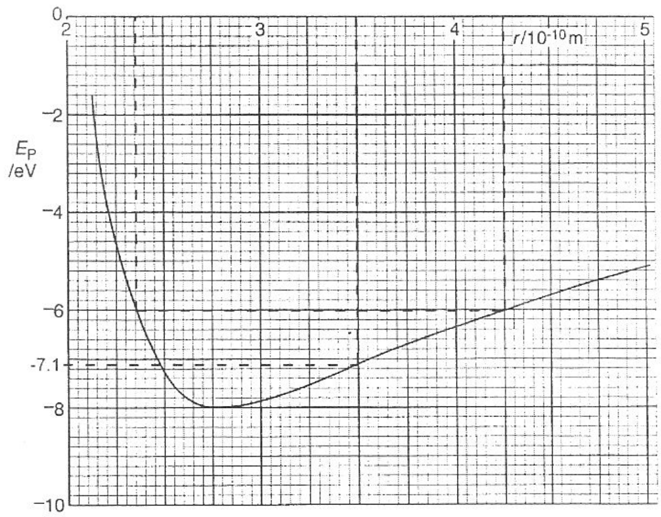


Fig. 7.6

1. Minimum value of $r = 2.35 \times 10^{-10}$ m.

   Maximum value of $r = 4.25 \times 10^{-10}$ m.

2. At distance $r = 3.5 \times 10^{-10}$ m, the potential energy of the ions is -7.1 eV.

Kinetic Energy = Total Energy – Potential Energy

= -6.0 eV – (-7.1 eV) = 1.1 eV.

(ii) The motion is not simple harmonic because the force F is not directly proportional to displacement r (in fact the graph is not even symmetrical about the equilibrium position at $r = 2.8 \times 10^{-10}$ m).

Since acceleration $a = F/m$, the acceleration of the ions is not directly proportional to displacement r.

This violates one of the two conditions for simple harmonic motion (which is that the acceleration of the body must be directly proportional to its displacement from the equilibrium position).

(d) As the lattice is heated, its total energy increases.

The total energy of the ions intersects the graph at higher levels, as shown in Fig. 7.7.

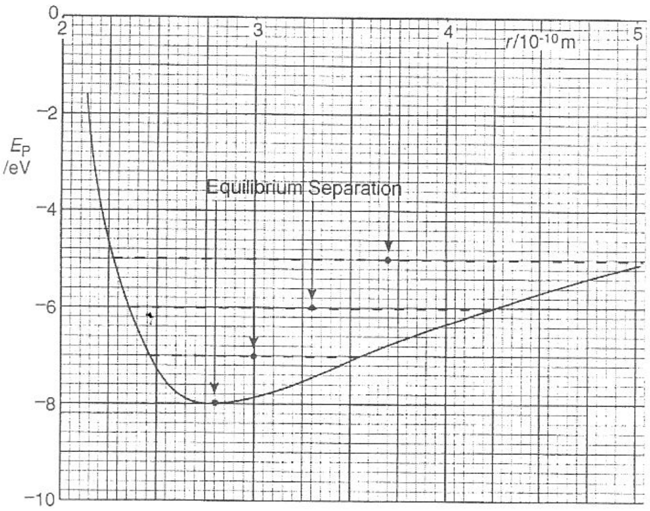


Fig. 7.7

As a result, the equilibrium position (mid-point) of the vibration becomes progressively greater.

In Fig. 7.7, the equilibrium positions of four different energy levels are shown:

Total Energy (eV)	Equilibrium Separation r (10^{-10} m)
-8.0	2.8
-7.0	3.0
-6.0	3.3
-5.0	3.7

With a greater equilibrium separation, the lattice expands.

7 A serious hazard for fire-fighters is the explosion of containers of 'liquefied gas' (butane) that have been heated in a fire. When the butane suddenly burns in an explosion, the fire spreads very rapidly in the form of a spherical fireball of increasing radius that is at very high temperature.

In order to study such fireballs, a series of experiments is carried out. Some butane of volume 12.5×10^{-3} m^3 is put in a sealed container and is then heated until it explodes. The variation with time t of the radius R of the fireball is determined. The results are shown in Fig. 7.1.

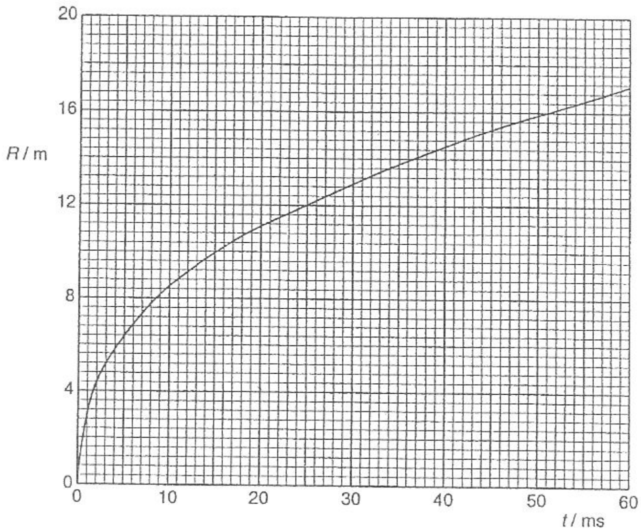


Fig. 7.1

(a) Use Fig. 7.1 to

(i) describe, without any calculation, the variation with time of the **rate** at which the radius of the fireball increases,

....

....

.... [2]

(ii) suggest why, in a room of length 12 m, width 5 m and height 3 m, such an explosion would be very hazardous.

....

....

....

.... [3]

(b) It is thought that, for a fixed volume of butane, the radius R of the fireball varies with time t according to the expression

$$R^n = k\,t^m,$$

where n and m are integers and k is a constant.

Some corresponding values of lg t and lg R for the data in Fig. 7.1 are plotted on the graph of Fig. 7.2.

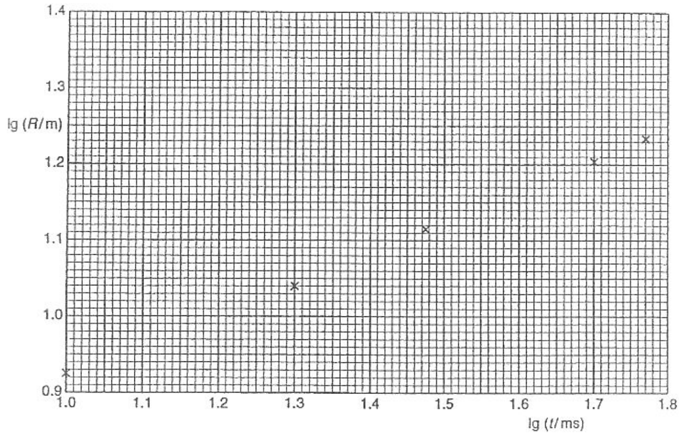


Fig. 7.2

(i) On Fig. 7.2,

1. plot the point corresponding to time t = 40 ms,

2. draw the best-fit line for all the plotted points. [2]

(ii) Determine the gradient of the line drawn in (i) part 2.

gradient = ... [2]

(iii) Hence suggest values for the integers n and m. Explain your working.

n = ...

m = ... [3]

(c) The experiment is repeated using similar containers but with different volumes of butane. The results are shown in Fig. 7.3.

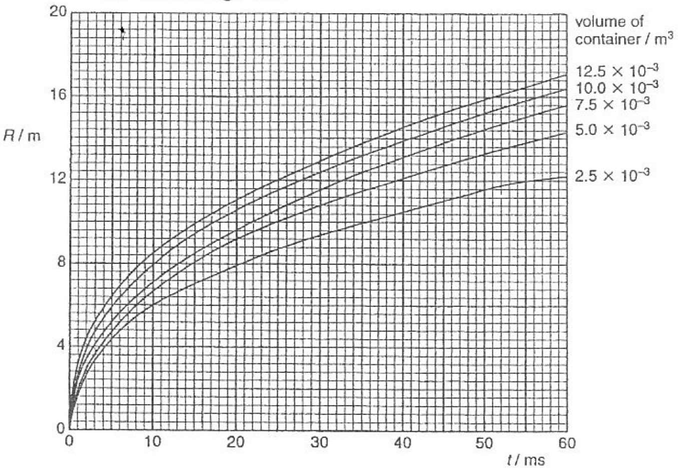


Fig. 7.3

Without drawing a further graph, show that, at time t = 40 ms, the radius R of the fireball is related to the volume V of butane by the expression

$$R^5 = cV,$$

where c is a constant. [3]

(d) (i) The equation in (c) may also be applied to other exploding gases.
Suggest **one** physical quantity on which the constant c will depend.

.... [1]

(ii) The data were collected for butane in a container in a room.
Suggest **one** other situation where the theory developed predicts a high level of hazard for fire-fighters.

.... [1]

N08/II/7

Solution

(a) (i) From 0 ms to 45 ms, the rate of increase decreases with time (decreasing gradient). From 45 ms to 60 ms, the rate of increase is constant (constant gradient).

(ii) With a length of 12 m, the fireball will take 25 ms or less (depending on where it starts) to fill up the room, leaving little time to react. With a ceiling height of only 3 m and width of 5 m, the fireball is not able to rise upwards and outwards, unlike those in open spaces. The intense heat trapped in the room is unable to dissipate quickly, and thus hazardous even to those with protective clothing.

(b) (i) 1. The log form of the equation is

$$\lg R = (m/n) \lg t + (1/n) \lg k$$

The x-axis co-ordinate is lg (40 ms) = 1.6.

The y-axis co-ordinate is interpolated between its two nearest neighbouring points.

It is plotted as a cross within a circle (Fig. 7.4).

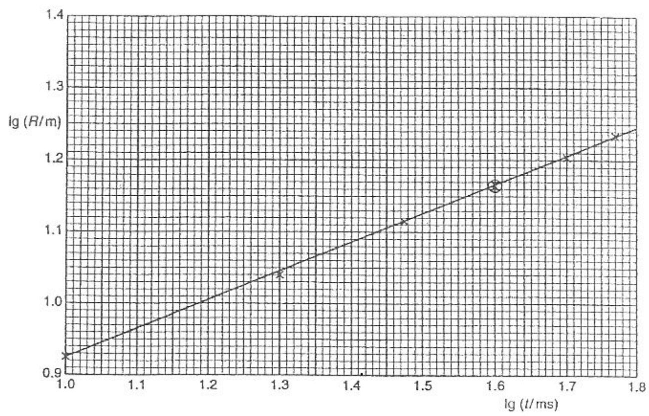


Fig. 7.4

2. The best-fit line is drawn in Fig. 7.4.

(ii) The gradient is (1.245 – 0.925)/(1.80 – 1.00) = 0.40.

(iii) Based on the log form equation in (b)(i)1.

$$\lg R = (m/n) \lg t + (1/n) \lg k$$

With lg R plotted as the y-axis co-ordinate and lg t the x-axis co-ordinate, the gradient is $m/n = 0.4$.

The two smallest integers which's ratio is 0.4 would be $n = 5$ and $m = 2$.

All other variations, e.g. $n = 10$ and $m = 4$ or any multiple of that, would simply be raising both sides of the equation by an integer power i, including the constant k.

The equation $R^{5i} = k^{i}\, t^{2i}$ is reducible to $R^5 = k\, t^2$.

Hence the suggested values are $n = 5$ and $m = 2$.

(c) That c is a constant can be proven by tabulating the values of R^5/V for the 5 data points at $t = 40$ ms.

Volume V (m^3)	R (m)	$c = R^5/V$ (m^2)
12.5×10^{-3}	14.5	5.13×10^7
10.0×10^{-3}	13.9	5.19×10^7
7.5×10^{-3}	13.1	5.14×10^7
5.0×10^{-3}	12.1	5.19×10^7
2.5×10^{-3}	10.5	5.11×10^7

For all five sets of reading, the value of c ($=R^5/V$) lies between 5.1×10^7 and 5.2×10^7.

Within the limits of experimental error, c is a constant.

(d) (i) Density of the liquefied gas.

If the constant c contains density within it, the density multiplied by volume V will determine the mass, and hence number of molecules available for reaction.

(ii) Even in an open space, the force of explosion could hurl fragments and objects at high speeds over large distances.

According to Newton's second law of motion, resultant force is proportional to rate of change of momentum. The expansion takes place in milliseconds, hence the rate of change of momentum is very high.

8 In order to study the sudden compression of a gas, some dry air is enclosed in a cylinder fitted with a piston, as shown in Fig. 7.1.

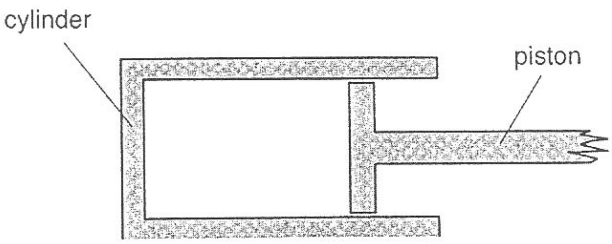


Fig. 7.1

The mass of air in the cylinder is constant. The material of the cylinder and the piston is an insulator so that no thermal energy enters or leaves the air.

The volume and pressure of the air are measured. The piston is then moved suddenly to compress the air and the new volume and pressure are measured.

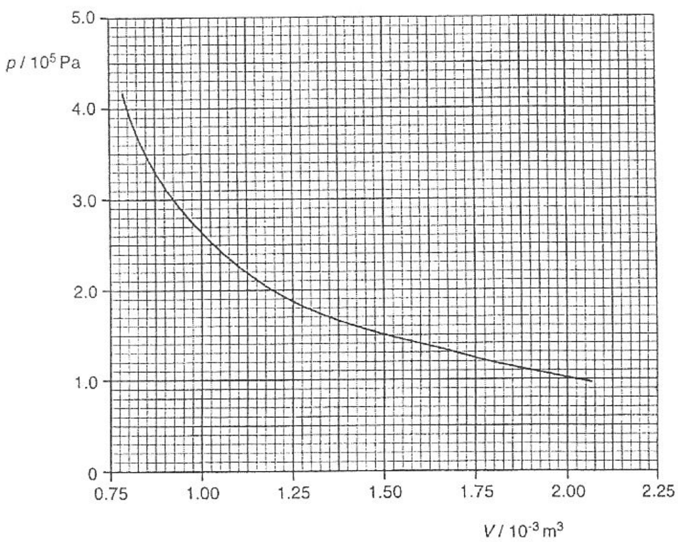


Fig. 7.2

It may be assumed that the dry air behaves as an ideal gas.

(a) By considering the air at volume 2.0×10^{-3} m^3 and at volume 8.0×10^{-4} m^3, and using the equation of state for an ideal gas, show that the temperature of the air increases when the air is compressed. [3]

(b) It is thought that the air in the cylinder obeys a relation of the form

$$pV^{\gamma} = c,$$

where γ and c are constants.
Explain how the relation may be tested by plotting a graph of lg p on the y-axis against lg V on the x-axis.

....

....

....

.... [3]

(c) Some data from Fig. 7.2 are used to plot the graph of Fig. 7.3.

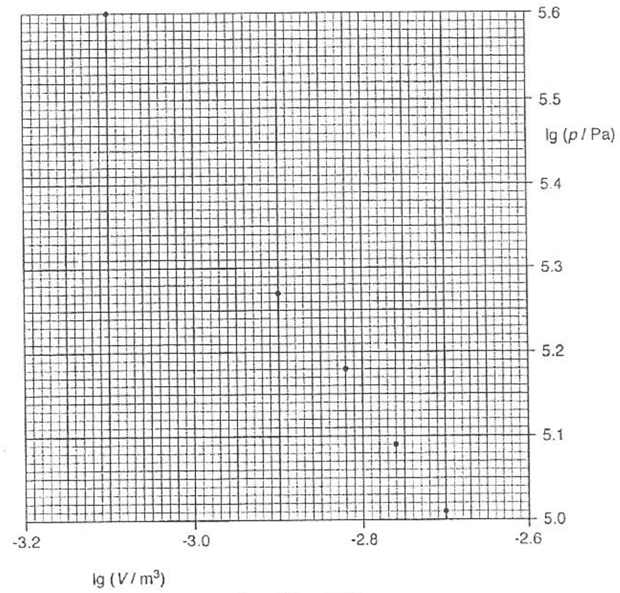


Fig. 7.3

(i) Use Fig. 7.2 to determine lg (p / Pa) for a volume V of 1.00 x 10^{-3} m^3.

lg (p / Pa) = ... [1]

(ii) On Fig. 7.3,

1. plot the point corresponding to V = 1.00 x 10^{-3} m^3, [1]
2. draw the line of best fit for the points. [1]

(iii) Use the line drawn in (ii) to determine the magnitudes of the constants γ and c in the expression in (b).

γ = ...

c = ...

[4]

(d) Fig. 7.4 shows the variation with volume V of the thermodynamic temperature T of the air.

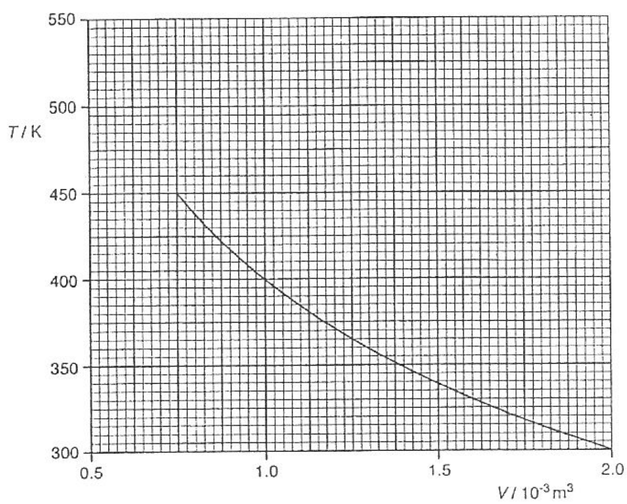


Fig. 7.4

The compression ratio for the air in the cylinder is given by the expression

$$\frac{\text{volume of gas before compression}}{\text{volume of gas after compression}}.$$

By extending the line of Fig. 7.4, estimate the final temperature of the air for an initial volume of 2.00 x 10^{-3} m^3 and a compression ratio of 3.85.

temperature = ... K [2]

(e) When a firework explodes, the powder in the firework burns rapidly to produce a small volume of gas at high temperature and pressure.

Use Fig. 7.4 to explain why the temperature of the gas falls rapidly.

....

....

.... [2]

N09/II/7

Solution

(a) In the ideal gas equation of state, the product of the pressure (P) and volume (V) of an ideal gas is directly proportional to its thermodynamic temperature (T).

$$pV = nRT$$

At volume 2.0 x 10^{-3} m^3, the pressure of the air was 1.0 x 10^5 Pa hence pV = 200 J.

At volume 8.0 x 10^{-4} m^3, the pressure of the air was 4.0 x 10^5 Pa hence pV = 320 J.

The product pV has increased from 200 J to 320 J, hence the temperature has increased.

(b) By taking the logarithm of both sides of the equation,

$$\lg p + \gamma \lg V = \lg c$$

$$\lg p = -\gamma \lg V + \lg c$$

If the relationship holds, then this graph will be a straight line with the equation in the form:

$$y = mx + c$$

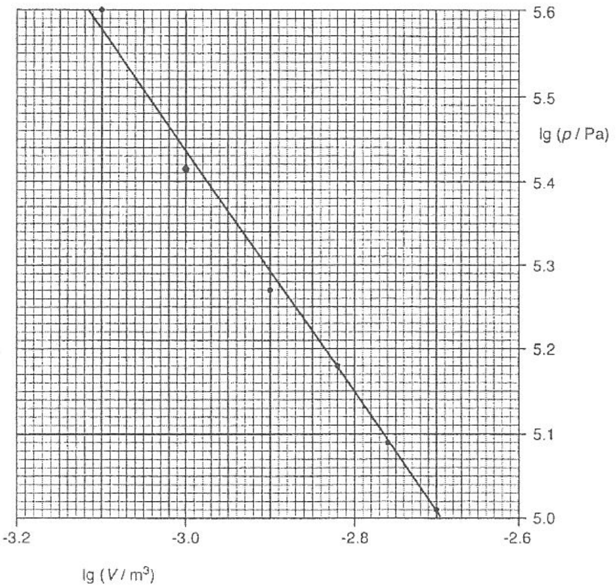
where the gradient $m = -\gamma$ and y-intercept lg c.

(c) (i) From Fig. 7.2, when volume V = 1.00 x 10^{-3} m^3, the pressure p = 2.6 x 10^5 Pa

lg (p / Pa) = <u>5.41</u>

(ii) 1. When V = 1.00 x 10^{-3} m^3, lg (V / m^3) = -3.0.

2. The best fit line is shown in the figure below.



(iii) The gradient

$-\gamma = (5.0 - 5.6)/(-2.69 - (-3.12)) = -0.6/0.43 = -1.4$

Substituting γ = 1.4 into the equation, using the data point (-2.69, 5.0),

$$5.0 = -1.4(-2.69) + \lg c$$

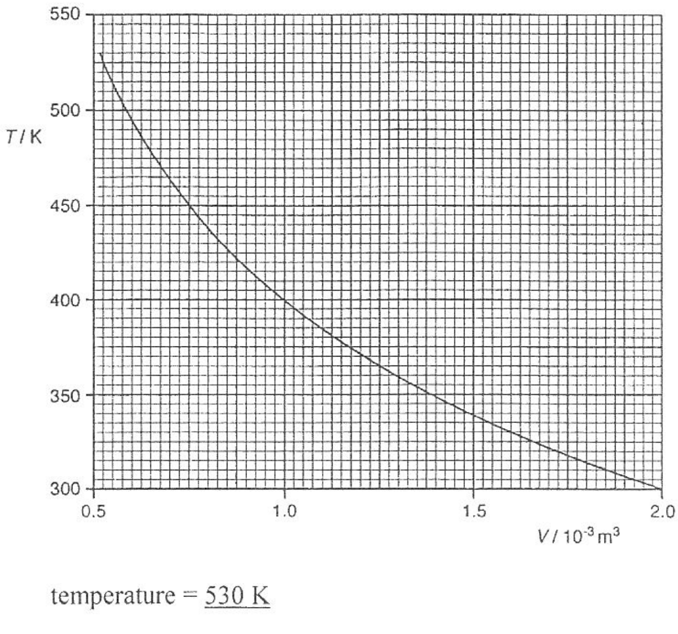
$$\lg c = 1.234$$

$$c = 17$$

γ = 1.4
c = 17 kg m^2 s^{-2}

(d) For a compression ratio of 3.85, the final volume of the air is 2.00 x 10^{-3} m^3 / 3.85 = 5.19 x 10^{-4} m^3.

The extended line is shown below.



temperature = 530 K

(e) The high pressure causes the gas to expand rapidly under adiabatic conditions, since there is no time for heat exchange between the gas and its surrounding.

Applying the first law of thermodynamics, the decrease in internal energy is equal to the work done by the gas in expanding.

From Fig. 7.4, the temperature decreases when the volume of the gas increases.

9 Thermal conduction is the transfer of thermal energy (heat) through a substance with no overall movement of the substance.

(a) Fig. 6.1 shows a solid metal rod of length about 50 cm that has one end maintained at a temperature of 100 °C using a bath of water.

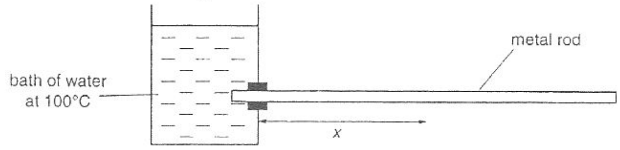


Fig. 6.1

The rod is in a draught-free room. The apparatus is left until the temperature at any point along the rod does not change. Fig. 6.2 shows the variation of the temperature θ of the rod with distance x from the bath of water.

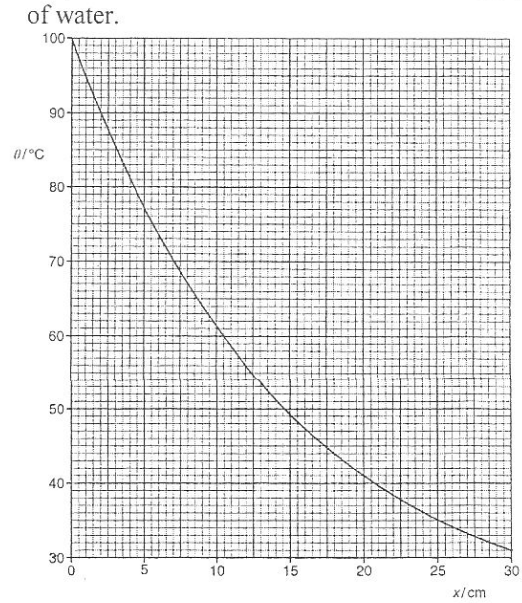


Fig. 6.2

Use Fig. 6.2 to determine the temperature of the rod at a distance x of 17.5 cm.

temperature = ... °C [1]

(b) The rod in (a) is shortened and placed between two baths of water, as shown in Fig. 6.3.

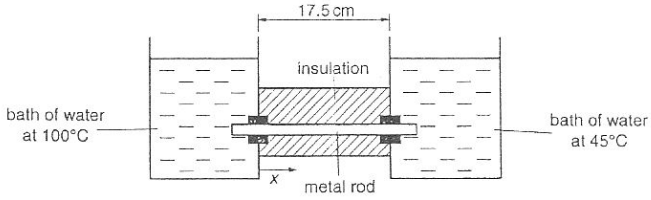


Fig. 6.3

The baths of water are maintained at temperatures of 100 °C and 45 °C. The length of the rod between the baths of water is 17.5 cm and the rod is surrounded by insulation. The apparatus is left until the temperature at any point along the rod does not change. Fig. 6.4 shows the variation of the temperature θ of the rod with distance x from the hotter bath of water.

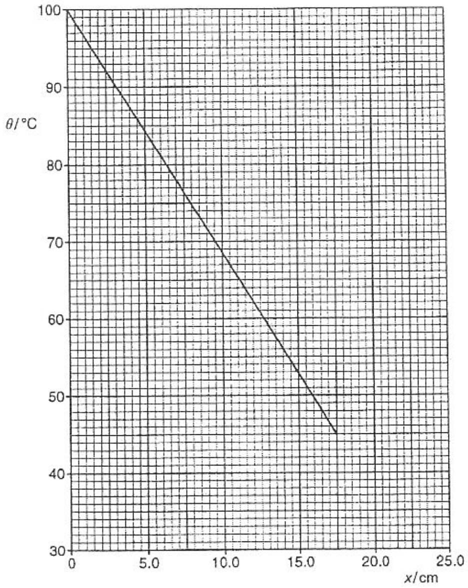


Fig. 6.4

(i) The insulation on the rod is perfect. State the relation between the rate at which thermal energy enters the rod at 100 °C and the rate at which it leaves the rod at 45 °C. [1]

(ii) Use Fig. 6.4 to determine the rate of change of temperature with distance along the rod (the temperature gradient). Give your answer to an appropriate number of significant figures.
Temperature gradient = ... °C cm^{-1} [2]

(iii) State why, for any value of x between 0.5 cm and 17 cm, the temperature, as shown in Fig. 6.4, is higher than that shown in Fig. 6.2. [1]

(c) A student thinks that the temperature θ of the insulated rod in (b) may be inversely proportional to the distance x along the rod. Show, without drawing a graph, that the proposal is not correct. [2]

(d) A second student thinks that the temperature θ of the non-insulated rod in (a) depends on the excess temperature of the rod above its surroundings.
Furthermore, the student thinks that the excess temperature reduces exponentially with distance along the rod.
In order to test the proposal, the student measures room temperature and then calculates the excess temperature θ_E and $\ln(\theta_E/°C)$ for different values of x. Fig. 6.5 shows some data for x, θ, θ_E and $\ln(\theta_E/°C)$.
room temperature = 20 °C

x/cm	θ/°C	θ_E/°C	$\ln(\theta_E/°C)$
0	100	80	4.38
2.0	90	70	4.25
5.0	77	....	....
8.0	67	47	3.85
12.0	56	36	3.58
15.0	49	29	3.37
17.5	45	25	3.22
20.0	41	21	3.04
25.0	35	15	2.70

Fig. 6.5

(i) Complete Fig. 6.5 for the distance x = 5.0 cm. [1]

(ii) Fig. 6.6 is a graph of some of the data of Fig. 6.5.

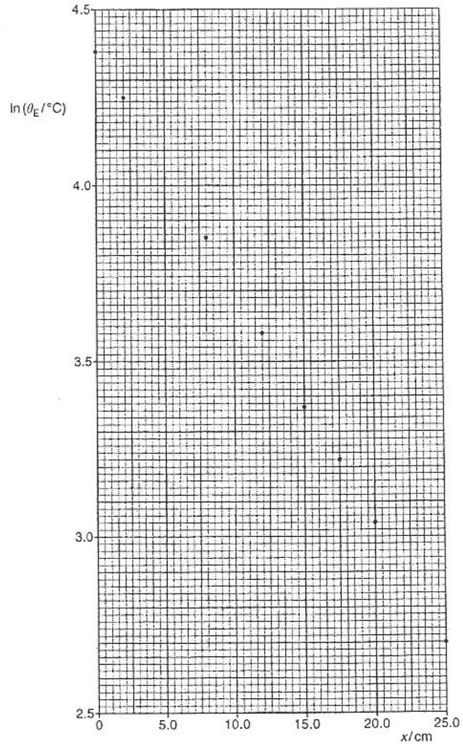


Fig. 6.6

1. Complete Fig. 6.6 using your data for the distance x = 5.0 cm. [1]
2. It is proposed that the excess temperature θ_E changes with distance x according to an expression of the form
$$\theta_E = \theta_0 e^{-\mu x}$$
where θ_0 and μ are constants.
Explain why the graph of Fig. 6.6 supports this proposal. [3]
3. Use Fig. 6.6 to determine the constants θ_0 and μ.
θ_0 = ... °C
μ = ... cm^{-1} [3]

(e) The material of the rod in (a) is a metal. A similar rod made of wood replaces the metal rod, under the same conditions.
On the axes of Fig. 6.2, sketch a graph to show a possible variation with distance x of the temperature θ of the wooden rod. [2]

N10/II/6

Solution

(a) From the graph, temperature at $x = 17.5$ cm is $\theta = 45$ °C.

(b) (i) Equal (since the rod is insulated).

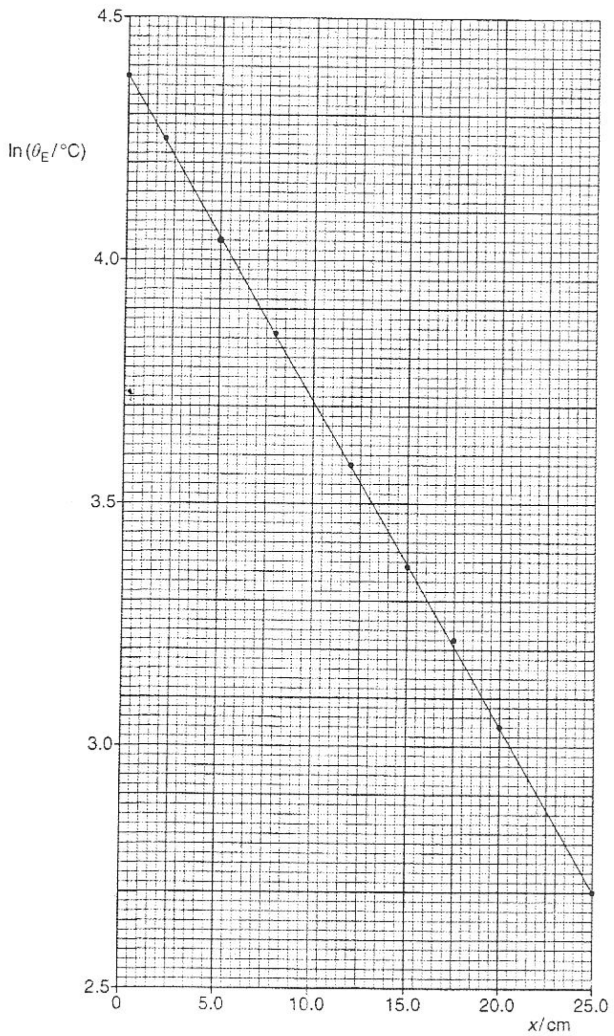
(ii) Temperature gradient
$= (45 - 100)/(17.5)$
$= (-65)/(17.5) = -3.1$ °C cm^{-1}.

(iii) The metal rod in Fig. 6.2 is a good conductor of heat and since it is not insulated, there is heat loss through its surface. The metal rod in Fig. 6.4, on the other hand, is insulated hence there is no heat loss through its surface. It is thus able to maintain a higher temperature at all points along its length.

(c) When the distance x tripled, e.g. from 5 cm to 15 cm, the temperature did not correspondingly fall to a third from 84 °C to 28 °C but instead to 53 °C. This violates an inverse-proportional mathematical relationship between temperature θ and distance x.

(d) (i) At $x = 5.0$ cm, θ_E/°C = 77 – 20 = 57 °C.
ln (θ_E/°C) = ln (57) = 4.04.

(ii) 1. The completed graph is shown below.



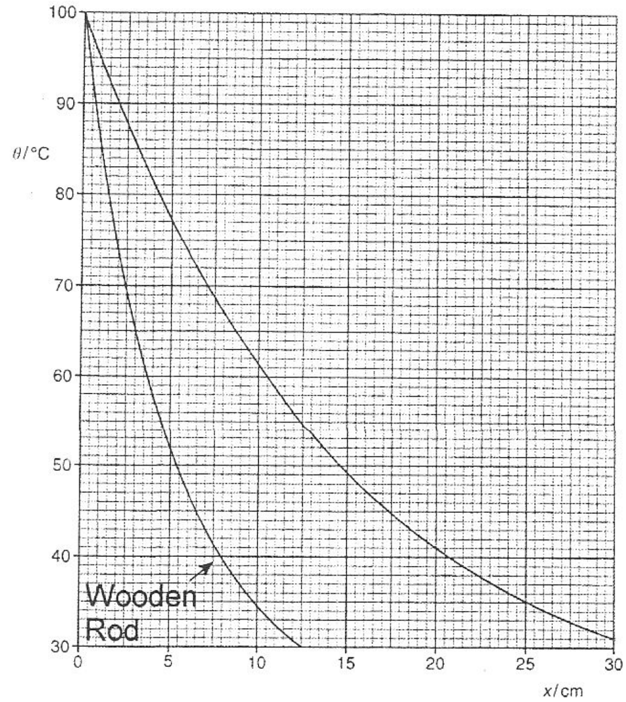
2. Taking the natural logarithm of both sides of the equation $\theta_E = \theta_0 e^{-\mu x}$, the equation becomes:

$$\ln(\theta_E) = \ln(\theta_0) - \mu x$$

which is the equation of a straight line graph of the form $y = mx + c$ if $\ln(\theta_E)$ is plotted against x, where the gradient $m = -\mu$ and y-intercept $c = \ln(\theta_0)$. Since the plotted graph is a straight line, this supports the exponential relationship between excess temperature θ_E and distance x.

3. From the graph, the y-intercept $\ln(\theta_0) = 4.38$ hence $\theta_0 = 80$ °C.
Gradient $-\mu = (2.70 - 4.38)/(25.0) = -0.067$ cm^{-1}.
Hence $\mu = 0.067$ cm^{-1}.

(e) As wood is a thermal insulator, the rate of heat transferred to the rod will be lower hence overall it will be at a lower temperature than the metal rod.



This is the reason frying pans come with wooden and not metal handles.

TOPIC 22 Planning

1 A solar panel heats water by absorbing infra-red radiation from the Sun. It consists of an array of pipes, through which water is passed. The array of pipes is contained in a flat box with a glass front, as shown in Fig. 7.1.

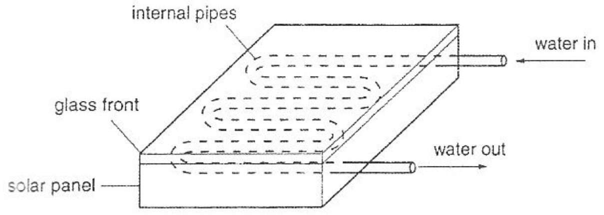


Fig. 7.1

Design an experiment to determine the efficiency of a model solar panel.

The following equipment is available: a high intensity lamp, a variable power supply, an intensity meter for infra-red radiation, a model solar panel with a glass front measuring 10 cm by 10 cm and any other equipment normally available in a school laboratory.

You should draw a labelled diagram to show the arrangement of your apparatus. In your account you should pay particular attention to

(a) the equipment you would use,
(b) the procedure to be followed,
(c) how the power output of the solar panel would be measured,
(d) the control of variables,
(e) any precautions that would be taken to improve the accuracy of the experiment. [12]

Diagram

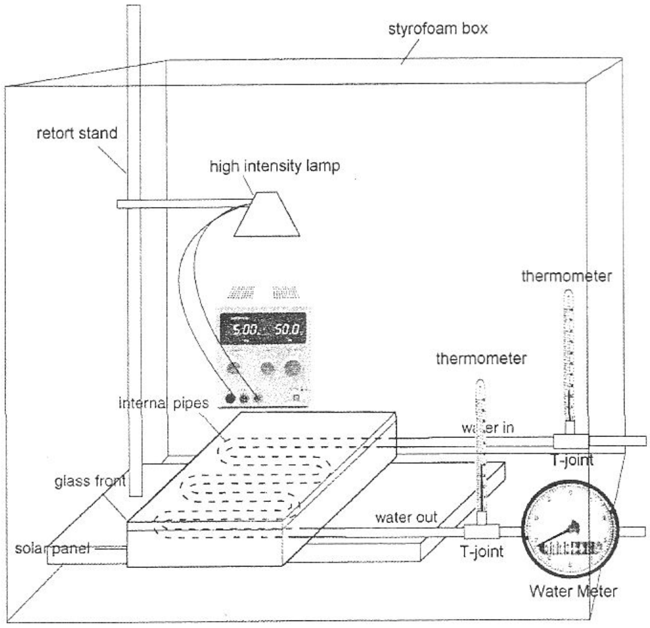
N10/II/7

Solution

Aim: To investigate the efficiency of a solar panel under different illumination and flow rate conditions.

Apparatus:
1. High intensity lamp
2. Variable power supply
3. Intensity meter for infra-red radiation
4. Solar panel 10 cm by 10 cm
5. Thermometers (Quantity: 2)
6. Styrofoam box with black-painted internal walls
7. Retort stand
8. Water flow meter (or alternatively a very large pail)
9. Stop watch
10. Electronic balance
11. Measuring cylinder

Experimental Setup:



Procedure:

1. Place the solar panel setup in the Styrofoam box to keep out ambient heat (the internal walls of the box are painted black to absorb the high intensity lamp's radiation and minimize reflection onto the solar panel).

2. Hold the high intensity lamp at a fixed distance above the solar panel using a retort stand.

3. Place one thermometer each at the inlet and outlet T-joints.

4. Collect some water in a measuring cylinder. Measure the mass (M) of the water with the electronic balance and volume (V) with the measuring cylinder to find its density $\rho = M/V$.

5. Connect a water meter to the outlet with the outflow emptied directly into the sink (if a water meter is not available then the outflow should be collected in a pail so that its mass can be measured later with an electronic balance. Then skip Step 4 above but the pail has to be very large since the water flow has to be stabilized for some time before temperature readings can be taken).

The Constants

The high intensity lamp should be held at a constant height above the solar panel. The height should be high enough to provide an even illumination of the entire solar panel, but not so high that too little radiation reaches the panel.

The inflow water should be kept at a constant temperature, preferably from an in house water tank since external roof top tanks tend to have uneven water temperatures.

The tap should provide a constant rate of water flow by ensuring that no other party is using the same water supply at the same time.

The Variables

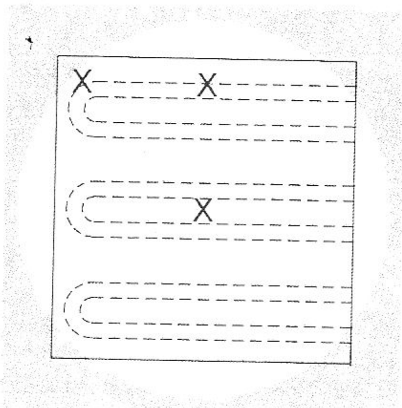
Varying the Intensity of the Infra-Red Radiation for a constant flow rate

Keeping the flow rate constant with the inlet tap at a fixed setting, the intensity of the high intensity lamp is varied by adjusting the output voltage of the variable power supply.

With each intensity setting, the intensity of the infra-red radiation arriving at the solar panel is measured by the intensity meter by removing the solar panel and placing in its place the intensity meter.

The intensity meter should be placed at the same vertical height above the table as that of the pipes.

The intensity meter should be placed at 3 different locations (marked with crosses 'X' below) corresponding to the centre, side and edge of the solar panel, and their readings weighted-averaged to obtain the weighted average intensity ((4 x corner intensity + 4 x side intensity + 1 x centre intensity) / 9).



The setup should be left alone for several minutes till the thermometer readings have stabilized, before each set of measurements is taken.

The mass flow rate can be obtained by taking the volume of water passing the water meter multiplied by the water's density calculated earlier, divided by the time (measured by a stop watch). If a water meter is not available, then the water has to be collected in a large pail and its mass measured with the electronic balance.

Measure six different intensity settings of infra-red radiation for each flow rate.

For each intensity setting, the outflow's temperature is measured after it has stabilized.

Varying the flow rate

After each set of readings, the tap's water flow rate is changed to a different value, to study the efficiency of the solar panel under different flow rates.

Three different flow rates are used to simulate low, medium and high flow rates.

Calculations

The thermal energy absorbed Q is related to the mass of water heated m, the specific heat capacity of water c and increase in temperature $\Delta\theta$ by the equation:

$$Q = mc\Delta\theta$$

Therefore,

$$dQ/dt = (dm/dt)\, c\Delta\theta$$

Where dQ/dt is the rate of thermal energy absorbed by the solar heater (i.e. power output P_{out}) and dm/dt the mass flow rate in kg s^{-1}.

Since intensity is defined as power per unit area, the incident infra-red power P can be obtained by multiplying the weighted-average intensity I (measured by the intensity meter) by the area of the solar panel A ($P = IA$).

Hypothesis

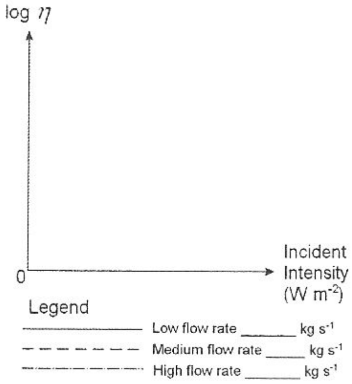
The efficiency η of the solar panel may be related to the incident intensity I by the equation $\eta = k I^Z$ where k and Z are constants. To verify whether the power Z has a value of one (in which case the equation is linear) or some other values, take the log of the equation: $\log \eta = \log k + Z \log I$.

Plot $\log \eta$ against $\log I$ to find Z (the gradient) and $\log k$ (the y-intercept.

Tabulation

The data collected can be tabulated (see below).
Repeat tabulation for medium and high flow rate.

Plot 3 graphs of $\log \eta$ vs average incident intensity of radiation for three different flow rates.



Precautions:

Accuracy

Ensure no air bubbles in the water in the pipes otherwise the water meter may overestimate the mass flow rate (this precaution is unnecessary if the water is collected in a pail and weighed).

Enclose the experimental setup in a Styrofoam box to minimize ambient heat.

Measure the infra-red radiation at several points on the solar panel and take the weighted average.

Water source should preferably be in-house water tank to ensure constant inflow water temperature.

Make sure no external party is using the same water source at the same time to avoid fluctuations in the water flow rate.

Wait for thermometer readings to stabilize before taking measurements.

Safety

Avoid touching the high intensity lamp or placing hands too near to it as it can be very hot.

Avoid shorting the leads of the variable power supply.

Avoid touching the leads of the variable power supply to prevent electric shock.

Mass flow rate (slow) = ________ kg s^{-1}						
Inflow water temperature = ________ ^{o}C						
Average Incident Intensity (I /W m^{-2})	Average Incident Power (P /W)	Outflow temperature (θ/ ^{o} C)	Increase in temperature ($\Delta\theta$/ ^{o} C)	Rate of Power Absorbed (P_{OUT} /W)	Efficiency η (P_{OUT} / P) x 100%	$\log \eta$

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