Level I
of the
CFA® Program

Quantitative Methods

TIME VALUE OF MONEY

Learning Objectives

- **LOS** : Interpret interest rates as **required rates of return**, **discount rates**, or **opportunity costs**.
- **LOS** : Explain an interest rate as the **sum of a real risk-free rate and premiums** that compensate investors for bearing distinct types of risk
- **LOS** : *Calculate* and interpret the **effective annual rate**, given the stated annual interest rate and the frequency of compounding
- **LOS** : *Calculate* the solution for time value of money problems for **different frequencies of compounding**
- **LOS** : Calculate and interpret the **future value** (FV) and **present value** (PV) of a single sum of money, an **ordinary annuity**, an **annuity due**, a **perpetuity** (PV only), and a **series of unequal cash flows**.
- **LOS** : Demonstrate the use of a **time line** in modeling and solving time value of money problems.

LOS : Interpret interest rates as required rates of return, discount rates, or opportunity costs

- Would you prefer $1,000 today or $1,000 in five years?
  - Obviously, you would rather have the $1,000 today.
  - This concept is called the **time value of money**.
  - Expressed more generally, 1 dollar **received today is worth more** than a similar amount be received some time in the future, due to earning potential.

- Time allows you the opportunity to **postpone consumption** and **earn interest**.

LOS : Interpret interest rates as required rates of return, discount rates, or opportunity costs

Interest rates can be thought of in three ways:

i. **Required rate of return:** The minimum rate of return an investor must receive in order **to accept an investment**.

ii. **Discount rate:** The rate used to discount future cash flows to allow for the time value of money (to determine the present value **equivalent** of some money to be received some time in future).

iii. **Opportunity cost:** The value of the **best forgone alternative**; the most valuable alternative investors give up when they choose what to do with money.

LOS : Explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk

What constitutes a given rate of interest?

$$\begin{aligned}\text{Interest rate } (r) = \; & \text{Real risk free interest rate} \\ & + \text{Inflation premium} \\ & + \text{Default risk premium} \\ & + \text{Liquidity premium} \\ & + \text{Maturity premium}\end{aligned}$$

- The real risk-free interest rate is the **single-period interest rate** for a completely **risk-free security** if no inflation was expected.
- The inflation premium compensates investors for **expected inflation**.

LOS : Explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk

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- The **default risk premium** compensates investors for the possibility that the borrower **will fail to make a promised payment** at the contracted time and in the contracted amount.
- The **liquidity premium** compensates investors for the risk of loss relative to an investment's fair value if the investment needs to be converted to cash quickly**converted to cash quickly**.

LOS : Explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk

What constitutes a given rate of interest?

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\end{aligned}
$$

- Maturity premium accounts for **additional risks** resulting from **longer time to maturity**.

LOS : Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding

The effective annual rate of interest (EAR) refers to the rate of return earned by an investor in a year, taking into account the effects of compounding.

$$\mathbf{EAR = (1 + Periodic\ rate)^m - 1 = \left(1 + \frac{Stated\ annual\ rate}{m}\right)^m - 1}$$

i. **Compounding** is the process of accumulating interest over a period of time. It can be **discrete**, in which the ***number of compounding periods*** within a year is finite, or **continuous**, if the number of compounding periods is ***infinite***.

ii. **Periodic rate of interest** is the rate of interest earned over a single compounding period. For example, a fixed bank account might pay a periodic quarterly interest rate of 3% that compounds 4 times a year.

iii. **The stated annual rate** is an often-quoted annual rate of interest that does not account for within-the-year compounding. It's equal to the **periodic rate multiplied by the number of compounding periods per year**.

LOS : Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding

Example

Suppose we are given a stated interest rate of 10%, **compounded quarterly**, here is what we get for EAR:

$$\text{EAR} = \left(1 + \frac{0.10}{4}\right)^{4} - 1 = 0.10381 \text{ or } 10.38\%$$

If the compounding was **monthly**,

$$\text{EAR} = \left(1 + \frac{0.10}{12}\right)^{12} - 1 = 0.10471 \text{ or } 10.47\%$$

- Therefore, **the more frequent the frequency of compounding, the higher the EAR**.
- The effective annual rate will always be higher than the stated rate if there is more than one compounding period.

LOS : Solve time value of money problems for different frequencies of compounding

An investment of $1 today will be worth a certain amount *N* years from now, if it is invested at an interest rate of *r* per year.

$$\mathbf{FV_N = PV \times (1 + r)^N}$$

Where *FV* = future value at time *N*, *PV* = present value, and *r* = interest rate per year.

Therefore, in order to a single cash flow of $1 *N* years from today, you must make a single investment today of the following amount:

$$\mathbf{PV = \frac{FV_N}{(1 + r)^N}}$$

LOS : Solve time value of money problems for different frequencies of compounding

When compounding periods are different than annual compounding, use the formula below.

$$\mathbf{PV = \frac{FV_N}{\left(1 + \frac{r}{m}\right)^{m \times N}}}$$

Where *m* is the number of compounding periods in a year, and *r* is the stated annual rate of interest.

Example

LOS : Solve time value of money problems for different frequencies of compounding

Example

Suppose **you need** to have **$10,000 in your savings account** at the **end of the next 3 years**. Assume that the account offers a return of **10 percent per year** with **quarterly compounding**. How much would you need to invest today?

LOS : Solve time value of money problems for different frequencies of compounding

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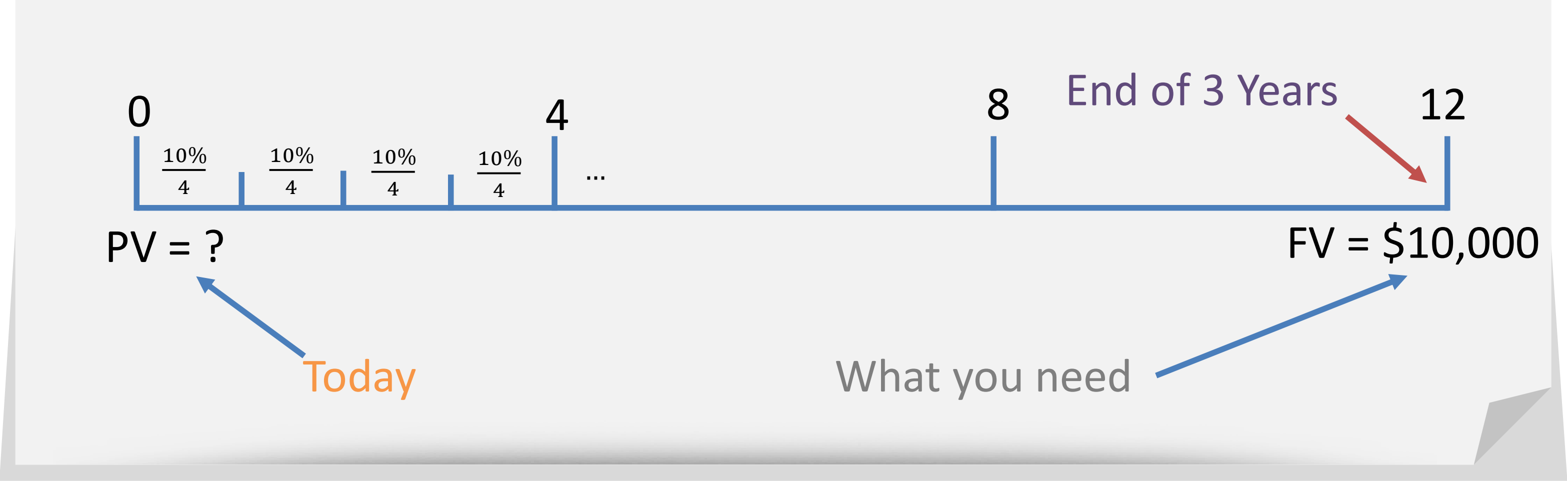

LOS : Solve time value of money problems for different frequencies of compounding

Example

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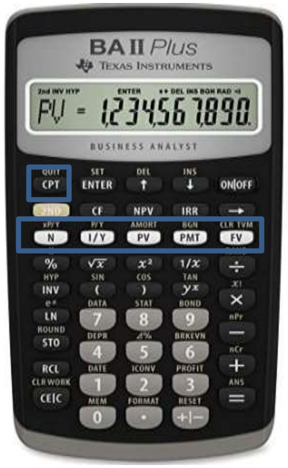
$$PV = \frac{FV_N}{\left(1 + \frac{r}{m}\right)^{m \times N}}$$

$$= \frac{\$10{,}000}{\left(1+\frac{0.10}{4}\right)^{4\times 3}} = \$7{,}435.56$$

LOS : Solve time value of money problems for different frequencies of compounding

Which can also be done on the financial calculator:



N: Number of periods

I/Y: Interest rate per period

PV: Present value

FV: Future value

Press 2nd FV to clear memory

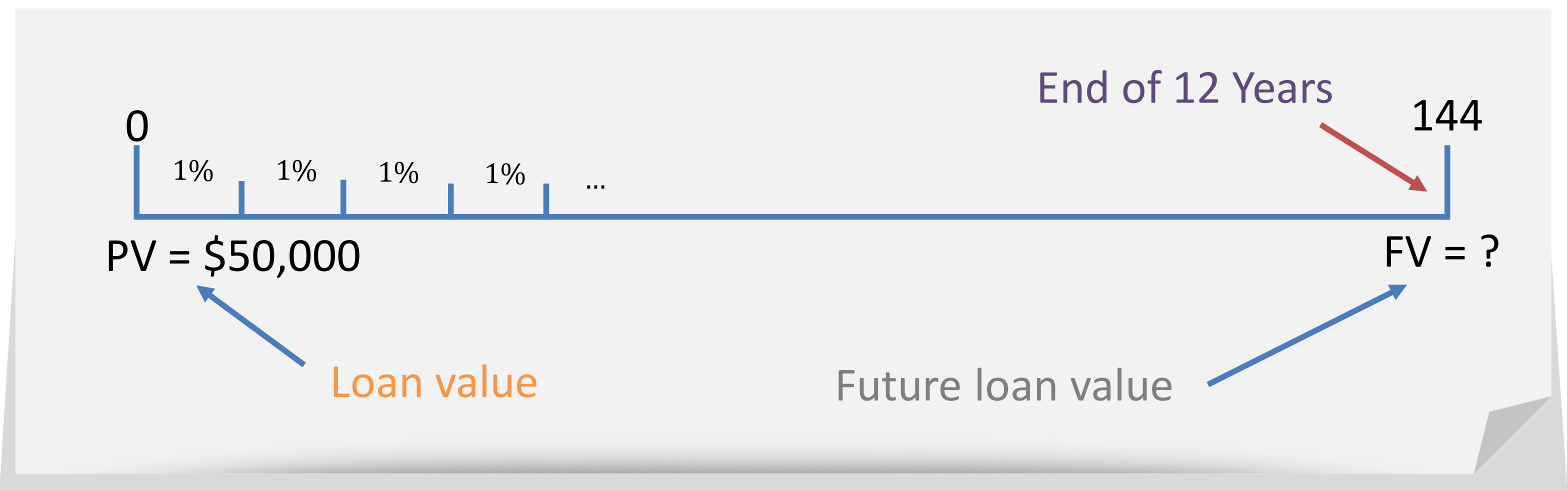
BA II Plus		HP 12C	
Press	**Display**	**Press**	**Display**
10,000 “FV”	FV= 10,000	10,000 CHS FV	10,000
4×3 “N”	N= 12	12 ENTER n	12
10/4 “I/Y”	I/Y= 2.5	2.5 ENTER i	2.5
“CPT” “PV”	**PV= -7,435.56**	**PV**	**-7,435.56**

LOS : Solve time value of money problems for different frequencies of compounding

Example

A **$50,000**, **12-year loan** has an interest rate of **1% per month**?

The **future value** of this loan is *closest* to:



LOS : Solve time value of money problems for different frequencies of compounding

Example

A **$50,000**, **12-year loan** has an interest rate of 1% per month?

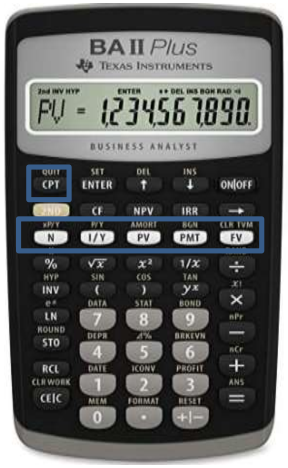
The **future value** of this loan is *closest* to:

$$\boldsymbol{FV = PV \times (1 + r)^{m\times N}}$$

$$\boldsymbol{FV = 50,000 \times (1 + 1\%)^{(12\times12)}}$$
$$\boldsymbol{= \$209,530.77}$$

LOS : Solve time value of money problems for different frequencies of compounding

Which can also be done on the financial calculator:



N: Number of periods

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PV: Present value

FV: Future value

Press 2nd FV to clear memory

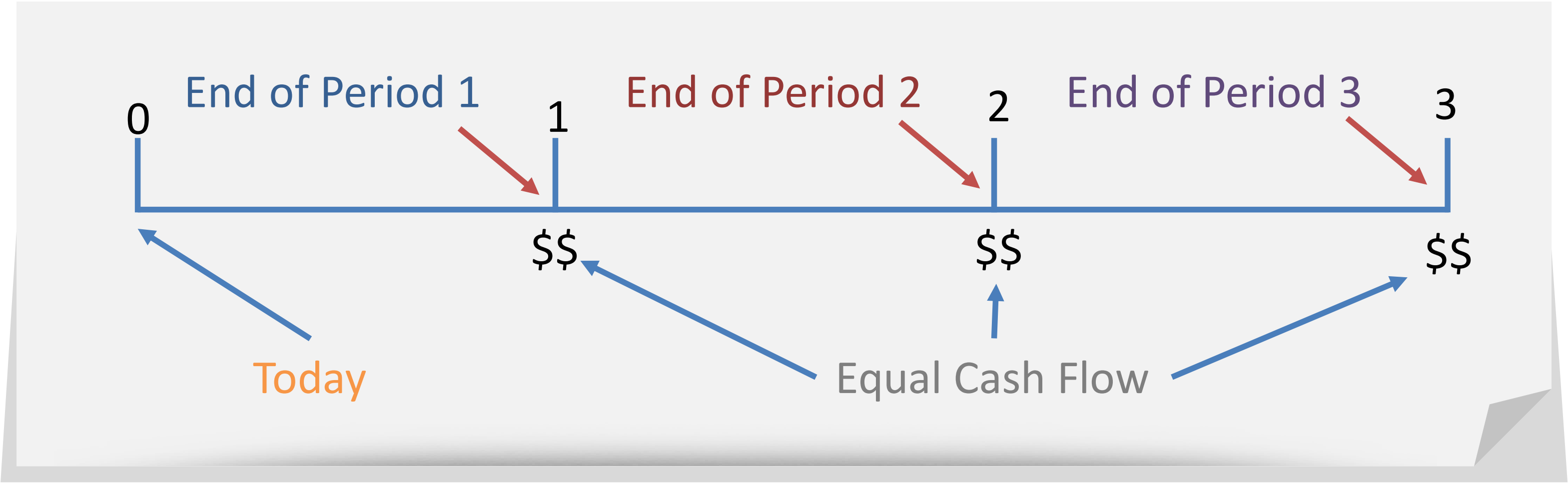
BA II Plus		HP 12C	
Press	**Display**	**Press**	**Display**
50,000 "PV"	FV = 50,000	50,000 CHS PV	50,000
12×12 "N"	N = 144	144 ENTER n	144
1 "I/Y"	I/Y = 1	1 ENTER i	1
"CPT" "FV"	**FV = -209,530.77**	**FV**	**-209,530.77**

LOS : Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows

An **annuity** is a finite series of cash flows, all with the same value.

In an **ordinary annuity**, payments are made at the end of each time period, usually a month or year, beginning at time *t = 1*.

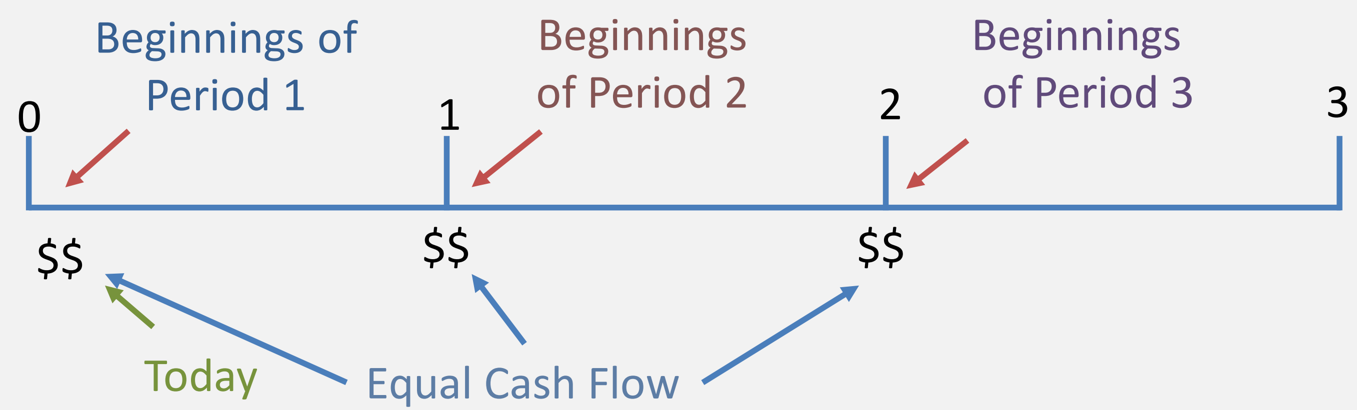
- Example: Mortgage payments.



LOS : Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows

In an **annuity due**, payments start immediately (in advance), beginning at time t = 0.

- Example: Insurance premiums.



LOS : Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows

The present value (PV) and future value (FV) of an ordinary annuity are given by the following:

$$\mathbf{PV} = \mathbf{A} \times \frac{1 - \frac{1}{(1+r)^N}}{r}$$

$$\mathbf{FV} = \mathbf{A} \times \frac{(1+r)^N - 1}{r}$$

Where:

A = annuity amount, r = interest rate per period, and N = number of annuity payments.

LOS : Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows

The present value of an annuity due is equal to the present value of an ordinary annuity multiplied by (1 + r).

$$\mathbf{PV} = \mathbf{A} \times \frac{1 - \frac{1}{(1+r)^N}}{r} \times (1 + r)$$

The future value of an annuity due is equal to the future value of an ordinary annuity multiplied by (1 + r).

$$\mathbf{FV} = \mathbf{A} \times \frac{(1+r)^N - 1}{r} \times (1 + r)$$

LOS : Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows

Example

Dave Maxwell buys an annuity that pays a **regular series of payments of $200 per year** for a period of **15 years**. He will receive equal payments at the ***beginning*** of every year. What premium should Maxwell be willing to pay for this annuity, assuming an **effective rate of interest of 13.5%**?

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Example

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Method 1:

Using the formula

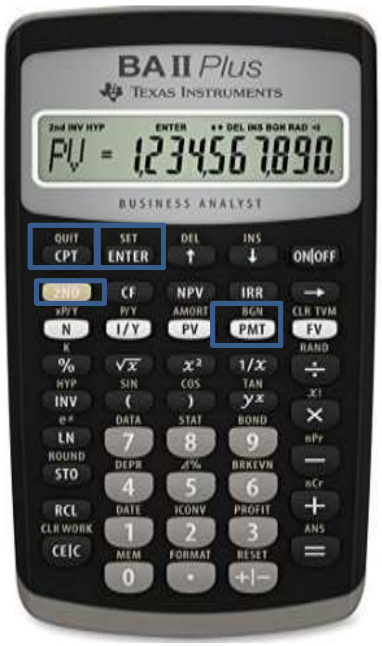
$$PV = A \times \frac{1 - \frac{1}{(1+r)^N}}{r} \times (1+r)$$

$$= 200 \times \frac{1 - \frac{1}{1.135^{15}}}{0.135} \times 1.135 = \$1,430$$

LOS e: Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows

Method 2:

Using the financial calculator. First set the calculator to ***beginning mode***.



- Press 2nd FV to clear all previously stored data.
- Set **BGN** since this is an annuity due. You do this by pressing 2nd **BGN** 2nd **SET** until **BGN** displays. After that, you must then press 2nd **QUIT** and then continue as below:

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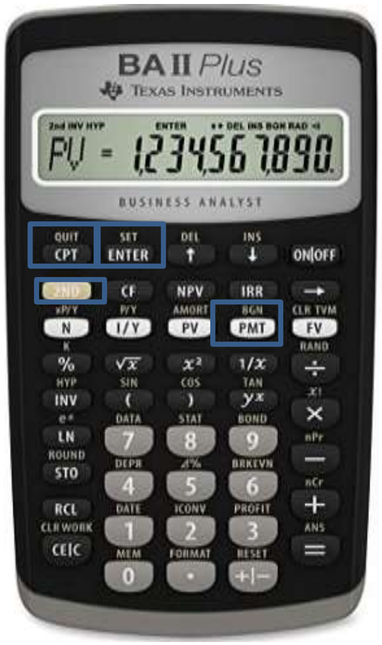

BA II Plus			
Press		**Display**	
15	"N"	N =	15
13.5	"I/Y"	I/Y =	13.50
200	"PMT"	PMT =	200
0	"FV"	FV =	0
"CPT"	**"PV"**	**PV =**	**-1,430**

LOS : Demonstrate the use of a time line in modeling and solving time value of money problems

A timeline is a physical illustration of the amounts and timing of cash flows associated with an investment project.

- It helps you to **visualize the cash flows** and therefore achieve a better understanding of the question at hand.
- Cash flows that are **regular and of equal amounts** can be modeled as **annuities**.
- But we need to create **timelines** for cash flows that are ***not* of equal amounts or timings**.

LOS : Demonstrate the use of a time line in modeling and solving time value of money problems

Example

A project has the following series of cash flows:

- \$100 at t = 1; \$150 at t = 2; \$250 at t = 3; \$300 at t = 4; and \$250 at t = 5

Assuming an annual interest rate is 5% per year, the present value of the series of cash flows is *closest* to:

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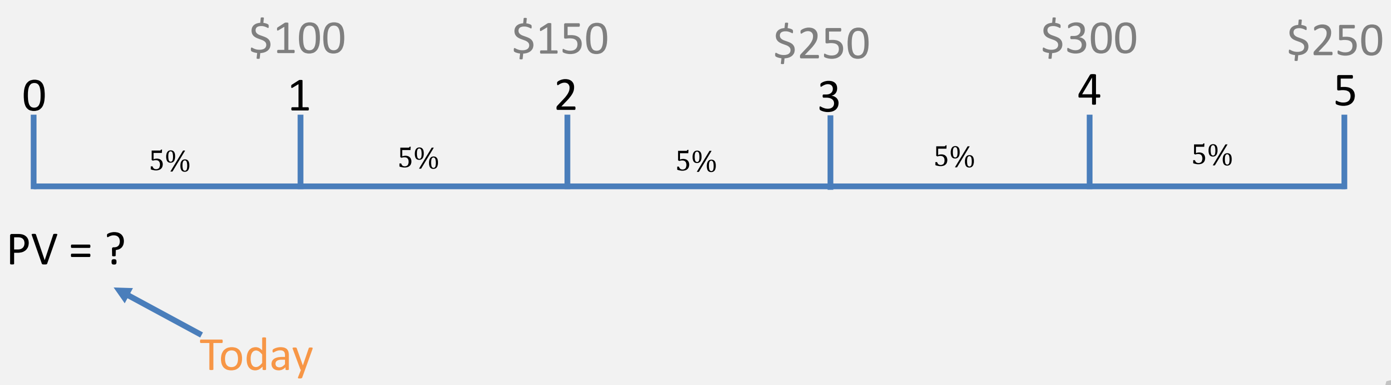

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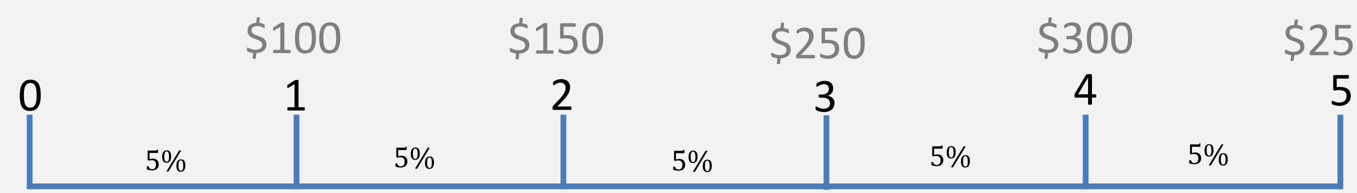

$$PV = \frac{\$100}{1.05^1} + \frac{\$150}{1.05^2} + \frac{\$250}{1.05^3} + \frac{\$300}{1.05^4} + \frac{\$250}{1.05^5}$$

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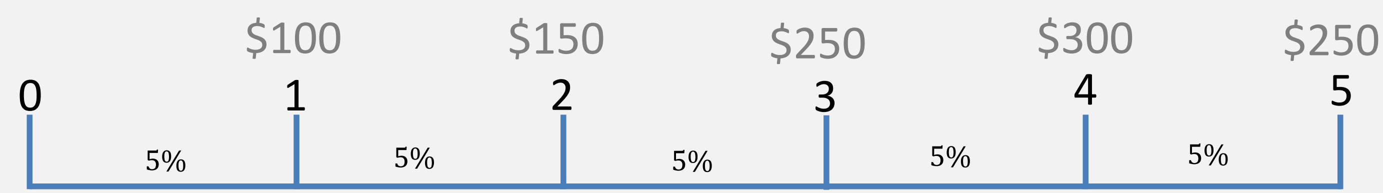

$$PV = \frac{\$100}{1.05^1} + \frac{\$150}{1.05^2} + \frac{\$250}{1.05^3} + \frac{\$300}{1.05^4} + \frac{\$250}{1.05^5}$$

$$\$95.24 + \$136.05 + \$215.96 + \$246.81 + \$195.88$$

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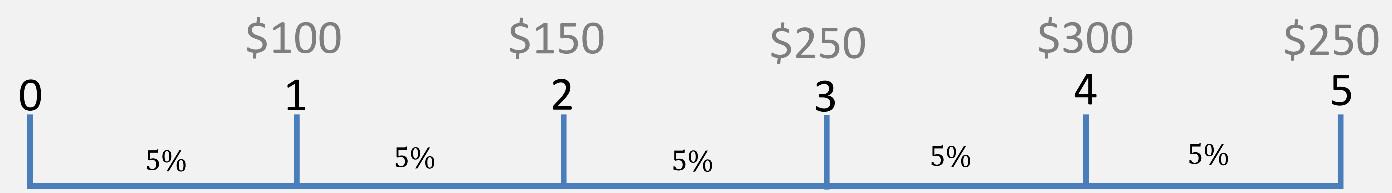

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$$\$889.94$$

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