CUBE NOTES
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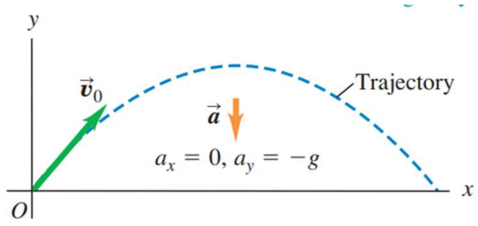
Projectile Motion & Circular Motion

Key Ideas

For a motion to be called projectile motion, it should meet 2 conditions:

- The object should be under the influence of gravity
- Undergoes two-dimensional motion only

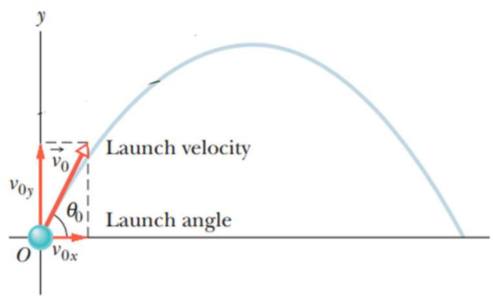


2D Motion under influence of gravity

Components of Velocity

For a body projected at an angle θ_0 with initial velocity v_0, the components of velocity are -

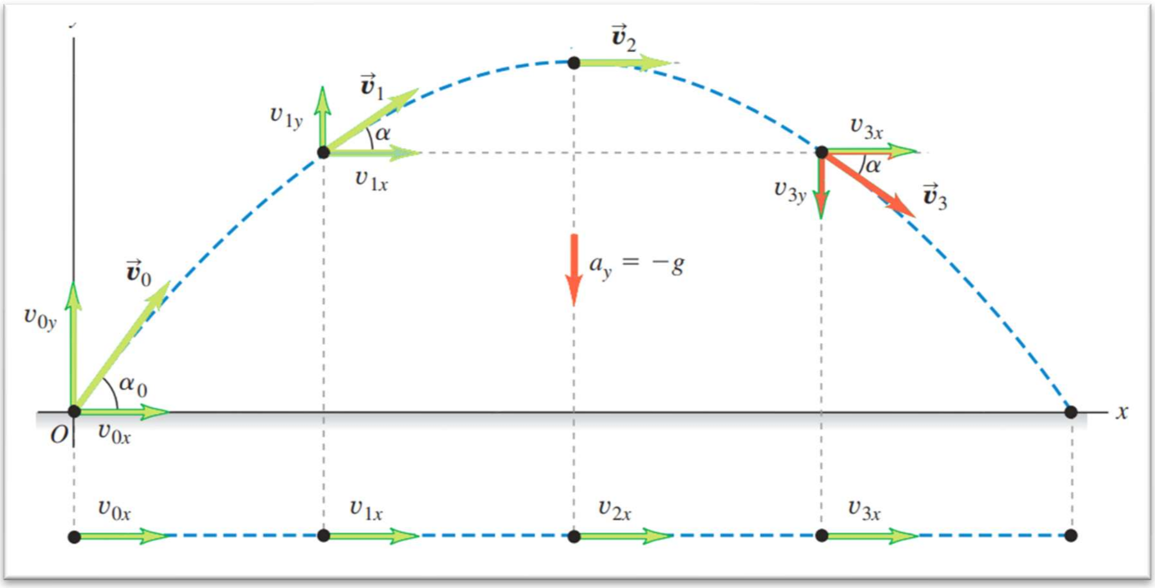
- Vertical Component: $v_{0y} = v_0 \sin\theta_0$
- Horizontal Component: $v_{0x} = v_0 \cos\theta_0$



Important Facts:

- Horizontal velocity remains constant.
  ...and why? because there is no force acting on the body in horizontal direction and therefore no reason for change in velocity
- Vertical component of velocity changes the same way as that of an object projected vertically up
  - ...and why? because the force of gravity continuously acts on the body changing the velocity.
  - acceleration due to gravity only affects the vertical motion, not the horizontal.
- Vertical and horizontal motions are independent
  - *Both components can be studied separately*

Velocity Components at Various Positions



- At t=0,
  - Initial velocity is v_0, at angle θ
  - Vertical component is v_{0y} and
  - Horizontal component is v_{0x}
- Upwards Motion,
  - v_y reduces,
  - v_x remains the same.
- At Maximum Height,
  - $v_y = 0$,
  - v_x is unchanged.
  - The vertical component v_y changes direction at the maximum height
- Downwards Motion,
  - v_y increases,
  - v_x remains the same.
- At Ground Level (when it hits the ground):
  - Magnitude of velocity same as initial, but direction may differ.
- Symmetry in Projectile Motion: The upward and downward parts of the motion are symmetrical in time and velocity, which can aid in solving problems.

Equations of Projectile Motion

- Horizontal Motion *(constant velocity):*
  - $x - x_0 = v_{0x}t$
  - $x - x_0 = (v_0 \cos \theta_0)t$ if x_0 is at the origin, position $x = (v_0 \cos \theta_0)t$

- Vertical Motion (freefall):
  - $y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$
  - $v_y = v_0 \sin(\theta_0) - gt$
  - $v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0)$

Caution

- If y_0 is at the origin, you can modify the above equations by putting $y_0 = 0$
- Choosing an appropriate coordinate system (e.g., origin at the launch point) can simplify the mathematical analysis
- However, do not ignore the initial conditions when they are not at the origin

Useful Equations (For Speed Solving of Problems)

- Equation Connecting x and y Coordinates:

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}$$

- The Horizontal Range

$$R = \frac{2v_0^2 \sin(\theta_0) \cos(\theta_0)}{g}$$

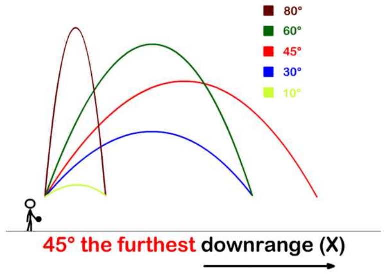
$$R = \frac{v_0^2 \sin(2\theta_0)}{g}$$

Complementary angles give the same range. That is, a projectile leaving at an angle θ_0 or $(180^0 - \theta_0)$ would have the same range if the initial velocity is the same

- Maximum Range:

  Attained at $\theta = 45^0$

  Maximum range = $\frac{v_0^2}{g}$



Summary of Equations

S.No	Equation	Description of Variables	Concept
1	$\Delta x = v_{0x} \cdot t$	Δx : Horizontal displacement, v_{0x} : Horizontal component of initial velocity,	Horizontal motion with constant velocity; (*Mistake often made*: Ignoring horizontal velocity is constant)
2	$\Delta y = v_{0y} \cdot t - \frac{1}{2} \cdot g \cdot t^2$	Δy : Vertical displacement, v_{0y} : Vertical component of initial velocity	Vertical motion under gravity; (*Mistake often made*: Mixing up vertical and horizontal components)
3	$v_{0x} = v_0 \cdot \cos(\theta)$	v_{0x} : Horizontal component of initial velocity, θ : Launch angle	Calculation of horizontal velocity component; (*Mistake often made*: Assuming it changes over time)
4	$v_{0y} = v_0 \cdot \sin(\theta)$	v_{0y} : Vertical component of initial velocity,	Calculation of vertical velocity component;
5	$T = \frac{2 \cdot v_0 \cdot \sin(\theta)}{g}$	T : Total time in air,	Time of flight; (*Mistake often made*: Ignoring the factor of 2 for the total time)
6	$H = \frac{v_0^2 \cdot \sin^2(\theta)}{2 \cdot g}$	H : Maximum height	Maximum height reached;
7	$R = \frac{v_0^2 \cdot \sin(2\theta)}{g}$	R : Horizontal range,	Horizontal range; (*Mistake often made*: Forgetting that the angle is doubled in the sine function for range)

9	$v_y = v_0 \cdot \sin(\theta) - g \cdot t$	v_y : Vertical velocity at any time,	Vertical velocity changes with time; (*Mistake often made*: Ignoring gravity's effect on vertical motion.
10	$v = \sqrt{v_x^2 + v_y^2}$	v : Resultant velocity, v_x : Horizontal velocity, v_y : Vertical velocity	Resultant velocity at any time;
11	$\theta = 45°$	θ : Angle of projection for maximum range	Maximizing range
12	$t_{\text{peak}} = \dfrac{v_0 \cdot \sin(\theta)}{g}$	t_{peak} : Time to reach peak,	Time to reach maximum height; (*Mistake often made*: Confusing this with total time of flight)
13	$v_{\text{impact}} = \sqrt{v_{0x}^2 + \left(v_{0y} - g \cdot t\right)^2}$	v_{impact} : Final velocity, v_{0x}, v_{0y} :	Impact velocity; (*Mistake often made*: Ignoring that the vertical component changes while horizontal remains constant
14	$t_{half} = \dfrac{v_0 \cdot sin(\theta)}{2g}$	t_{half} : Half of the total time of flight,	Time to reach peak and return;

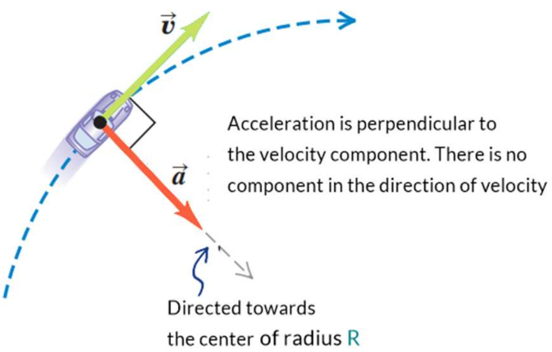
Uniform Circular Motion

Key Idea

When an object travels in a circle at a constant speed, it is said to be undergoing uniform circular motion

Conditions for Circular Motion

- Velocity Vector: Always tangent to the circle at that point. The magnitude remains constant, but the direction changes continuously.
  - This tangential velocity helps maintain the object in a circular path without affecting its speed.
- Acceleration Vector: Always perpendicular to the velocity vector and pointing towards the centre.
  - it ensures that the object stays on its circular path.



- Magnitude: The magnitude of both the velocity and acceleration vectors remains constant.
  - The constancy in magnitude allows uniform circular motion. *Any change would cause non-uniform circular motion.*

Formulas in Circular Motion

Formula	Description	Common Mistakes	How to Use Correctly
$a = \frac{v^2}{R}$	Centripetal acceleration	Confusing v with speed	Ensure v is the magnitude of velocity
$T = \frac{2\pi R}{v}$	Time period for one revolution	Mixing up radius and diameter	Use radius, not diameter, in the formula
$f = \frac{v}{2\pi R}$	Frequency of revolution (revolutions/second)	Ignoring units of R	Ensure the radius is in consistent units