Given that $m = \frac{1}{x}$ and $n = \frac{1}{y}$, solve the simultaneous equations.

$\frac{1}{x} + \frac{2}{y} + 1 = 0$ — ①

$\frac{3}{x} + \frac{5}{y} = 2$ — ②

Given $m = \frac{1}{x}$ and $n = \frac{1}{y}$:

from ①: $m + 2n + 1 = 0$

$m + 2n = -1$ — ③

from ②: $3m + 5n = 2$ — ④

③×3: $3m + 6n = -3$ — ⑤

⑤−④: $(3m+6n) - (3m+5n) = -3-2$

$n = -5$

Substitute $n = -5$ into ③:

$m + 2(-5) = -1$

$m = 9$

Since $m = 9$ and $n = -5$:

$\therefore \frac{1}{x} = 9$ and $\frac{1}{y} = -5$

$x = \frac{1}{9}$ $y = -\frac{1}{5}$

Solutions

A fraction becomes $\frac{2}{9}$ when 1 is subtracted from its numerator and the denominator. It becomes $\frac{3}{4}$ when the numerator is tripled and 11 is added to the denominator. Find the fraction.

Let the fraction be $\frac{x}{y}$.

$\frac{x-1}{2y} = \frac{2}{9}$

$4y = 9x - 9$

$9x - 4y = 9$ — ①

$\frac{3x}{y+11} = \frac{3}{4}$

$3y + 33 = 12x$

$12x - 3y = 33$ — ②

①×3: $27x - 12y = 27$ — ③

②×4: $48x - 12y = 132$ — ④

④−③: $(48x - 12y) - (27x - 12y) = 132 - 27$

$48x - 27x = 105$

$21x = 105$

$x = 5$

Substitute $x = 5$ into ①:

$9(5) - 4y = 9$

$36 = 4y$

$y = 9$

$\frac{x}{y} = \frac{5}{9}$

The school needs to buy new sets of tables and chairs to replace damaged ones in the hall. The school found that at least 100 sets of tables and chairs need to be replaced. Each chair costs \$16 and each table costs \$28. The budget allocated for the replacement is \$4800.

(a) Form two inequalities in x, the number of tables and chairs to be purchased.

(b) Solve the inequality and represent the solution on a number line.

(c) Find the maximum sets of tables and chairs the school can buy.

(a) $x \geq 100$ — ①

$16x + 28x \leq 4800$

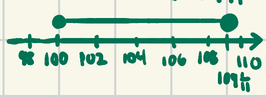
(b) $44x \leq 4800$

$x \leq \frac{4800}{44}$

$x \leq 109\frac{1}{11}$ — ②

Combining ① and ②:

$100 \leq x \leq 109\frac{1}{11}$



(c) maximum $x = 109$

Given that $(3x-2)(x+p) = 3x^2 + qx - 6$, find p and q.

$(3x-2)(x+p) = 3x^2 + 3px - 2x - 2p$

$= 3x^2 + (3p-2)x - 2p$

$3x^2 + qx - 6 = 3x^2 + (3p-2)x - 2p$

$-6 = -2p$ — ①

$p = 3$

$q = 3p - 2$ — ②

Substitute $p = 3$ into ②:

$q = 3(3) - 2$

$= 7$

Given that $(7x+2)(3x-8) + px(x+1) = 30x^2 + qx + r$, find p, q, r.

$(7x+2)(3x-8) + px(x+1)$

$= 21x^2 - 56x + 6x - 16 + px^2 + px$

$= 21x^2 + px^2 - 56x + 6x + px - 16$

$= 21x^2 + px^2 - 50x + px - 16$

$= (21+p)x^2 + (p-50)x - 16$

$(21+p)x^2 + (p-50)x - 16 = 30x^2 + qx + r$

$r = -16$

$21 + p = 30$

$p = 9$ — ①

$p - 50 = q$ — ②

Sub ① in ②: $q = -41$

Expand & simplify

(a) $4p - 3(3p+2)(2p-1)$

(b) $-8b(1-2b) - (2b+1)(4b-5)$

(c) $\frac{1}{2}(3-x)(2x-6) - 5(x^2-x-1)$

(d) $(3d-6)(6d-\frac{1}{3}) - (4d-8)(\frac{1}{4}d+3)$

37. (a) $4p - 3(3p+2)(2p-1)$

$= 4p - 3(6p^2 - 3p + 4p - 2)$

$= 4p - 3(6p^2 + p - 2)$

$= 4p - 18p^2 - 3p + 6$

$= -18p^2 + p + 6$

(b) $-8b(1-2b) - (2b+1)(4b-5)$

$= -8b + 16b^2 - (8b^2 - 10b + 4b - 5)$

$= -8b + 16b^2 - (8b^2 - 6b - 5)$

$= -8b + 16b^2 - 8b^2 + 6b + 5$

$= 8b^2 - 2b + 5$

(c) $\frac{1}{2}(3-x)(2x-6) - 5(x^2-x-1)$

$= \frac{1}{2}(6x - 18 - 2x^2 + 6x) - 5x^2 + 5x + 5$

$= \frac{1}{2}(-2x^2 + 12x - 18) - 5x^2 + 5x + 5$

$= -x^2 + 6x - 9 - 5x^2 + 5x + 5$

$= -6x^2 + 11x - 4$

(d) $(3d-6)(6d-\frac{1}{3}) - (4d-8)(\frac{1}{4}d+3)$

$= 18d^2 - d - 36d + 2 - (d^2 + 12d - 2d - 24)$

$= 18d^2 - d - 36d + 2 - (d^2 + 10d - 24)$

$= 18d^2 - d - 36d + 2 - d^2 - 10d + 24$

$= 17d^2 - 47d + 26$

(a) $(4c+5d)(5d-3c) + (7c-d)(2d+c)$

(b) $(3g-2h)(2g-h) - 2g(g-2h)$

(c) $(5e-3h)(2h+e) - (3e-h)(2h-4e)$

(d) $(8m-3n)(2n+m) - 3m(2m-4n)$

(e) $4xy - (2x-y)(3y+5x)$

(f) $5mn + 3(m-2n)(2m-3n)$

12. (a) $(4c+5d)(5d-3c) + (7c-d)(2d+c)$

$= 20cd - 12c^2 + 25d^2 - 15cd + 14cd + 7c^2 - 2d^2 - cd$

$= 23d^2 + 18cd - 5c^2$

(b) $(3g-2h)(2g-h) - 2g(g-2h)$

$= 6g^2 - 3gh - \cancel{4gh} + 2h^2 - 2g^2 + \cancel{4gh}$

$= 4g^2 - 3gh + 2h^2$

(c) $(5e-3h)(2h+e) - (3e-h)(2h-4e)$

$= 10eh + 5e^2 - 6h^2 - 3eh - (6eh - 12e^2 - 2h^2 + 4eh)$

$= 10eh + 5e^2 - 6h^2 - 3eh - 6eh + 12e^2 + 2h^2 - 4eh$

$= 17e^2 - 3eh - 4h^2$

(d) $(8m-3n)(2n+m) - 3m(2m-4n)$

$= 16mn + 8m^2 - 6n^2 - 3nm - 6m^2 + 12mn$

$= 2m^2 + 25mn - 6n^2$

(e) $4xy - (2x-y)(3y+5x)$

$= 4xy - (6xy + 10x^2 - 3y^2 - 5xy)$

$= 4xy - 6xy - 10x^2 + 3y^2 + 5xy$

$= 3y^2 + 3xy - 10x^2$

(f) $5mn + 3(m-2n)(2m-3n)$

$= 5mn + (3m-6n)(2m-3n)$

$= 5mn + 6m^2 - 9mn - 12mn + 18n^2$

$= 6m^2 - 16mn + 18n^2$