CUBE NOTES

Class 11/12 | AP Physics | IIT JEE | NEET

PHYSICS
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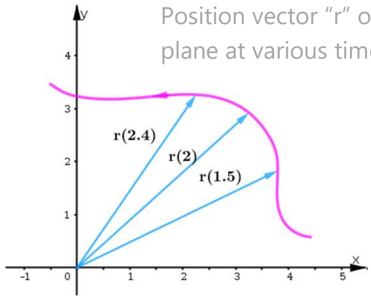
Motion in Two Dimensions: Position, Displacement & Velocity

Position Vectors in space

1. In 2D space, a position vector $\vec{r}$ is written as:

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

Position vector "r" on XY plane at various times

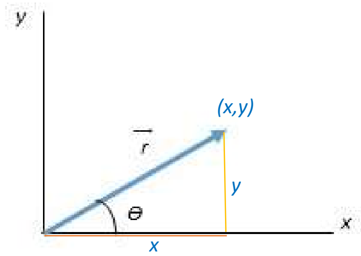


where x & y are the scalar components of the vector.

(in 3D space, include the Z component such that: $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$)

2. Position vector $\vec{r}$ is represented as a line *from origin* to point (x, y) in the XY plane. (See different r representations in the diagram above)

3. *Magnitude* of the position vector



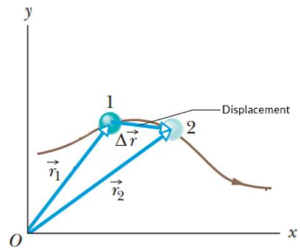
$$|\vec{r}| = \sqrt{x^2 + y^2}$$

4. *Direction*: The angle θ that $\vec{r}$ makes with the positive x-axis can be found as:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Displacement $\Delta\vec{r}$

1. Displacement, denoted as $\Delta\vec{r}$ is the change in position from $\vec{r_1}$ to $\vec{r_2}$. It is a vector quantity



$$\Delta\vec{r} = \vec{r_2} - \vec{r_1} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}$$

where Δx and Δy are the changes in the x and y components.

(in 3D, include displacement in Z direction as well $\Delta\vec{r} = \vec{r_2} - \vec{r_1} = \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$)

Magnitude of Displacement: $$|\Delta\vec{r}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Direction of Displacement: $$\tan\theta = \frac{\Delta y}{\Delta x}$$

Average Velocity

1. $\overrightarrow{V_{\text{avg}}} = \frac{Displacement}{Time}$

$$\overrightarrow{v_{\text{avg}}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{\imath}+\Delta y\hat{\jmath}}{\Delta t}$$

$$\overrightarrow{v_{\text{avg}}} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$$

(In 3D, just add $\Delta Z\ \hat{k}$ component)

The direction of the average velocity is the same as the direction of displacement vector.

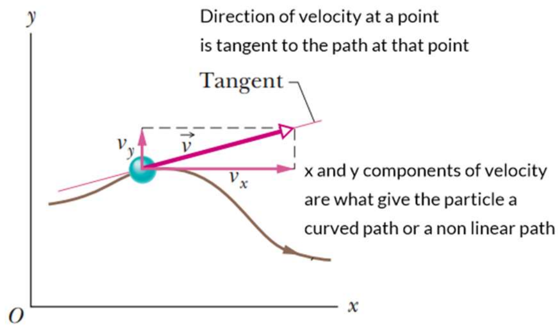
2. *Magnitude of average velocity*:

$$\left|\overrightarrow{v_{\text{avg}}}\right| = \frac{|\Delta\vec{r}|}{\Delta t}$$

3. The magnitude of average velocity *is the average speed*
4. Zero Displacement: If the displacement is zero (i.e., starting and ending at the same point), the average velocity will also be zero, regardless of the path taken.

Instantaneous Velocity

1. Velocity at an instant of time. It represents the actual velocity of the particle at a given instant, reflecting how its position is changing with time



2. If position vector changes from $\vec{r_1}$ to $\vec{r_2}$, with displacement $\Delta\vec{r}$ in $time\ \Delta t$.

   Instantaneous velocity is approached as Δt shrinks towards zero

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

3. Components:

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}$$

Direction: Same as the tangent at that point.

(in 3D, include displacement in Z direction as we

4. Magnitude: The magnitude of instantaneous velocity can be found as:

$$|\vec{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$