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Calculation of the Ship's Vertical Center of Gravity, *KG*

For the ship's officer, stability is mainly a problem of finding the position of the vessel's vertical center of gravity, *KG*, because the position of metacenter, *M*, is readily available to him by using the hydrostatic data aboard the ship. In this chapter we will discuss the methods of finding the height of the vertical center of gravity of the ship when it is loaded with its fuel, water, stores, and cargo.

What is the Center of Gravity?

The center of gravity of the ship is that point through which all the vertically downward forces of weight are considered to act together. When we talk about the *KG*, we are only considering the vertical height of *G* above the keel. For transverse stability calculations it is assumed that *G* will be on the centerline when *KG* is used. The use of off the centerline positions of *G* and of the longitudinal position of *G* (the longitudinal center of gravity LCG) will be discussed later in this book. An important point to note is that for a ship to be at rest or in equilibrium the center of gravity must be vertically in line with the center of buoyancy.

The Light Ship *KG*

Before a ship's officer can begin stability calculations, he must know the position of the center of gravity for the vessel in a light condition (a condition prior to loading any cargo, fuel, water, or stores). This light *KG* should be found by performing the inclining experiment (discussed at length in Chapter 5). Fortunately, the inclining experiment is performed at the shipyard when the ship is built or by a naval architect dockside after an alteration to the ship's structure which could change the value of the light ship *KG*. If the light ship *KG* information is not aboard, the ship's officer should communicate directly with his company's office to obtain such data.

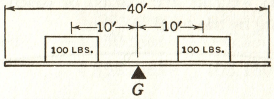
After loading begins, every weight that is added to that of the vessel will affect the original center of gravity. In order to find the change in the

position of *G*, the officer must employ the theory of moments. In practice he must estimate as accurately as possible the positions of the centers of gravity of every consignment of cargo, fuel, water, and stores and multiply each weight by the height of its *G* above the keel. Then he must divide the sum of all these products or moments by the total weights, including the weight of the light ship, which will establish the new center of gravity, expressed as a number of feet above the keel, the *KG* for the loaded condition.

Using Moments to Find *KG*

It is convenient in discussing moments to recall a seesaw. Referring to Figure 12 let us suppose the seesaw is 40 feet in length. Its center of gravity is at the midpoint of its length. A 100-pound weight is placed 10 feet from the fulcrum at *G*. How far must another 100-pound weight be placed from the fulcrum on the other side in order that the seesaw will balance? The distance is 10 feet. Perhaps the student has subconsciously used the theory of moments to obtain the answer.

The *moment* of 1,000 foot-pounds (100 pounds × 10 feet = 1,000 foot-pounds) must be balanced on the other side of the fulcrum by the same moment. Therefore we divide 1,000 foot-pounds by 100 pounds to obtain a distance of 10 feet. If the weight on the other side had been 200 pounds, the distance would have been 5 feet. In other words, the moments around the center of gravity of any member must be equal.



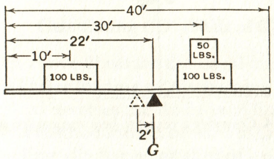


Figure 13. We cannot revolve moments about a shifting center of gravity.

Now let us consider Figure 13 where 150 pounds are placed 10 feet from the fulcrum. The seesaw will not balance. How far will the fulcrum have to move toward the 150-pound weight? In other words, how far has G shifted due to the additional 50 pounds? We cannot revolve moments about a shifting center of gravity. Let us revolve them instead around the left end of the seesaw.

Weights	Distance		Moment
100 pounds ×	10 feet	=	1,000 foot-pounds
150 pounds ×	30 feet	=	4,500 foot-pounds
250 pounds total weights			5,500 foot-pounds total moments

Dividing total moments by total weights (ignoring weight of seesaw)

$$\frac{5{,}500 \text{ foot-pounds}}{250 \text{ pounds}} = 22 \text{ feet (from left end of seesaw)}$$

G, therefore, has shifted 2 feet to the right.

Now let us check with the former method of using moments on the seesaw.

8 feet × 150 pounds = 1,200 foot-pounds on one side of fulcrum
12 feet × 100 pounds = 1,200 foot-pounds on other side of fulcrum

The seesaw balances. We have now seen that, where there is an addition of weights with a consequent shifting of G, it is better to revolve the moments around a fixed point. On a vessel this point is the keel, K. If we rotate the seesaw to a vertical position as shown in Figure 14 and consider it to be a vessel, with a vertical center of gravity (KG) at the fulcrum, we can determine the KG as weights are loaded or discharged by finding the moments and dividing by the total weights.

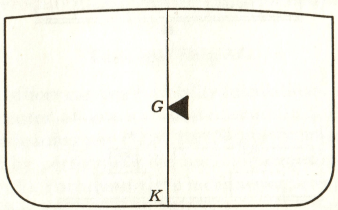


Figure 14. Consider a vessel to be a vertical seesaw.

In using the example of a seesaw, we have not included the weight of the seesaw since it was negligible. With a vessel, however, the weight of the light ship and her KG must be considered and their moment added to the moments of all weights loaded.

Using examples, let us now observe how moments are used to find the height of a vessel's center of gravity above the keel, KG.

EXAMPLE 1. A vessel floating at her light draft displaces (weighs) 5,000 tons. The light ship KG is 20 feet above the keel. Two hundred tons are loaded 10 feet above the keel and 300 tons 5 feet above the light ship KG. What will be the position of the new KG?

Weights in tons	*KG* in feet	Moments in foot-tons
5,000	20	100,000
200	10	2,000
300	25	7,500
5,500 total weight		109,500 total moments

Dividing total moments by total weights, KG = 19.9 ft *Ans.*

EXAMPLE 2. The vessel in Example 1 now has the following weights removed from the locations listed below. Find the new KG after discharging.

Weights in tons	*KG* in feet	Moments in foot-tons
700	5	3,500
300	2	600
150	15	2,250
1,150 total weight		6,350 total moments

Original	5,500 tons	109,500 foot-tons
Discharged	−1,150 tons	−6,350 foot-tons
Final	4,350 tons	103,150 foot-tons

$$\text{New } KG = \frac{103{,}150 \text{ foot-tons}}{4{,}350 \text{ tons}} \times 23.7 \text{ feet } Ans.$$

Calculating *GG′* (Shift of *G*)

The two examples above illustrate the usual method of calculating the vertical position of G after loading or discharging several different items of weight. The method involves the division of final vertical moments by final weight. But what method should be used to find the *shift* of the center of gravity due to loading, discharge, or shift of a *single* item of weight? Of course, even in the case of a shift of weight we can use our basic principle of final moments divided by final weight to calculate the position of G, this final position then being compared with the initial position to find GG' (the shift of G to a new position G').

EXAMPLE 3. Suppose that on a 10,000-ton vessel with a *KG* of 25 feet, 200 tons are shifted vertically upwards a distance of 20 feet:

	Weight	KG	Moments
Initial condition	10,000	25	250,000
Moment due to shift			+4,000
Final condition	10,000		254,000

$$KG = \frac{254{,}000}{10{,}000} = 25.4 \text{ feet (or a shift of 0.4 foot up)}$$

This method of solving the problem is somewhat cumbersome. It involves steps which can be summarized as follows:

$$\frac{\text{Initial vertical moments} \pm \text{moment due to shift}}{\text{Displacement}} = \text{initial } KG \pm GG'$$

But since

$$\frac{\text{Initial moments}}{\text{Displacement}} = \text{Initial } KG$$

It follows that

$$\frac{\text{Moment due to shift } (w \times d)}{\text{Displacement}} = GG'$$

This is an extremely important formula, expressing the relationship between the shift of weight on a vessel and the corresponding shift of the ship's center of gravity. It can be used to calculate not only a vertical shift of *G*, but a shift in any direction whatever. Constant reference will be made to the formula throughout the remainder of the text.

Let us use the formula to solve the problem in Example 3.

EXAMPLE 4. $GG' = \dfrac{w \times d}{\Delta} = \dfrac{(200 \text{ tons}) \times (20 \text{ feet})}{(10{,}000 \text{ tons})} = 0.4 \text{ foot (up)}$

The simplicity of calculation of *GG'* when the shift of a single item of weight is involved is apparent in Example 4. The formula may be used with equal simplicity to solve for *GG'* when a single item is loaded or discharged. Two cautions must be observed, however. The displacement is that which the vessel possesses *after* the act of loading or discharging. Also, the distance *d* is the vertical distance between the initial position of *G* and the position of the loaded or discharged weight. The validity of these observations may be proved by noting that if weight is loaded at (or discharged from) the center of gravity of a vessel, *G* does not shift. Only the displacement has been increased (or decreased). The problem is now simply a shifting problem. Shift the weight from *G* to where it actually has been loaded (or shift the weight to *G* from where it was actually discharged). The next two examples will illustrate the procedure:

EXAMPLE 5. Three hundred tons of salt water are loaded in a starboard deep tank, the center of the water 10 feet above the keel. The displacement the vessel before loading was 9,700 tons, the *KG* 25 feet. Find the vertical shift of *G*.

$$GG' = \frac{w \times d}{\Delta} = \frac{300 \text{ tons} \times 15 \text{ feet}}{10{,}000 \text{ tons}} = 0.45 \text{ foot (down)}$$

EXAMPLE 6. A military tank weighing 60 tons is discharged from the port side of the upper deck. Its *KG* is 45 feet. Displacement of the vessel before discharging was 6,060 tons, *KG* 20 feet. Find the vertical shift of *G*.

$$GG' = \frac{w \times d}{\Delta} = \frac{60 \text{ tons} \times 25 \text{ feet}}{6{,}000 \text{ tons}} = 0.25 \text{ foot (down)}$$

Calculating *GG'* with Suspended Weight

A suspended weight problem is a special type of a *GG'* problem. Whether a ship is upright or inclined in either direction, when a weight is lifted by the ship's cargo gear, the center of gravity of that weight is transferred to the point from which it is suspended. This is a very important and greatly overlooked situation. Imagine that the jumbo boom of your ship is lifting a 30-ton container. The head of the jumbo boom is 80 feet above the container which is in the lower hold. As soon as you lift the container a fraction of an inch clear of the deck, the *KG* of the 30-ton container shifts from the bottom of the lower hold to the point of suspension, the head of the boom. If the displacement of the ship were 10,000 tons, the following *GG'* would result.

EXAMPLE 7. The case of the suspended cargo.

$$GG' = \frac{w \times d}{\Delta} = \frac{30 \text{ tons} \times 80 \text{ feet}}{10{,}000 \text{ tons}} = 0.24 \text{ foot (up)}$$

Note that as the container is raised to a two-blocked position (i.e., its highest possible position) the *GG'* will not increase any more than it did when initially lifted clear of the deck. Many fishing vessels and small work boats have been capsized because their crew did not realize what actually happens with a suspended weight. With a shore-side crane there is no suspended weight problem. It is just a loading or a discharge type problem. Merchant marine cargo vessels have such large displacement that it is not usually necessary to be concerned about loss of stability due to lifting cargo with their own gear.

To summarize the GG' formula:

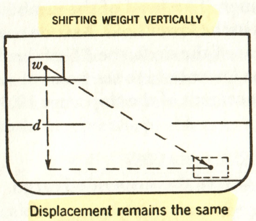
1. The center of gravity of a ship will shift from its original position G to a new position G' a total distance of GG' when a weight is added, removed, or shifted.

$$GG' = \frac{w \times d}{\Delta}$$

Where: w = the weight in tons added, removed, or shifted
d = a. for loading or discharging, the distance in feet between center of gravity of the cargo and the center of gravity of the ship or
b. for shifting cargo, the distance in feet of the shift or
c. for suspended cargo, the distance in feet between the point of suspension and the center of gravity of the vessel
Δ = the final displacement of the ship in tons.

2. When a weight is loaded, GG' will move in the direction toward the added weight.
3. When a weight is discharged, GG' will move directly away from the discharged weight.
4. When a weight is shifted, GG' will move directly parallel to the shifted weight.

See Figure 15.



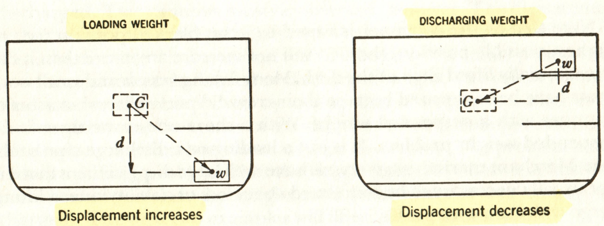


Figure 15. Using the GG' formula to find the vertical shift of G.

Finding *KG* When Loading or Discharging

So far we have dealt only with the effect on the center of gravity of loading or discharging one or a few items. In finding KG for a vessel that is to be completely loaded, the following steps are to be taken:

1. Find KG for every cargo, fuel, water, and stores compartment or tank on the vessel.
2. Multiply these distances by the weights in the respective locations.
3. Add total weights including weight of light ship.
4. Add total moments including the moment of light ship (light ship displacement × light ship KG).
5. Divide total moments by total weights to produce final KG.

The question now arises: How is the center of gravity of a compartment found? There are two cases: (1), where the compartment is completely filled with a homogeneous cargo (cargo that has the same density) and (2), where the compartment is partly filled with cargo, homogeneous or otherwise, or the compartment is filled completely with heterogeneous cargo (cargo of varied nature, general cargo).

CASE 1. Compartment filled with homogeneous cargo.

The centers of gravity for all compartments and tanks are found on the ship's capacity plan. It is possible that this plan might not be available. In this case, it would be well to contact the ship's company office and try to obtain it. Sufficient data is required to be supplied in the ship's stability booklet so that KG for each tank, compartment or level can be readily obtained by the ship's officer.

CASE 2. Compartment partially filled or completely filled with heterogeneous cargo.

Finding the centers of gravity in holds where general cargo is loaded or where the hold is only partially filled, is entirely a matter of estimation. If the cargo is homogeneous, an estimation of G is made. The moment is found by multiplying the weight of the cargo by the distance of the estimated G above the keel, KG. If the cargo is of general nature, heterogeneous, an estimate of G may also be made, giving careful attention to the distribution of heavy and light cargoes. One should remember that the center of gravity of the weight of an entire hold will never be above the geometric center of the hold. For general cargo it is almost inevitable that the actual G of the weight in the hold will be well below the geometric center of the hold because heavy cargo is not loaded over a lighter less dense cargo.

In rare cases when extreme accuracy is desired, the mere estimate of the position of the center of gravity of the weight in a hold may not be sufficient. The general cargo may have to be broken down into its components and a calculation made as in the example shown in Figure 16. We

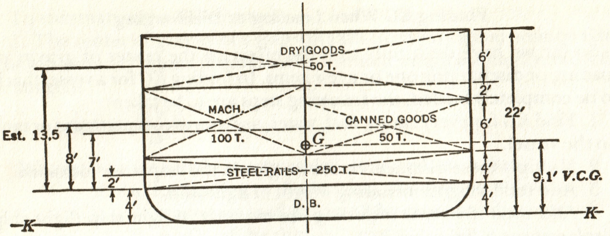


Figure 16. Finding the center of gravity of No. 2 LH (lower hold) when stowed with general cargo.

can see from the example in Figure 16 that finding the center of gravity of a compartment is similar in method to finding the *G* of a vessel. The moments are taken from the bottom of the compartment instead of the keel. Note that the centers of gravity of the various commodities are taken at half their height. In the case of the dry goods in Figure 16 an estimate of 13.5 feet had to be made.

When dealing with modern ships such as containerships, barge carriers, and roll-on-roll-off ships, the distribution of weights is calculated by levels. It is assumed that all units on a level have the same *KG*. Thus the sum of the weights per level multiplied by the *KG* of the level gives the vertical moment. In turn the sum of the vertical moments divided by the total displacement yields the *KG* for the ship in that loaded condition. Unitized loading as found on modern ships allows the officer to pigeonhole cargo weights much the same way as weights in double bottoms were done in earlier days.

Required Accuracy of *KG*

In general we want to calculate *GM* of a vessel to the closest tenth of a foot. By using the *GG'* formula we can easily see that for approximately every 10,000 tons of displacement we can allow ourselves 1,000 foot-tons of error in our total vertical moment. It is true that chances for error are many, but it is also true that, if care and diligence are practiced, the errors will be negligible when the center of gravity of the whole vessel is obtained. As an illustration, let us say that an officer has miscalculated the center of gravity of a hold by one foot. The weight of the cargo in the hold is 500 tons. The error will be 500 foot-tons added to the total moments of the vessel. If the weight of the vessel and cargo were 10,000 tons and the true center of gravity for the vessel were 20 feet above the keel, the true moments would be 200,000 foot-tons. Now add 500 foot-tons which are in

error. The resulting *KG* is 20.05 feet. In other words, an error of only one twentieth of a foot has resulted from this miscalculation. This type of error is negligible. However, if an error of 1 foot were made in every compartment, and if the errors were all above or below the true position of the center of gravity of each compartment, the error of the final *KG* would be nearly a foot, and a serious error. Obviously, however, it would be difficult to err so consistently on one side. Morever, an error of one foot will be made if only carelessness prevails.

Questions

1. Which of the following quantities does the ship's officer calculate in doing a transverse stability problem to determine *GM*? **I. *KG*. II. *KM*.**
   A. I B. II C. Either I or II D. Neither I nor II
2. The point through which all the vertically downward forces of weight are considered to act is known as:
   A. The metacenter
   B. The center of buoyancy
   C. The center of gravity
   D. None of the above
3. The light ship condition is:
   A. A condition prior to loading cargo, but with fuel, water, and stores aboard.
   B. A condition prior to loading cargo and fuel, but with water and stores aboard.
   C. A condition prior to loading cargo, fuel, and water, but with stores aboard.
   D. A condition prior to loading cargo, fuel, water, and stores.
4. A moment is: **I. A weight. II. A distance.**
   A. I B. II C. Either I or II D. Neither I nor II
5. A seesaw will not balance. We wish to determine where to put the fulcrum in order to make it balance. Which of the following methods would you use? You are not allowed to change the distribution of weight. **I.** Revolve moments about a shifting fulcrum. **II.** Revolve moments about one end of the seesaw only.
   A. I B. II C. Either I or II D. Neither I nor II
6. The *GG'* Formula is used to solve for:
   A. The movement of *G* when a weight is shifted aboard a vessel.
   B. The movement of *G* when a weight is loaded aboard a vessel.
   C. The movement of *G* when a weight is discharged aboard a vessel.