2D MOTION

Variables and Kinematic Equations

Variables		SI Unit
t	time	s
x	horizontal position	m
y	vertical position	m
v	velocity	$\frac{m}{s}$
a	acceleration	$\frac{m}{s^2}$

Variable → v_x ← Subscript

"horizontal velocity"

Subscripts		
i	0	initial
f	_	final
x		horizontal
y		vertical

delta

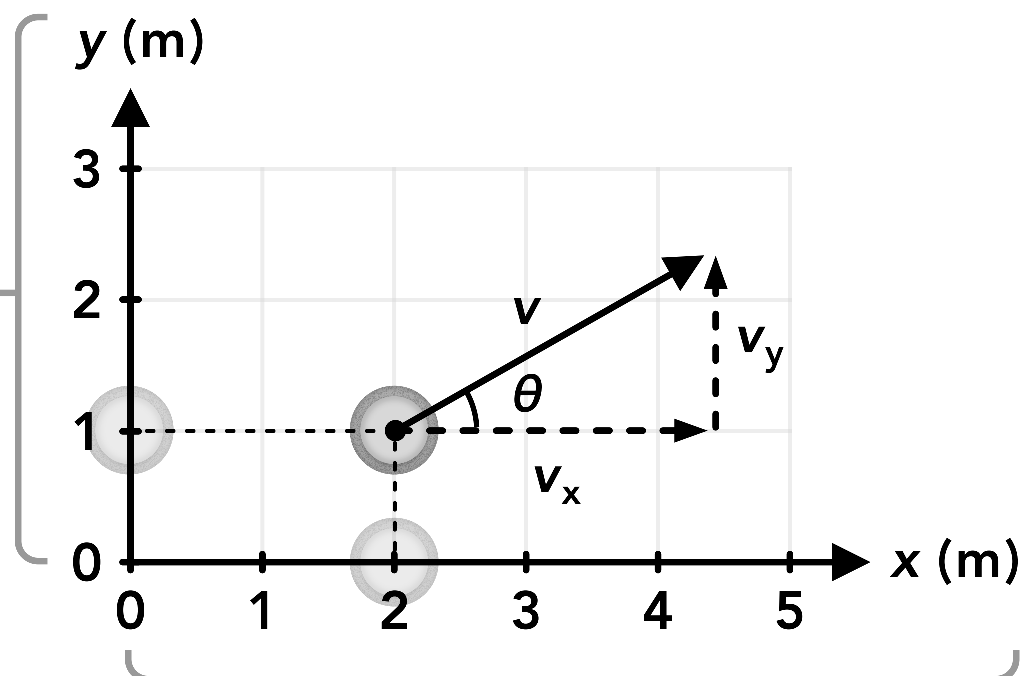
Δ = final − initial

$\Delta x = x_f - x_i$

or

$\Delta x = x - x_0$

An object's x motion and y motion are independent of each other



	Vertical motion	Horizontal motion
Displacement:	$\Delta y = y_f - y_i$	$\Delta x = x_f - x_i$
Velocity:	$v_y = \frac{\Delta y}{\Delta t}$	$v_x = \frac{\Delta x}{\Delta t}$
Velocity (rearranged):	$y_f = y_i + v_y \Delta t$	$x_f = x_i + v_x \Delta t$
Acceleration:	$a_y = \frac{\Delta v_y}{\Delta t}$	$a_x = \frac{\Delta v_x}{\Delta t}$
Acceleration (rearranged):	$v_{yf} = v_{yi} + a_y \Delta t$	$v_{xf} = v_{xi} + a_x \Delta t$
Kinematic equations for constant acceleration:	$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$	$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$
	$v_{yf}^2 = v_{yi}^2 + 2 a_y (y_f - y_i)$	$v_{xf}^2 = v_{xi}^2 + 2 a_x (x_f - x_i)$

- An object's x motion and y motion are completely independent of each other, so the x and y motions can be described separately. The kinematic equations for an object in 2D motion are just a combination of the 1D kinematic equations for horizontal motion and vertical motion.
- When starting with a 2D displacement, velocity or acceleration vector, the vector can be broken down into its x and y components and the kinematic equations apply to the x and y motions separately.
- When starting with separate x and y motions, a 2D displacement, velocity or acceleration vector can be found by combining the x and y motion components.

2D Position and Coordinates

Coordinates

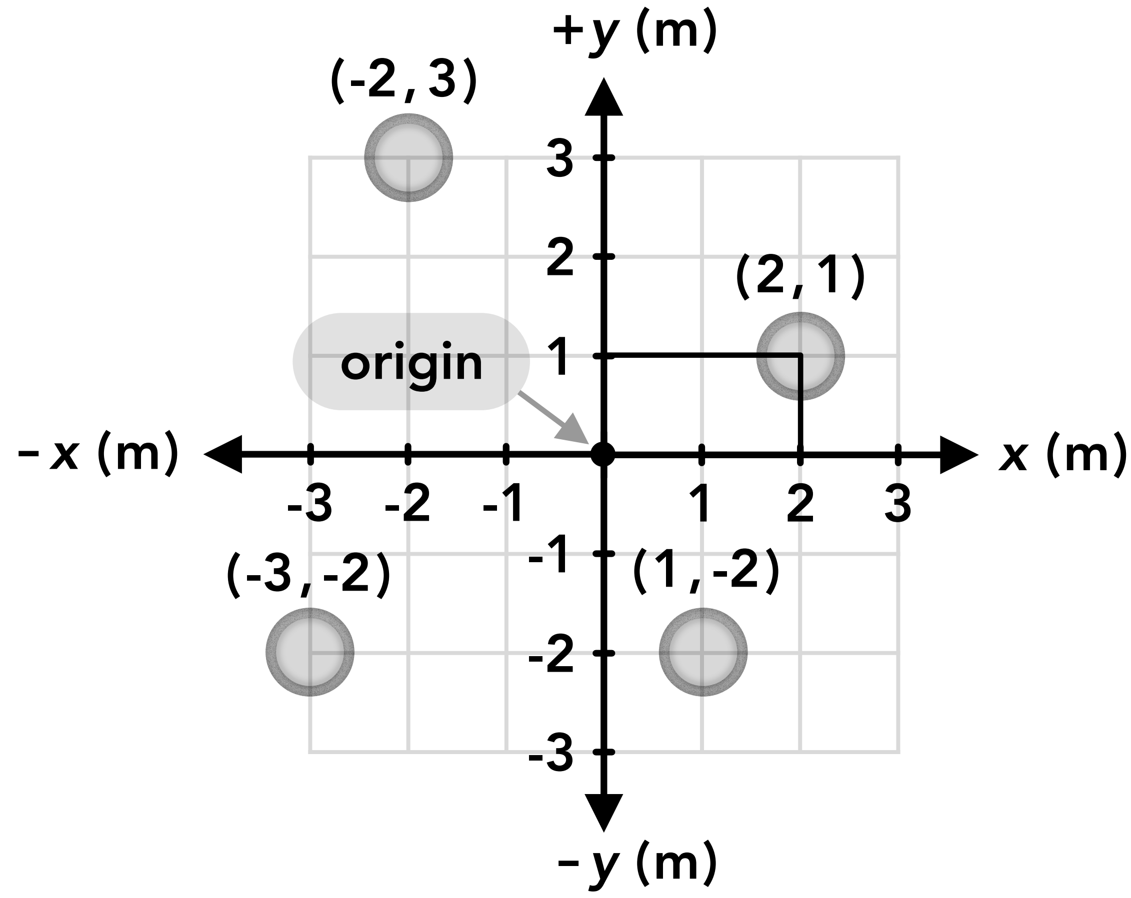
(x, y)

(x position, y position)

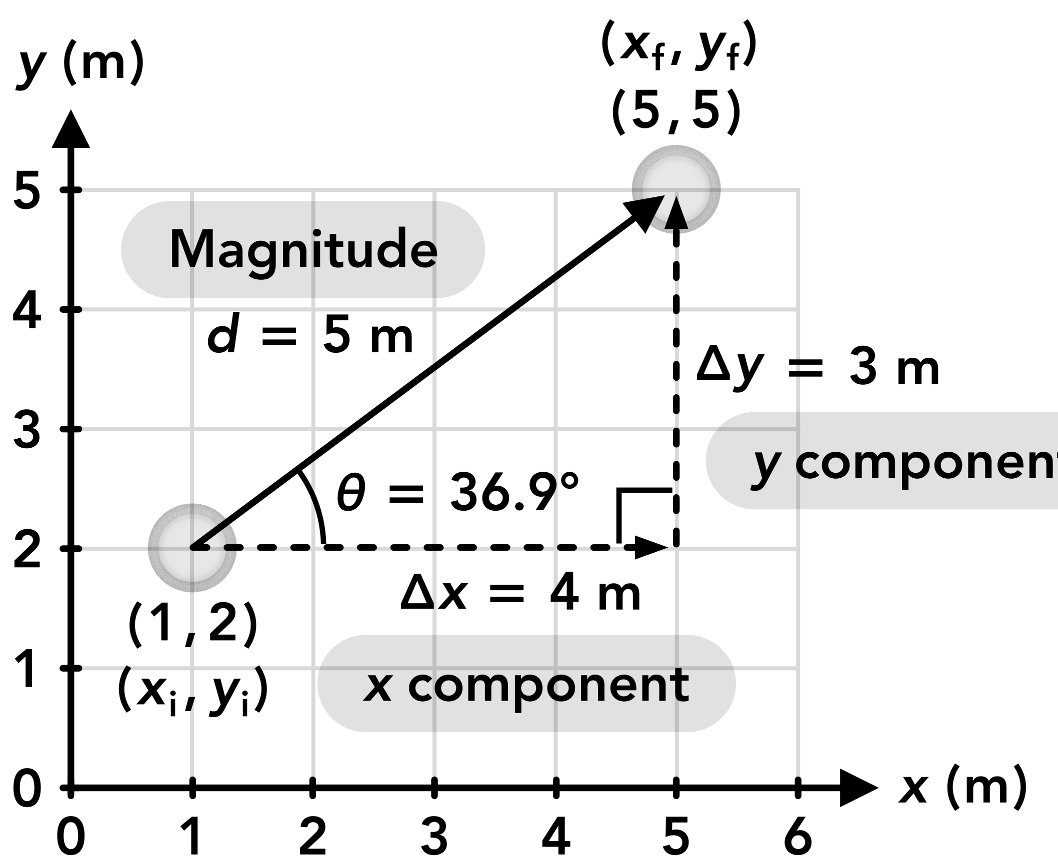
- If an object is in two-dimensional (2D) motion it has an x position and a y position at every moment in time.
- The position of an object or a point in 2D space is described using coordinates on a 2D plane (known as a Cartesian coordinate system).
- **Coordinates** are a pair of values: the first value represents the position of the object along the x axis, the second value represents the position along the y axis.
- The axes of the 2D coordinate system are just like the x and y axes from linear (1D) motion.
- The **origin** of the coordinate system has coordinates (0, 0).

2D coordinate system

(Cartesian coordinate system)



2D Displacement Vectors



Displacement vector

Magnitude and angle:

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

5 m, 36.9°

Components:

$$\Delta x = d\cos(\theta)$$

$$\Delta y = d\sin(\theta)$$

$\Delta x = 4$ m, $\Delta y = 3$ m

x component

x displacement

$$\Delta x = x_f - x_i$$

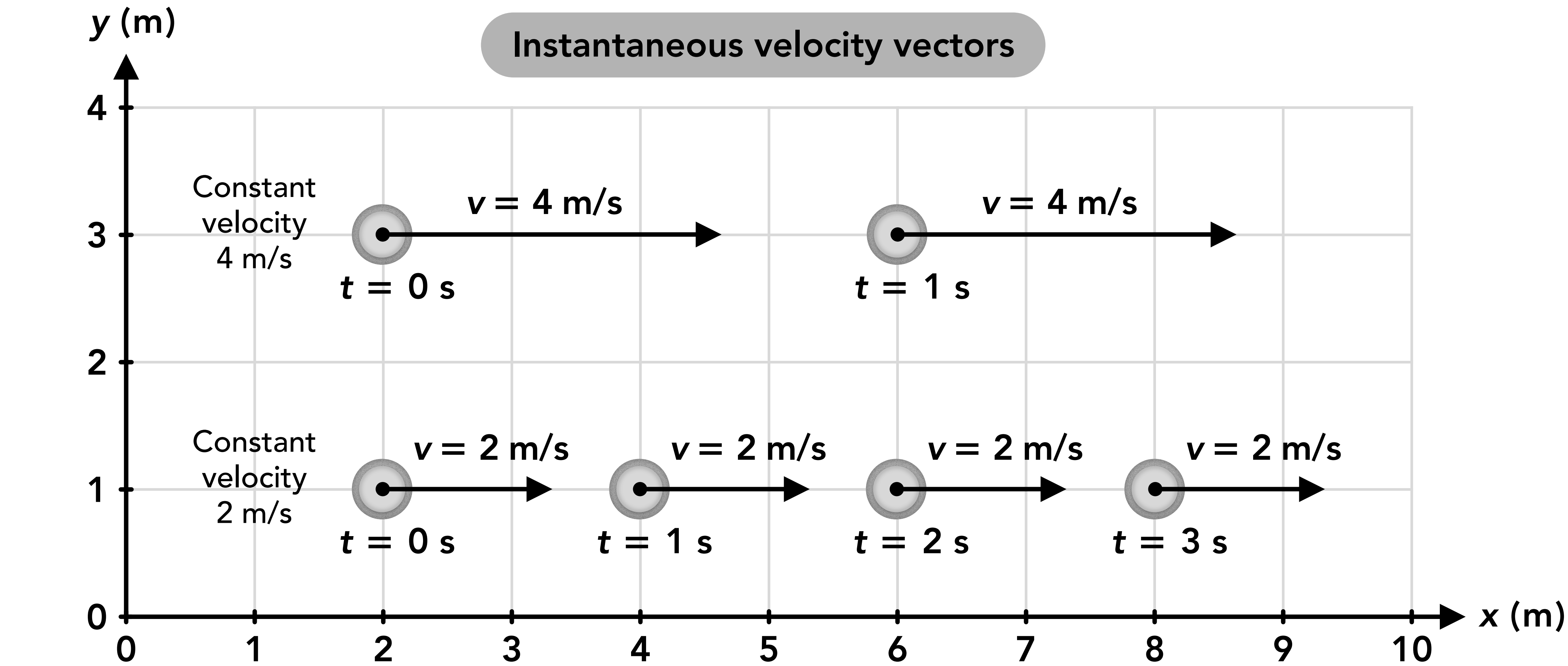
y component

y displacement

$$\Delta y = y_f - y_i$$

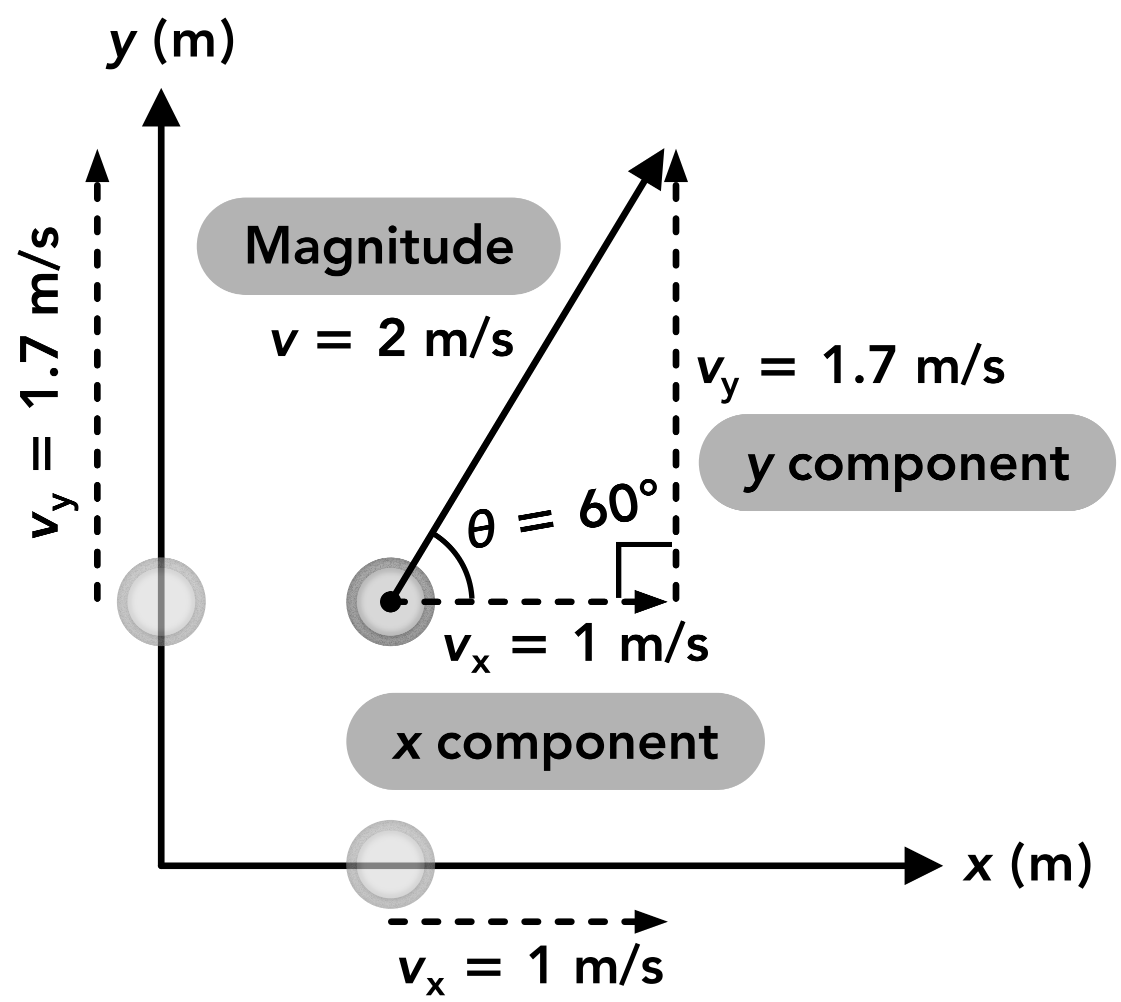
- When an object moves in 2D its x position and y position both change, so it has an x displacement and a y displacement at the same time. The 2D displacement is represented with a vector connecting the initial and final positions, and the x and y displacements are the components of the displacement vector.
- The components of the displacement vector (the x and y displacements) can be calculated using the initial and final coordinates, or using the magnitude and angle of the vector. The magnitude and angle of the vector can be calculated using the x and y components.

Velocity Vectors



- In 1D and 2D motion, the velocity of an object can be represented using a velocity vector.
- This is usually representing the instantaneous velocity of the object: the magnitude (speed) and direction of the velocity at an instant in time, as opposed to an average velocity.
- When comparing the lengths of several vectors, the length of the vector represents the magnitude of the velocity (the speed). Otherwise, the length of the velocity vector is arbitrary.

2D Velocity Vectors



Velocity vector

Magnitude and angle:

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

2 m/s, 60°

Components:

$$v_x = v\cos(\theta)$$

$$v_y = v\sin(\theta)$$

v_x = 1 m/s, v_y = 1.7 m/s

x component

x velocity

$$v_x = \frac{\Delta x}{\Delta t}$$

y component

y velocity

$$v_y = \frac{\Delta y}{\Delta t}$$

- When an object moves in 2D its ***x*** and ***y*** positions are changing at the same time, so it has an ***x*** velocity and a ***y*** velocity at every moment. The velocity is represented as a vector, and the ***x*** and ***y*** velocities are the components.
- The velocity components can be thought of as the velocities of the object's shadows along the ***x*** and ***y*** axes.
- The components of the velocity vector (the ***x*** and ***y*** velocities) can be calculated using the magnitude and angle of the vector. The magnitude and angle of the vector can be calculated using the ***x*** and ***y*** components.