AP PHYSICS 1

UNIT 6

Energy and Momentum of Rotating Systems

AP

5–8%
AP EXAM WEIGHTING

~8–14
CLASS PERIODS

Remember to go to **AP Classroom** to assign students the online **Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Progress Check** provides each student with immediate feedback related to this unit's topics and science practices.

Progress Check 6

Multiple-choice: ~18 questions

Free-response: 4 questions

- Mathematical Routines
- Translation Between Representations
- Experimental Design and Analysis
- Qualitative/Quantitative Translation

UNIT 6

5–8% AP EXAM WEIGHTING

~8–14 CLASS PERIODS

Energy and Momentum of Rotating Systems

ESSENTIAL QUESTIONS

- What keeps a bicycle balanced?
- Why do planets move faster when they travel closer to the sun?
- What do satellites and projectiles have in common?
- What do ice skaters do with their arms when they want to spin faster? Why?

Developing Understanding

In Unit 6, students will apply their knowledge of energy and momentum to rotating systems. Similar to the approach used for translational energy and momentum concepts in Units 3 and 4, it is important that students have conceptual understanding of how angular momentum and rotational energy change due to external torque(s) on a system. Additionally, articulating the conditions under which the rotational energy and/or angular momentum of a system remains constant is foundational to working through more complex scenarios. Students will use the content and skills presented in both Units 5 and 6 to further study the motion of orbiting satellites and rolling without slipping in this unit.

Building the Science Practices

2.C 2.D 3.B 3.C

Unit 6 provides opportunities for students to compare physical quantities between scenarios or at different times in a single scenario (**2.C**), as well as determine new values of quantities using functional dependencies between variables (**2.D**). From there, students can also make and justify claims based on these physical principles and functional relationships (**3.B**, **3.C**). For example, students could describe conceptually what happens to the rotational inertia of a system when the pivot point is moved, and then justify what impact that change will have on the angular acceleration of the system. By the end of the unit, it is important for students to be comfortable with making claims about the reasonableness of their claims and justifications made with functional dependence (**2.D**, **3.C**), starting with first principles of physics.

Preparing for the AP Exam

On both the multiple-choice and free-response sections of the AP Physics 1 Exam, students need to be able to describe the relationships between physical quantities in order to articulate the effects of changing the value of a specific physical quantity in a scenario. Therefore, students will benefit from opportunities to investigate changes in systems, including practicing using fundamental principles of physics to decide whether a quantity will increase, decrease, or remain the same when another quantity is changed. Additionally, when writing justifications for claims, simply referencing an equation, law, or physical principle is not sufficient. For example, stating that one disk is rolling faster than another because of “conservation of energy” is not a complete enough answer to earn credit on the free-response section of the exam. Students must clearly and concisely explain the steps in their reasoning that lead from the equation, law, or physical principle to the justification of their claim in FRQ #1, the MR question.

UNIT AT A GLANCE

Topic	Suggested Skills
6.1 Rotational Kinetic Energy	1.A Create diagrams, tables, charts, or schematics to represent physical situations. 2.B Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway. 2.C Compare physical quantities between two or more scenarios or at different times and locations in a single scenario. 3.B Apply an appropriate law, definition, theoretical relationship, or model to make a claim.
6.2 Torque and Work	1.B Create quantitative graphs with appropriate scales and units, including plotting data. 2.A Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway. 2.C Compare physical quantities between two or more scenarios or at different times and locations in a single scenario. 2.D Predict new values or factors of change of physical quantities using functional dependence between variables. 3.A Create experimental procedures that are appropriate for a given scientific question.
6.3 Angular Momentum and Angular Impulse	1.B Create quantitative graphs with appropriate scales and units, including plotting data. 2.A Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway. 2.D Predict new values or factors of change of physical quantities using functional dependence between variables. 3.B Apply an appropriate law, definition, theoretical relationship, or model to make a claim.
6.4 Conservation of Angular Momentum	1.B Create quantitative graphs with appropriate scales and units, including plotting data. 2.D Predict new values or factors of change of physical quantities using functional dependence between variables. 3.A Create experimental procedures that are appropriate for a given scientific question. 3.B Apply an appropriate law, definition, theoretical relationship, or model to make a claim. 3.C Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

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UNIT AT A GLANCE *(cont'd)*

Topic	Suggested Skills
6.5 Rolling	**1.A** Create diagrams, tables, charts, or schematics to represent physical situations. **2.A** Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway. **2.C** Compare physical quantities between two or more scenarios or at different times and locations in a single scenario. **3.C** Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.
6.6 Motion of Orbiting Satellites	**1.C** Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system. **2.A** Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway. **2.C** Compare physical quantities between two or more scenarios or at different times and locations in a single scenario. **3.C** Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

Go to **AP Classroom** to assign the **Progress Check** for Unit 6.
Review the results in class to identify and address any student misunderstandings.

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate various instructional approaches in the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 153 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	6.1	**Desktop Experiment Task** Have students release a yo-yo from rest, calculate its acceleration from distance and time measurements, and then determine the yo-yo's rotational inertia (which requires the yo-yo's mass and the radius at which the string connects to the yo-yo). Next, have them roll the yo-yo down a ramp and use distance and time data to construct a conservation of energy equation that can be solved for the yo-yo's rotational inertia.
2	6.3	**Predict and Explain** Allow students to play with a set of fidget spinners. Ask them to explain why it is difficult to change the plane of rotation of a spinner while it is rotating.
3	6.5	**Concept-Oriented Demonstration** Obtain a ring and a disk of equal mass and radius and load up a low-friction cart with weights to make it the same mass. "Race" the three objects from rest down identical inclines to show students the cart wins, then the disk, and then the ring. Have students explain why the objects win in this order, with forces and then with energy.
4	6.5	**Ranking Tasks** Present students with the following scenario and its accompanying three cases: A wheel rolls down an incline from rest and across a flat surface. Case 1: Tracks are rough enough that there is no slipping. Case 2: Tracks have some friction, but there is slipping. Case 3: Tracks have negligible friction. Have students rank translational kinetic energies at the end, rotational kinetic energies at the end, and total mechanical energies of the wheel at the end as three separate tasks. $(K_{T3} > K_{T2} > K_{T1})$, $(K_{R1} > K_{R2} > K_{R3})$, and $(E_1 = E_3 > E_2)$.
5	6.5	**Construct an Argument** Have students roll a hoop and a disk (of equal mass and radius) down identical ramps. Then have them explain why the disk reached the bottom in less time using energy bar charts and to-scale free-body diagrams.

TOPIC 6.1
Rotational Kinetic Energy

Required Course Content

LEARNING OBJECTIVE

6.1.A

Describe the rotational kinetic energy of a rigid system in terms of the rotational inertia and angular velocity of that rigid system.

ESSENTIAL KNOWLEDGE

6.1.A.1

The rotational kinetic energy of an object or rigid system is related to the rotational inertia and angular velocity of the rigid system and is given by the equation

$$K = \frac{1}{2}I\omega^2.$$

6.1.A.1.i

The rotational inertia of an object about a fixed axis can be used to show that the rotational kinetic energy of that object is equivalent to its translational kinetic energy, which is its total kinetic energy.

6.1.A.1.ii

The total kinetic energy of a rigid system is the sum of its rotational kinetic energy due to its rotation about its center of mass and the translational kinetic energy due to the linear motion of its center of mass.

6.1.A.2

A rigid system can have rotational kinetic energy while its center of mass is at rest due to the individual points within the rigid system having linear speed and, therefore, kinetic energy.

6.1.A.3

Rotational kinetic energy is a scalar quantity.

SUGGESTED SKILLS

1.A

Create diagrams, tables, charts, or schematics to represent physical situations.

2.B

Calculate or estimate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2.C

Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

3.C

Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

SUGGESTED SKILLS

1.B
Create quantitative graphs with appropriate scales and units, including plotting data.

2.A
Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

2.C
Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

2.D
Predict new values or factors of change of physical quantities using functional dependence between variables.

3.A
Create experimental procedures that are appropriate for a given scientific question.

TOPIC 6.2

Torque and Work

Required Course Content

LEARNING OBJECTIVE

6.2.A
Describe the work done on a rigid system by a given torque or collection of torques.

ESSENTIAL KNOWLEDGE

6.2.A.1
A torque can transfer energy into or out of an object or rigid system if the torque is exerted over an angular displacement.

6.2.A.2
The amount of work done on a rigid system by a torque is related to the magnitude of that torque and the angular displacement through which the rigid system rotates during the interval in which that torque is exerted.

Relevant equation:

$$W = \tau \Delta\theta$$

6.2.A.3
Work done on a rigid system by a given torque can be found from the area under the curve of a graph of torque as a function of angular position.

TOPIC 6.3

Angular Momentum and Angular Impulse

Required Course Content

LEARNING OBJECTIVE

6.3.A

Describe the angular momentum of an object or rigid system.

ESSENTIAL KNOWLEDGE

6.3.A.1

The magnitude of the angular momentum of a rigid system about a specific axis can be described with the equation

$L = I\omega.$

6.3.A.2

The magnitude of the angular momentum of an object about a given point is

$L = rmv \sin\theta.$

6.3.A.2.i

The selection of the axis about which an object is considered to rotate influences the determination of the angular momentum of that object.

6.3.A.2.ii

The measured angular momentum of an object traveling in a straight line depends on the distance between the reference point and the object, the mass of the object, the speed of the object, and the angle between the radial distance and the velocity of the object.

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SUGGESTED SKILLS

1.B Create quantitative graphs with appropriate scales and units, including plotting data.

2.A Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

2.D Predict new values or factors of change of physical quantities using functional dependence between variables.

3.B Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

LEARNING OBJECTIVE

6.3.B

Describe the angular impulse delivered to an object or rigid system by a torque.

ESSENTIAL KNOWLEDGE

6.3.B.1

Angular impulse is defined as the product of the torque exerted on an object or rigid system and the time interval during which the torque is exerted.

Relevant equation:

$$\text{angular impulse} = \tau\Delta t$$

6.3.B.2

Angular impulse has the same direction as the torque exerted on the object or system.

6.3.B.3

The angular impulse delivered to an object or rigid system by a torque can be found from the area under the curve of a graph of the torque as a function of time.

LEARNING OBJECTIVE

6.3.C

Relate the change in angular momentum of an object or rigid system to the angular impulse given to that object or rigid system.

ESSENTIAL KNOWLEDGE

6.3.C.1

The magnitude of the change in angular momentum can be described by comparing the magnitudes of the final and initial angular momenta of the object or rigid system:

$$\Delta L = L - L_0$$

6.3.C.2

A rotational form of the impulse–momentum theorem relates the angular impulse delivered to an object or rigid system and the change in angular momentum of that object or rigid system.

6.3.C.2.i

The angular impulse exerted on an object or rigid system is equal to the change in angular momentum of that object or rigid system.

Relevant equation:

$$\Delta L = \tau\Delta t$$

6.3.C.2.ii

The rotational form of the impulse–momentum theorem is a direct result of the rotational form of Newton's second law of motion for cases in which rotational inertia is constant:

$$\tau_{\text{net}} = \frac{\Delta L}{\Delta t} = I\frac{\Delta\omega}{\Delta t} = I\alpha$$

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LEARNING OBJECTIVE

6.3.C

Relate an object's or rigid system's change in angular momentum to the angular impulse given to that object or rigid system.

ESSENTIAL KNOWLEDGE

6.3.C.3

The net torque exerted on an object is equal to the slope of the graph of the angular momentum of an object as a function of time.

6.3.C.4

The angular impulse delivered to an object is equal to the area under the curve of a graph of the net external torque exerted on an object as a function of time.

BOUNDARY STATEMENT

While AP Physics 1 expects that students can mathematically manipulate the magnitude of angular momentum using one-dimensional vector conventions, the direction of angular momentum and angular impulse is beyond the scope of the course.

SUGGESTED SKILLS

1.B
Create quantitative graphs with appropriate scales and units, including plotting data.

2.D
Predict new values or factors of change of physical quantities using functional dependence between variables.

3.A
Create experimental procedures that are appropriate for a given scientific question.

3.B
Apply an appropriate law, definition, theoretical relationship, or model to make a claim.

3.C
Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

TOPIC 6.4
Conservation of Angular Momentum

Required Course Content

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
6.4.A Describe the behavior of a system using conservation of angular momentum.	**6.4.A.1** The total angular momentum of a system about a rotational axis is the sum of the angular momenta of the system's constituent parts about that axis. **6.4.A.2** Any change to a system's angular momentum must be due to an interaction between the system and its surroundings. **6.4.A.2.i** The angular impulse exerted by one object or system on a second object or system is equal and opposite to the angular impulse exerted by the second object or system on the first. This is a direct result of Newton's third law. **6.4.A.2.ii** A system may be selected so that the total angular momentum of that system is constant. **6.4.A.2.iii** The angular speed of a nonrigid system may change without the angular momentum of the system changing if the system changes shape by moving mass closer to or further from the rotational axis. **6.4.A.2.iv** If the total angular momentum of a system changes, that change will be equivalent to the angular impulse exerted on the system.

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LEARNING OBJECTIVE

6.4.B

Describe how the selection of a system determines whether the angular momentum of that system changes.

ESSENTIAL KNOWLEDGE

6.4.B.1

Angular momentum is conserved in all interactions.

6.4.B.2

If the net external torque exerted on a selected object or rigid system is zero, the total angular momentum of that system is constant.

6.4.B.3

If the net external torque exerted on a selected object or rigid system is nonzero, angular momentum is transferred between the system and the environment.

SUGGESTED SKILLS

1.A
Create diagrams, tables, charts, or schematics to represent physical situations.

2.A
Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

2.C
Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

3.C
Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

TOPIC 6.5
Rolling

Required Course Content

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
6.5.A Describe the kinetic energy of a system that has translational and rotational motion.	**6.5.A.1** The total kinetic energy of a system is the sum of the system's translational and rotational kinetic energies. *Relevant equation:*. $K_{tot} = K_{trans} + K_{rot}$
6.5.B Describe the motion of a system that is rolling without slipping.	**6.5.B.1** While rolling without slipping, the translational motion of a system's center of mass is related to the rotational motion of the system itself with the equations: $\Delta x_{cm} = r\Delta\theta$ $v_{cm} = r\omega$ $a_{cm} = r\alpha$ **6.5.B.2** For ideal cases, rolling without slipping implies that the frictional force does not dissipate any energy from the rolling system.

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LEARNING OBJECTIVE

6.5.C

Describe the motion of a system that is rolling while slipping.

ESSENTIAL KNOWLEDGE

6.5.C.1

When slipping, the motion of a system's center of mass and the system's rotational motion cannot be directly related.

6.5.C.2

When a rotating system is slipping relative to another surface, the point of application of the force of kinetic friction exerted on the system moves with respect to the surface, so the force of kinetic friction will dissipate energy from the system.

BOUNDARY STATEMENT

Rolling friction is beyond the scope of AP Physics 1.

BOUNDARY STATEMENT

The precise mathematical relationships between linear and angular quantities while a rigid body is rolling while slipping are beyond the scope of AP Physics 1 and 2, and students will not be expected to model those relationships quantitatively. However, students are expected to qualitatively explain the changes to linear and angular quantities while a rigid body is rolling while slipping.

SUGGESTED SKILLS

1.C
Create qualitative sketches of graphs that represent features of a model or the behavior of a physical system.

2.A
Derive a symbolic expression from known quantities by selecting and following a logical mathematical pathway.

2.C
Compare physical quantities between two or more scenarios or at different times and locations in a single scenario.

3.C
Justify or support a claim using evidence from experimental data, physical representations, or physical principles or laws.

TOPIC 6.6
Motion of Orbiting Satellites

Required Course Content

LEARNING OBJECTIVE

6.6.A
Describe the motions of a system consisting of two objects interacting only via gravitational forces.

ESSENTIAL KNOWLEDGE

6.6.A.1
In a system consisting only of a massive central object and an orbiting satellite with mass that is negligible in comparison to the central object's mass, the motion of the central object itself is negligible.

6.6.A.2
The motion of satellites in orbits is constrained by conservation laws.

6.6.A.2.i
In circular orbits, the system's total mechanical energy, the system's gravitational potential energy, and the satellite's angular momentum and kinetic energy are constant.

6.6.A.2.ii
In elliptical orbits, the system's total mechanical energy and the satellite's angular momentum are constant, but the system's gravitational potential energy and the satellite's kinetic energy can each change.

6.6.A.2.iii
The gravitational potential energy of a system consisting of a satellite and a massive central object is defined to be zero when the satellite is an infinite distance from the central object.

Relevant equation:

$$U_g = -G\frac{m_1 m_2}{r}$$

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LEARNING OBJECTIVE

6.6.A

Describe the motions of a system consisting of two objects interacting only via gravitational forces.

ESSENTIAL KNOWLEDGE

6.6.A.3

The escape velocity of a satellite is the satellite's velocity such that the mechanical energy of the satellite–central-object system is equal to zero.

6.6.A.3.i

When the only force exerted on a satellite is gravity from a central object, a satellite that reaches escape velocity will move away from the central body until its speed reaches zero at an infinite distance from the central body.

6.6.A.3.ii

The escape velocity of a satellite from a central body of mass M can be derived using conservation of energy laws.

Derived equation:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$