Types of Control Charts

Alright, time to jump into the meat of the chapter!

Let's start with control charts for **variable data**, then move on to control charts for **attribute data**.

We cover the **3 most common control charts for variable data - X-bar & R Chart,** the **X-Bar & S chart** and the **I-MR chart**.

Then, we will review the **4 most common control charts for attribute data** - the **p-chart, np-chart, c-chart and u-chart.**

Besides being two different types of data, these two groups of control charts also have another key different.

Variable control charts always work in pairs while attribute charts are a single chart.

For example, an **X-bar and R chart is two charts** – an X-bar chart monitors the average value of the process and a Range (R) chart that monitors the variation of the process.

However, a **c-chart is simple a single chart** that monitors **defects** over time.

There are **pros and cons** to both types of charts, and as a Quality Engineer you'll have to weigh these against each other to pick the right chart.

In general, variable data control charts tend to be more sensitive to process changes (😊) but can also be more expensive and difficult to administer (more math ☹).

Variable Control Charts

The 3 Most Common Variable Control Charts

Let's start with variable control charts, which include the **X-bar & R Chart,** the **X-Bar & S chart** and the **I-MR chart**.

Like I said above, Variable control charts always work in pairs, with the first chart monitoring the process average, and the second chart monitoring the process variability.

For example, the **X-bar chart** monitors the **central tendency** of your **process**. The **Individuals (I) Chart** is also a representation of the **central tendency** of your **process**.

Similarly, the **R (Range) Chart**, **S (Standard Deviation) Chart** and the **MR (Moving Range) Chart** all reflect the variability in your process.

How to Choose the Right Variable Control Chart

When deciding which control chart to use, the one factor to consider is the sample size of the rational sub-group.

If you're **rational sub-group size is a single value (1)**, then you'll use the **I-MR (Individual and Moving Range) Chart.**

If you're **rational sub-group size is between 2 – 9**, then you'll use the **X-Bar and R Chart.** When the sample size is less than 10, the **Range** of the sample data is a better estimator of the **process variability** than the standard deviation.

If you're **rational sub-group size is equal to or greater than 10**, then you'll use the **X-Bar and S Chart.** When you've got 10 or more samples in a rational sub-group, then the **best estimator of the process variability is the standard deviation**.

Ok, let's jump into the X-bar and R Chart to see how to construct and analyze this chart.

X-bar & R Charts

the **X-bar and R chart** is the **workhorse of control charts**. It is the most common control chart for variable data.

This control chart should be used anytime your rational subgroup size (n) is between 2 & 9, (**2 ≤ n ≤ 9**).

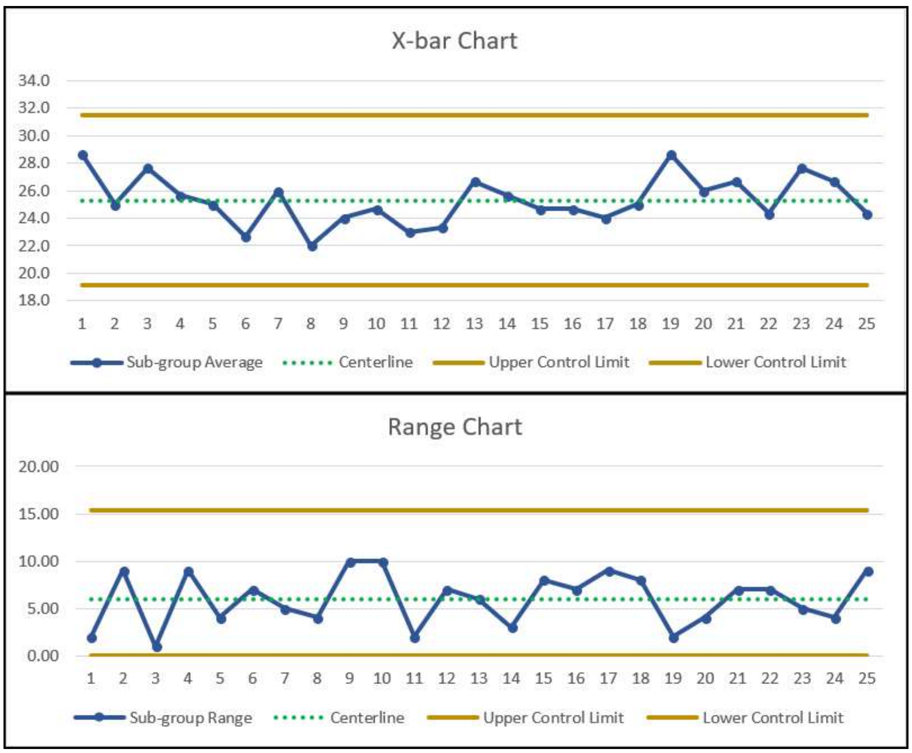
Like other variable control charts, it works in a pair.

The first chart is the **X-bar chart**, which monitors the **subgroup mean** of your **process**.

The second chart is the R chart, where **R** stands for **Range**.

As you know, the **Range** of a data set is one way to estimate the **variability or spread** associated with a process. So, the **Range chart** monitors the **variability** of the process.

Below is an example of a **X-bar and R Chart** where our **sub-group size is 3**.



As you can see, both charts have the three elements discussed above, the **CL** (Centerline), the **UCL** (Upper Control Limit) and the **LCL** (Lower Control Limit).

X-Bar Chart Critical Elements

Below are the calculations for those **critical elements on the X-bar chart**:

$$\boldsymbol{X-bar\ (\bar{X})\ Centerline} = \ Grand\ Average = \bar{\bar{X}} = \frac{\sum \bar{X}_i}{k}$$

$$\boldsymbol{UCL_{\bar{X}}} = \bar{\bar{X}} + A_2\bar{R} \qquad \boldsymbol{LCL_{\bar{X}}} = \bar{\bar{X}} - A_2\bar{R}$$

The **centerline of the X-bar chart**, is also called the **grand average**, and is also called **X-double bar**. This is the average of the averages, and it is the best estimate of the population mean (μ) for your process.

The UCL and LCL are calculated using the **grand average**, the average range (**R-bar**), and a **factor** (**A_2**), which varies depending on the size of your subgroup.

The **A_2** factor is below, and in this instance with a **sub-group size of 3**, A_2 is equal to 1.023.

Range (R) Chart Critical Elements

Moving on to the **Range Chart**, below are the calculations for the **centerline, UCL & LCL** for the **range chart**.

$$\boldsymbol{Average\ Range = \bar{R} = \frac{\sum R_i}{k}} = \frac{Sum\ of\ Subgroup\ Ranges}{\#\ of\ Subgroups}$$

To calculate the control limits for the **range chart**, we multiply the average range (R-bar) by two factors (**D_4 & D_3**), which are based on the subgroup size (n) and can be found on the table below.

$$\boldsymbol{UCL_R = D_4\bar{R}} \qquad\qquad \boldsymbol{LCL_R = D_3\bar{R}}$$

Just like the grand average is a good estimate of the population mean (μ), you can use your **R-bar value** to calculate an **unbiased estimate of the population standard deviation** using a constant (d_2).

$$\boldsymbol{Estimate\ of\ Population\ Standard\ Deviation = \hat{\sigma} = \frac{\bar{R}}{d_2}}$$

X-Bar and R Chart Constants

Below are the constants that must be used to calculate the critical elements (CL, UCL, LCL) of the X-bar and R chart.

X-Bar and R Chart				
Subgroup Sample Size	X-Bar Factor	Range Factors		Variance Factor
n	A_2	D_3	D_4	d_2
2	1.880	-	3.267	1.128
3	1.023	-	2.575	1.693
4	0.729	-	2.282	2.059
5	0.577	-	2.115	2.326
6	0.483	-	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
15	0.223	0.347	1.653	3.472
20	0.180	0.415	1.585	3.735
25	0.153	0.459	1.541	3.931

X-Bar and R Chart Example

Let's use the following data to calculate the **centerline, UCL and LCL for an X-bar and R Chart.**

Normally, you'd want to use at least 25 sub-groups from a stable process to calculate your control limits, but this limited data set of 10 sub-groups makes the math a little easier.

Sub-group	Sample 1	Sample 2	Sample 3	Sub-group Average	Sub-group Range
Sub-group 1	7	4	4	5.0	3
Sub-group 2	6	3	8	5.7	5
Sub-group 3	9	10	12	10.3	3
Sub-group 4	7	8	4	6.3	4
Sub-group 5	11	6	10	9.0	5
Sub-group 6	7	4	7	6.0	3
Sub-group 7	10	8	11	9.7	3
Sub-group 8	10	11	10	10.3	1
Sub-group 9	5	11	9	8.3	6
Sub-group 10	8	4	6	6.0	4
			Average	**7.7**	**3.70**

First, the 3 samples from each of the sub-groups (k = 10) are averaged to calculate the sub-group average. Then all of the sub-group averages are averages to calculate the grand average (7.7). Same for the Range of each sub-group.

$$\boldsymbol{X-bar\ (\bar{X})Centerline = \ Grand\ Average = \bar{\bar{X}} = \frac{\sum \bar{X}_i}{k}} = \frac{5+5.7+10.3+6.3+9+6+9.7+10.3+8.3+6.0}{10} = 7.7$$

$$\boldsymbol{Range\ Centerline} = \ \bar{R} = \frac{\sum R_i}{k} = \frac{3+5+3+4+5+3+3+1+6+4}{10} = 3.7$$

We can also calculate the control limits for the Range Chart:

$$\boldsymbol{UCL_R} = \ D_4\bar{R} = 2.575 * 3.7 = 9.53 \qquad \boldsymbol{LCL_R} = \ D_3\bar{R} = 0 * 3.7 = 0$$

We can now use the **grand average** (7.7) and **R-bar** (average range value) to calculate the control limits for the x-bar chart.

$$\boldsymbol{UCL_{\bar{X}}} = \bar{\bar{X}} + A_2\bar{R} = 7.7 + 1.023 * 3.7 = 11.5 \qquad \boldsymbol{LCL_{\bar{X}}} = \bar{\bar{X}} - A_2\bar{R} = 7.7 - 1.023 * 3.7 = 3.9$$