Advanced RC Structures

Two-way Slab

Shear Design

PART 3

- Shear strength of slab with beam
- Shear strength of slab without beam
- EX1 : Shear strength of flat plate
- Shear reinforcement
- EX2, 3 : Shear reinforcement design

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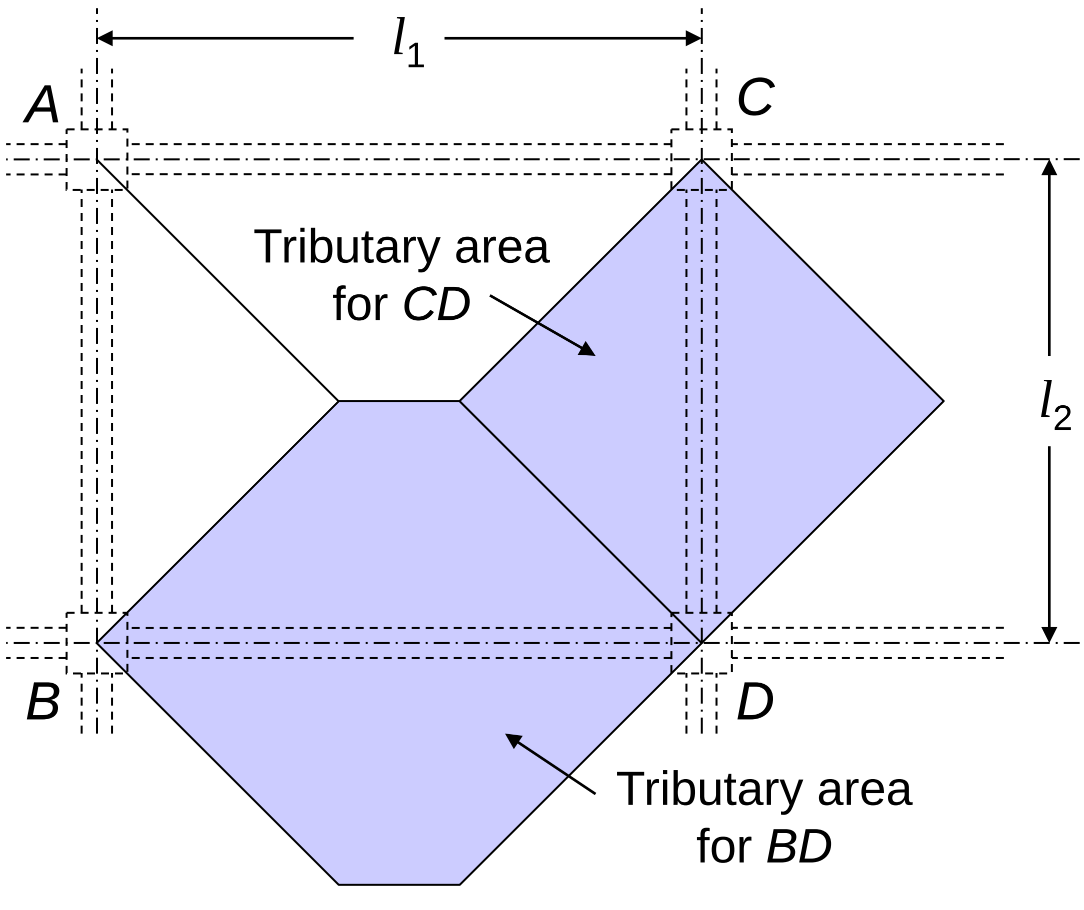
Shear in Slab Systems with Beams

Beams with $\alpha\, l_1/l_2 \geq 1.0$ shall be proportioned to resist shear caused by load on tributary area as shown.

In proportioning beams with $0 \leq \alpha\, l_1/l_2 < 1.0$ to resist shear, linear interpolation shall be permitted.



Shear Strength of Slab Systems with Beams

The critical location is found at ***d*** distance from the column, where

$$\phi V_c = \phi\left(0.53\sqrt{f_c'}\,bd\right)$$

The supporting beams are stiff and are capable of transmitting floor loads to the columns.

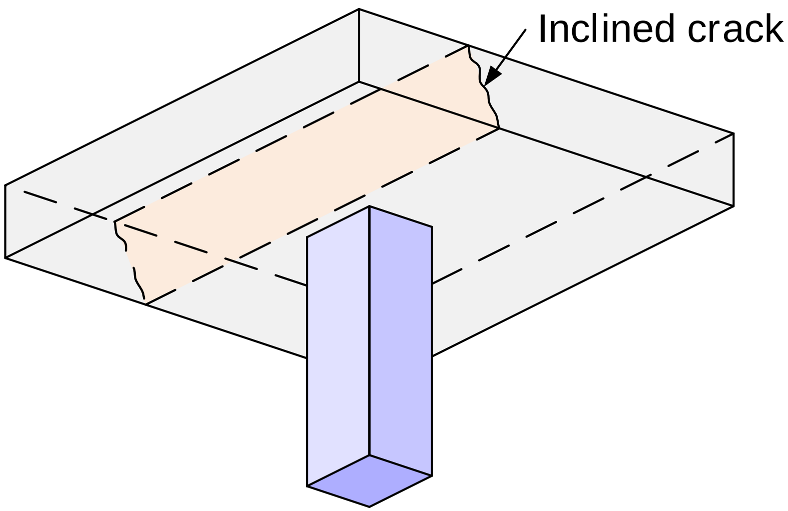
The shear force is calculated using the triangular and trapezoidal areas. If no shear reinforcement is provided, the shear force at a distance d from the beam must equal

$$V_{ud} \leq \phi V_c$$

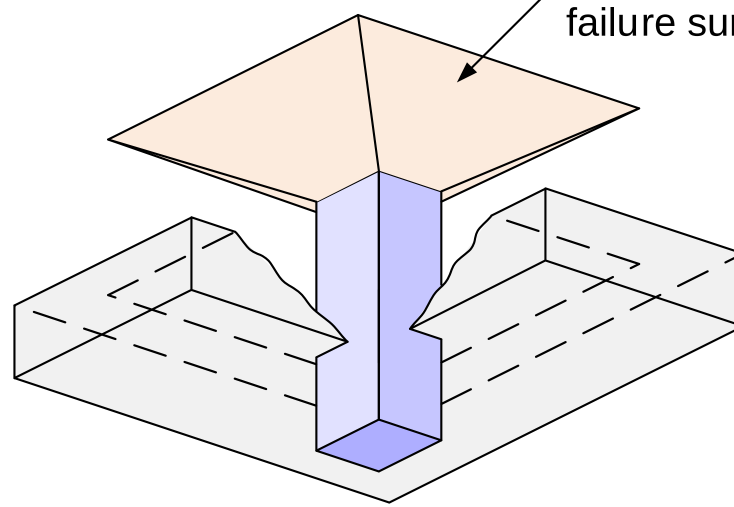
where

$$V_{ud} = w_u\left(\frac{l_2}{2} - d\right)$$

Shear Strength of Slab Systems without Beams



One-way shear



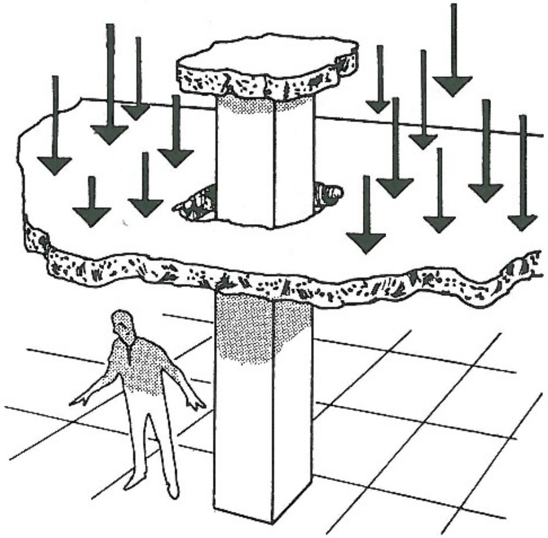
Two-way shear

There are two types of shear that need to be addressed:

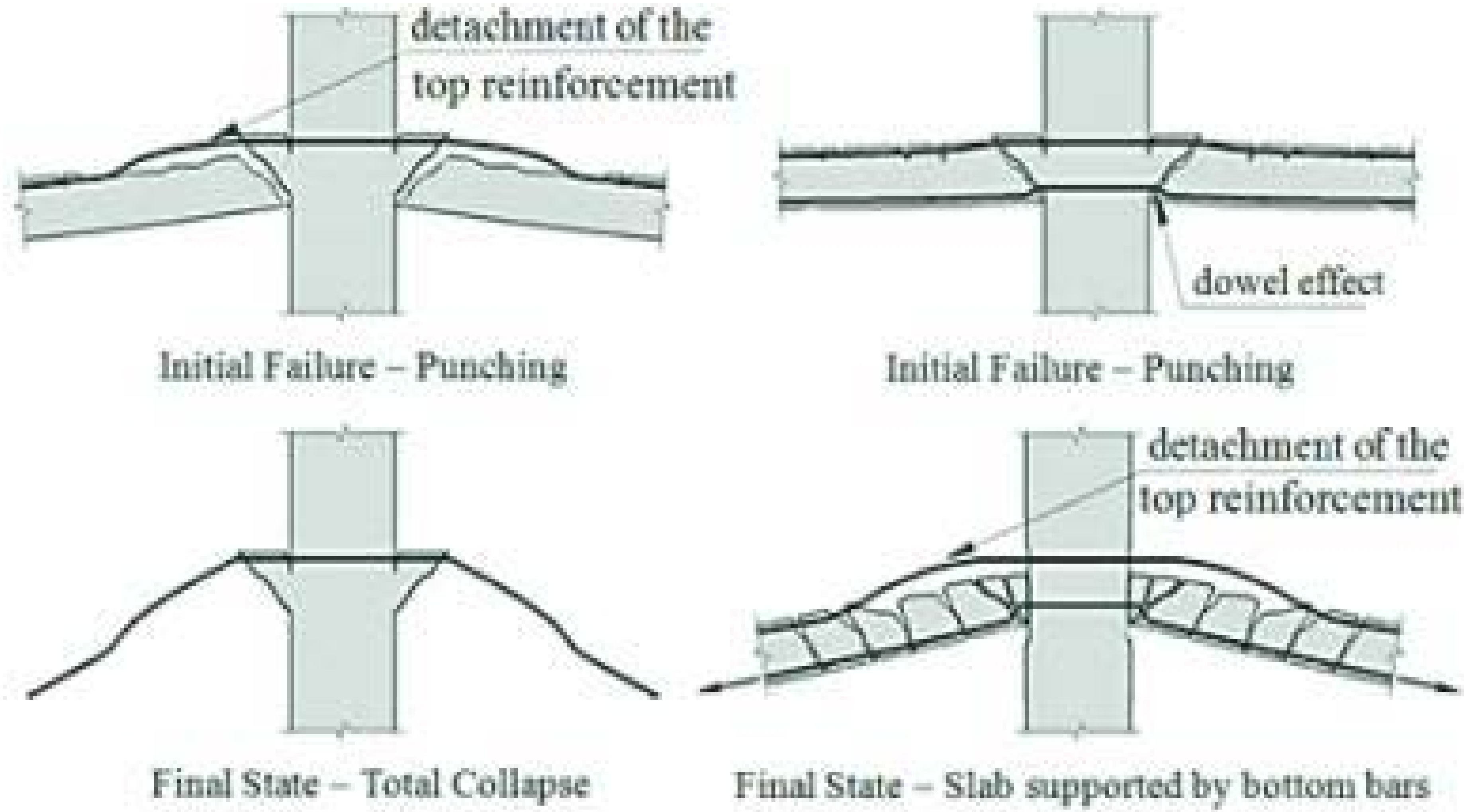
1) One-way shear or beam shear at distance **d** from the column

2) Two-way or punch out shear which occurs along a truncated cone.

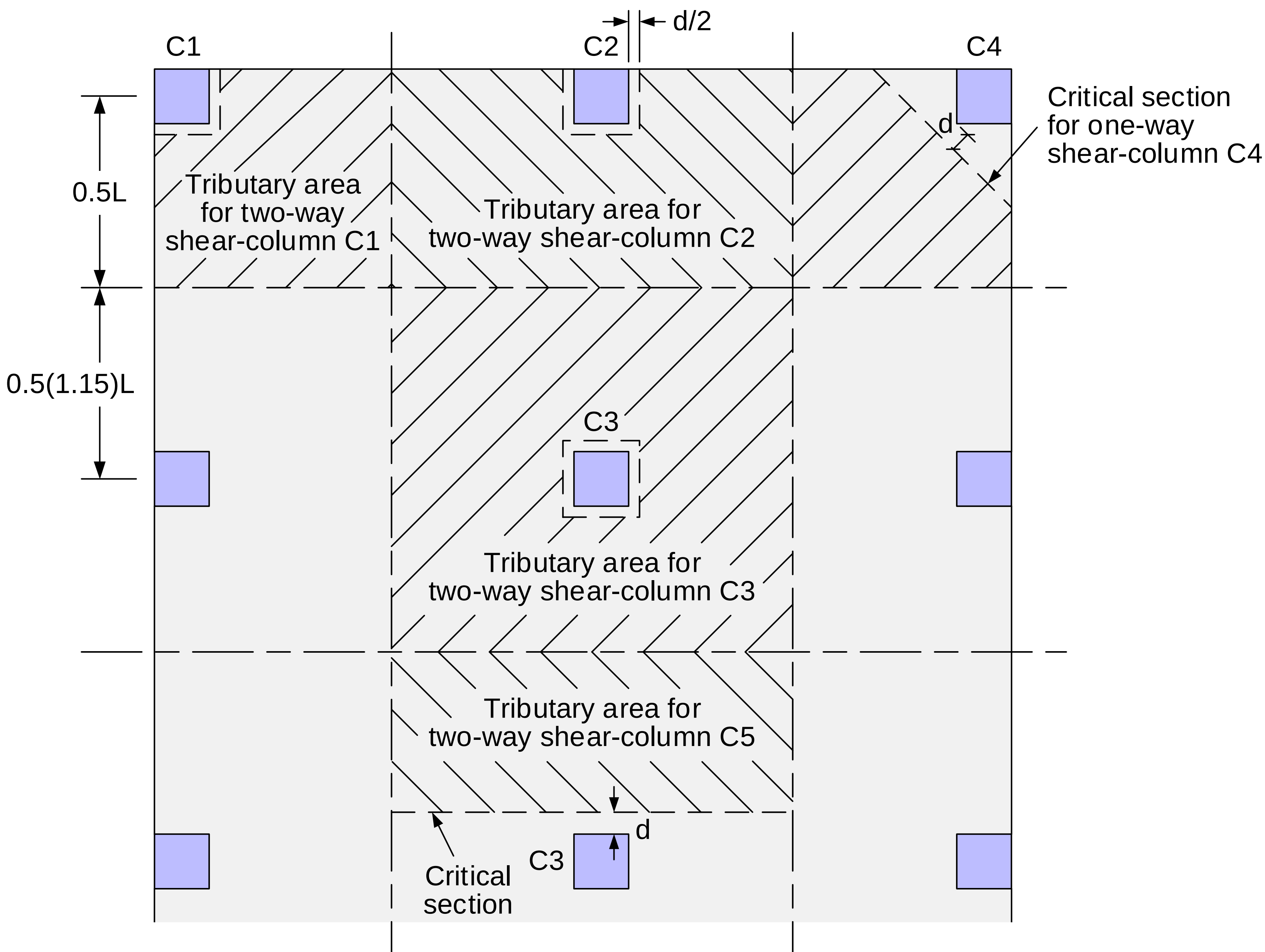
Punching Shear Failure



Post Punching Reinforcement



Shear Tributary Area



Shear Strength of Slab Systems

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Beam Shear : critical section at distance d from column face

$$V_c = 0.53\sqrt{f'_c}\, b_w\, d$$

Puching Shear : critical section occur around column with periphery b_o at distance d/2 outside column. V_c is the smallest of

$$V_c = 1.06\sqrt{f'_c}\, b_o\, d \quad \text{(a)}$$

$$V_c = 0.27\left(2 + \frac{4}{\beta}\right)\sqrt{f'_c}\, b_o\, d \quad \text{(b)}$$

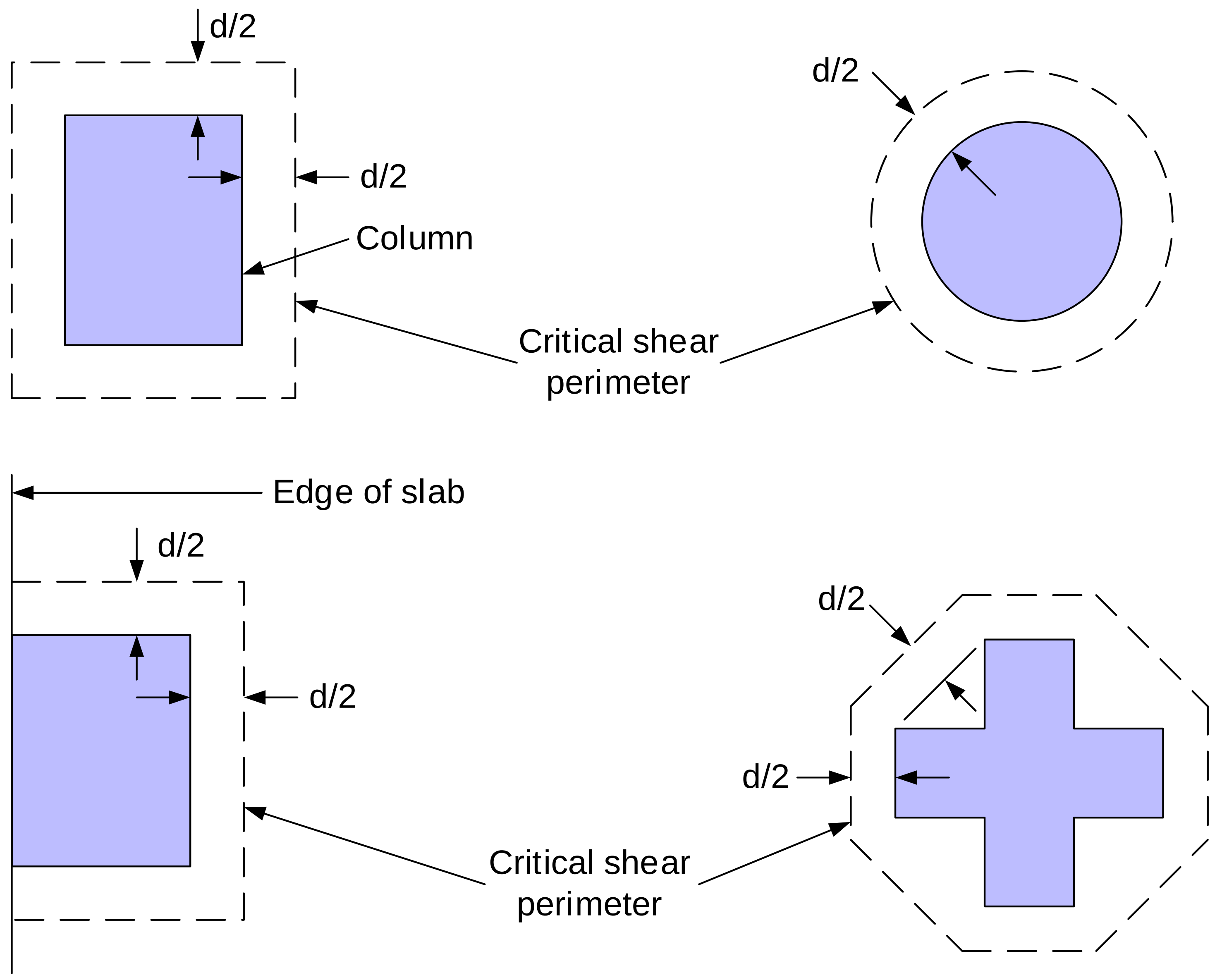
$$V_c = 0.27\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f'_c}\, b_o\, d \quad \text{(c)}$$

where b_o = perimeter of critical section at distance d/2 outside column

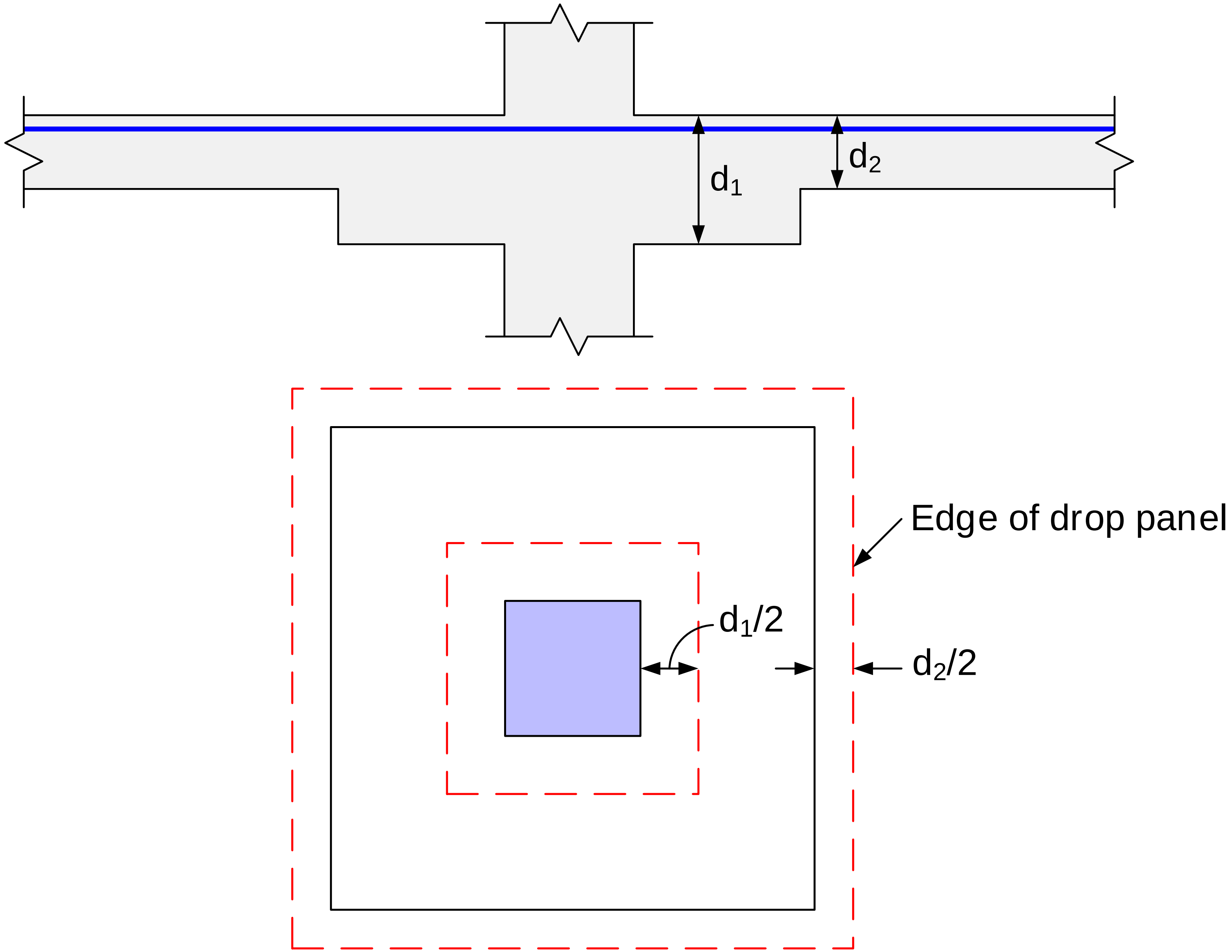
β = ratio of long side to short side of column

α_s = 40 for interior columns, 30 for edge columns and 20 for corner columns

Punching shear perimeter



Punching shear perimeter



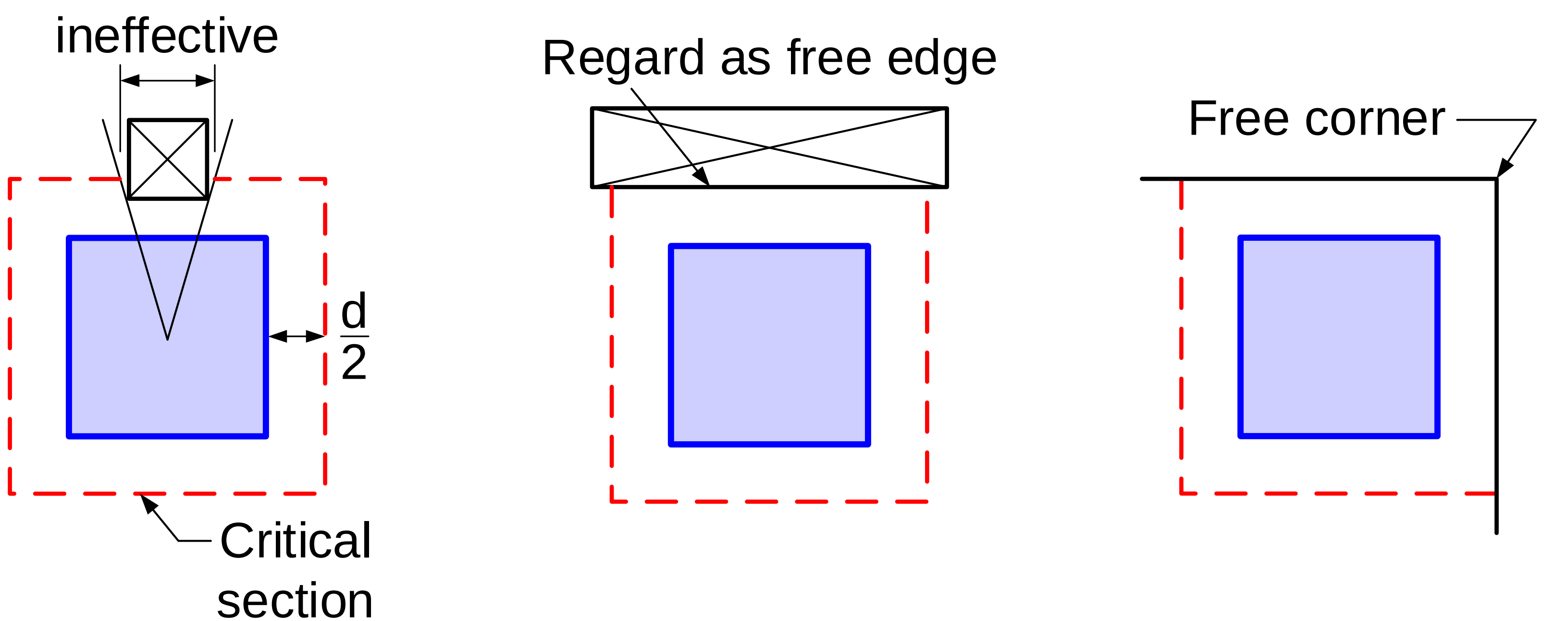
Effect of Openings and Free Edges

ACI318-14

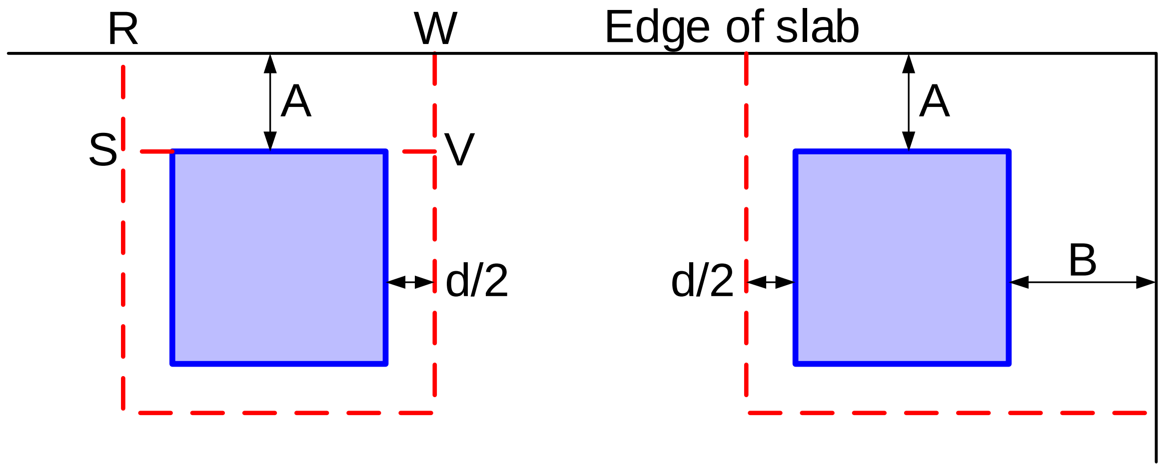
22.6.4.3 If an opening is located within a column strip or closer than **10h** from a concentrated load or reaction area, a portion of b_o enclosed by straight lines projecting from the centroid of the column, concentrated load or reaction area and tangent to the boundaries of the opening shall be considered ineffective.

where **h** = overall thickness of slab

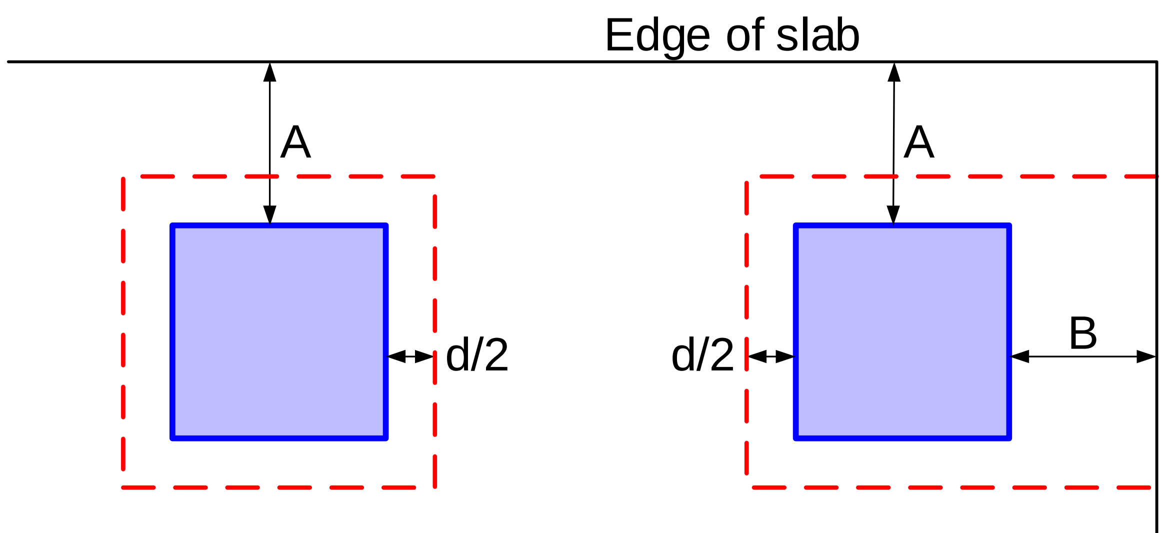
b_o = perimeter of critical section for two-way shear



Column Near Slab Edge



(ก) เส้นรอบรูปวิกฤตเมื่อ A และ B ไม่เกินค่าที่มากกว่าของ 4h หรือ $2L_d$



(ข) เส้นรอบรูปวิกฤตเมื่อ A เกินค่าที่มากกว่าของ 4h หรือ $2L_d$ แต่ B ไม่เกิน

β for a Nonrectangular Loaded Area

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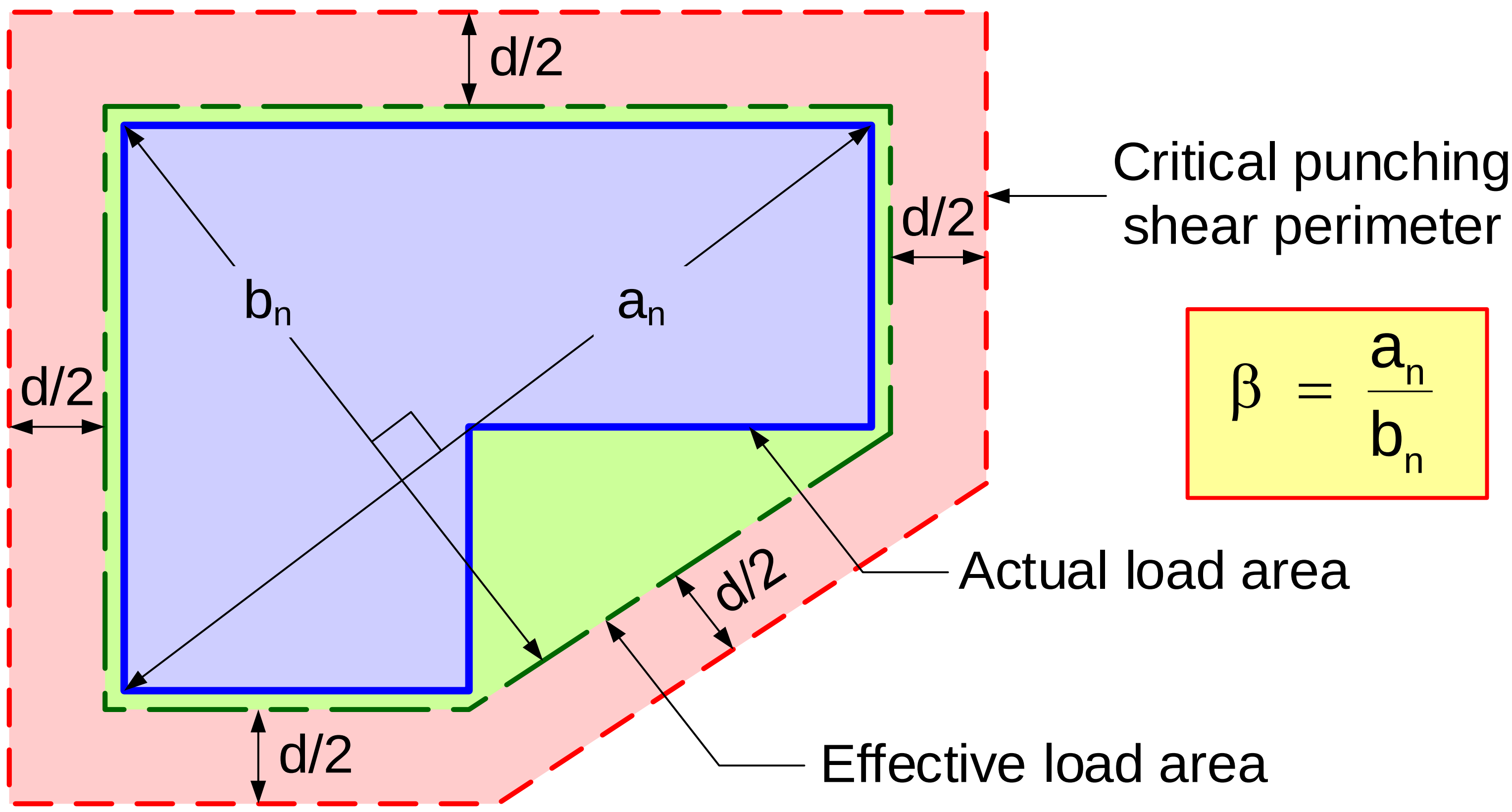
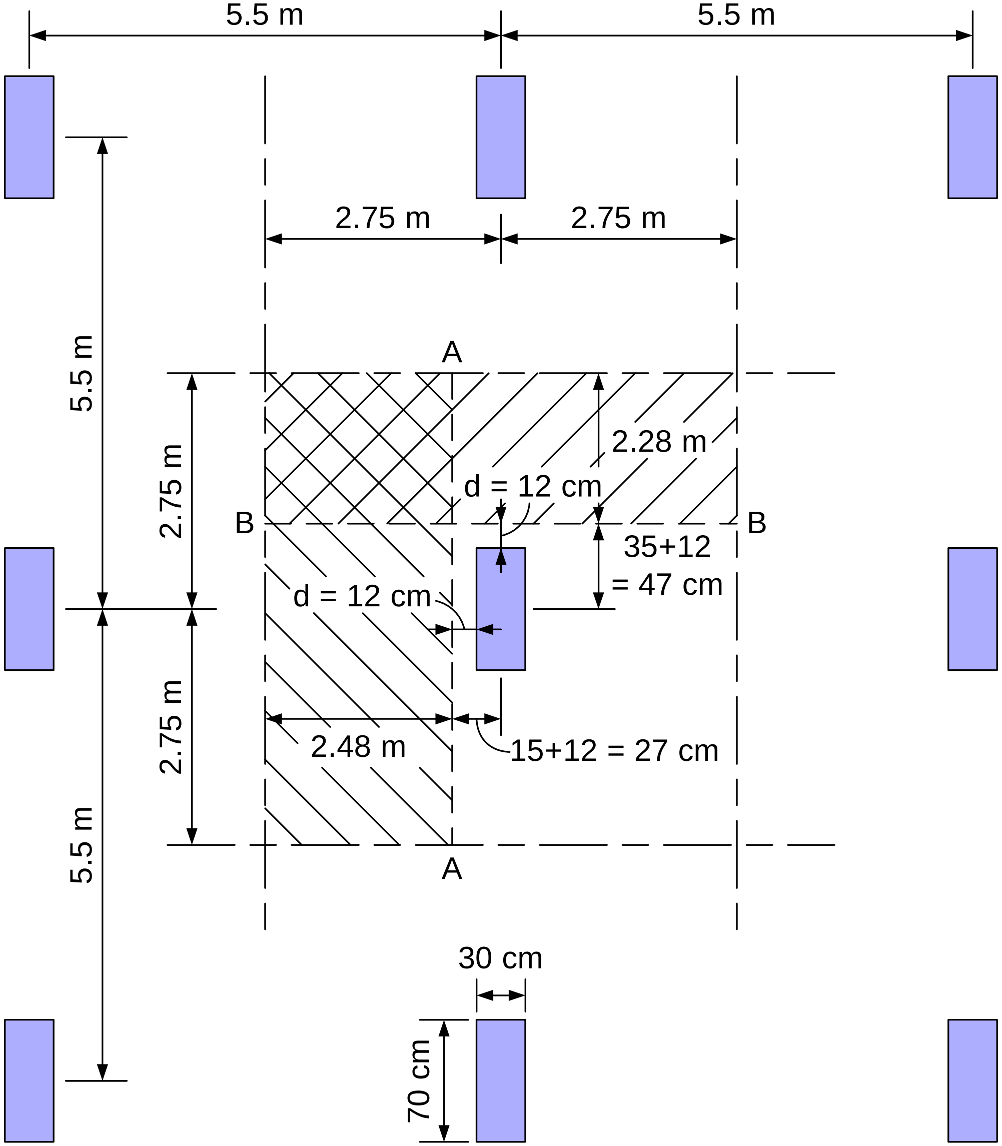
β = ratio of longest overall dimension to the largest overall perpendicular dimension of effective load area



Effective load area is an area totally enclosing the **actual loaded area** with minimum perimeter.

Example 1 : Shear strength of flat plate



Slab Data:

f'_c = 240 kg/cm^2

f_y = 4,000 kg/cm^2

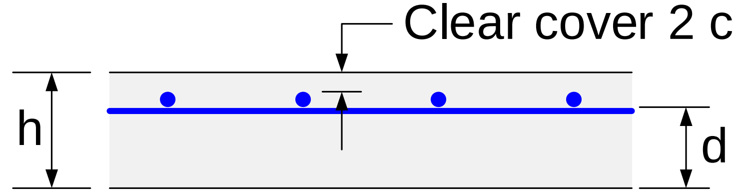
SDL = 100 kg/m^2

LL = 300 kg/m^2

Slab thickness h = 15 cm

Solution:

Assume using rebar DB12



$$d = 15 - 2 - 1.2 \approx 12 \text{ cm}$$

$$w_u = 1.4(0.15 \times 2,400 + 100) + 1.7 \times 300$$

$$= 1,154 \text{ kg/m}^2$$

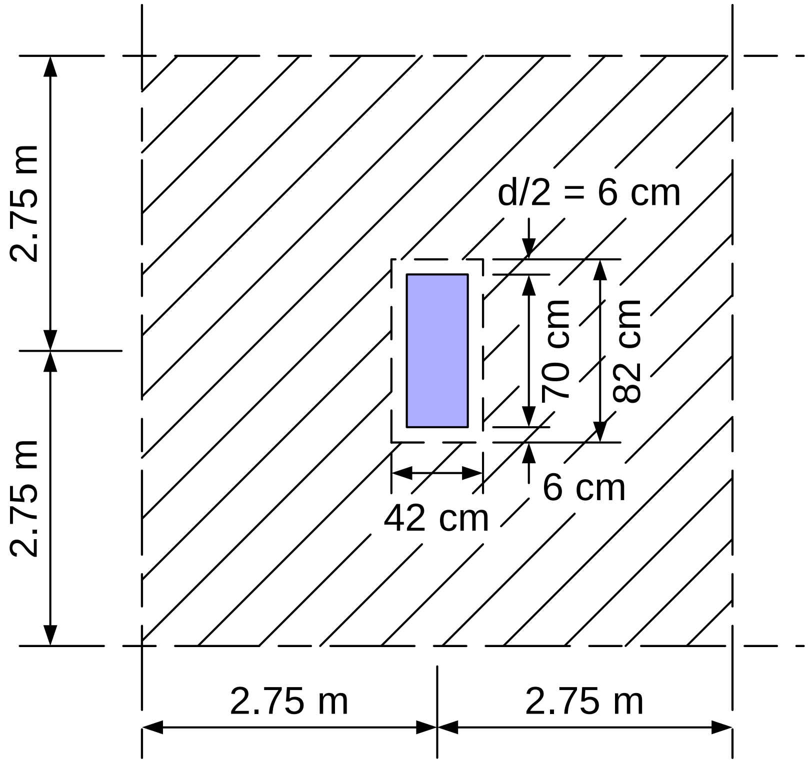
One-way Shear Strength : at distance **d** from column face along line A-A and B-B

At section A-A: $V_u = 1.154 \text{ t/m}^2 \times 2.48 \text{ m} \times 5.5 \text{ m} = 15.7 \text{ tons}$ **control**

At section B-B: $V_u = 1.154 \text{ t/m}^2 \times 2.28 \text{ m} \times 5.5 \text{ m} = 14.5 \text{ tons}$

$$\phi V_c = 0.85(0.53\sqrt{240} \times 550 \times 12)/1{,}000 = 46.1 \text{ tons} > V_u \quad \textbf{OK}$$

Two-way Shear Strength : critical shear perimeter at distance **d/2** from column face



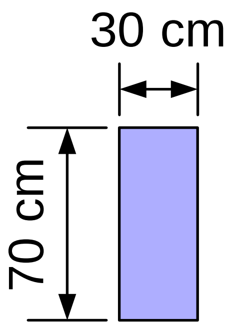
$$V_u = 1.154 \text{ t/m}^2 \times (5.5^2 - (0.42 \times 0.82))$$

$$= 34.5 \text{ tons}$$

$$b_0 = 2(42 + 82) = 248 \text{ cm}$$

$$\phi V_c = 0.85(1.06\sqrt{240} \times 248 \times 12)/1{,}000$$

$$= 41.5 \text{ tons}$$ —— Eq.(a)



$$\beta = \frac{70}{30} = 2.33$$

$$\phi V_c = 0.85(0.27\left(2 + \frac{4}{2.33}\right)\sqrt{240} \times 248 \times 12) / 10^3$$

$$= 39.3 \text{ tons} \;\text{———}\; \text{Eq.(b)}$$

For interior column $\alpha_s = 40,$

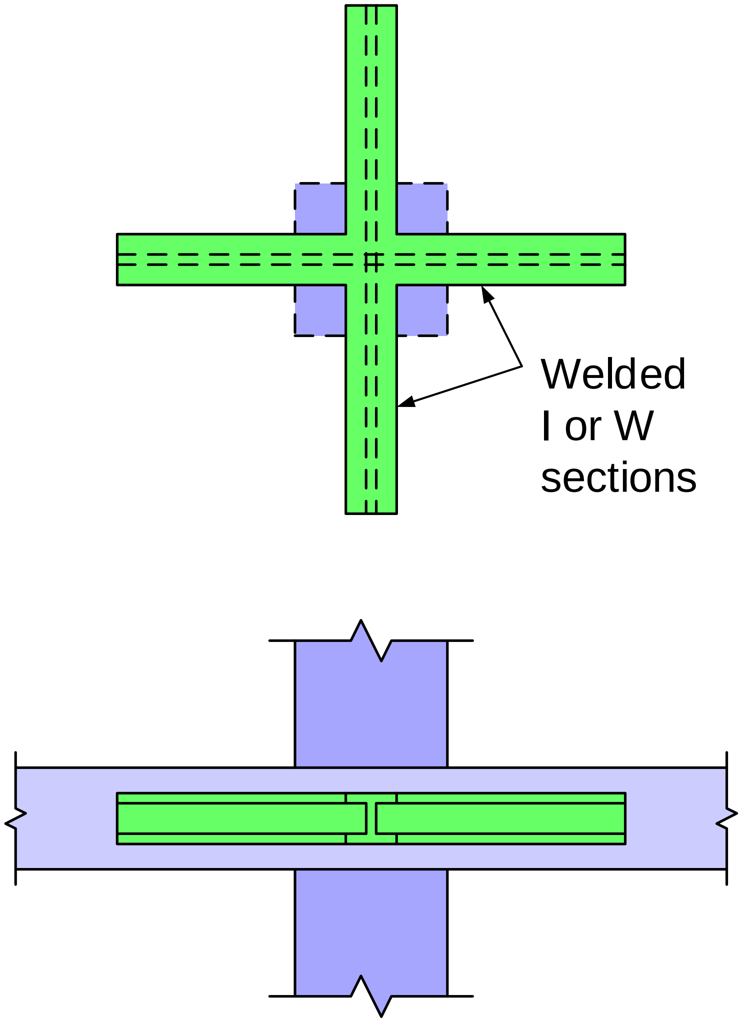
$$V_c = 0.27\left(\frac{\alpha_s d}{b_0} + 2\right)\sqrt{f'_c}\, b_0\, d$$

$$\phi V_c = 0.85(0.27\left(\frac{40 \times 12}{248} + 2\right)\sqrt{240} \times 248 \times 12) / 10^3$$

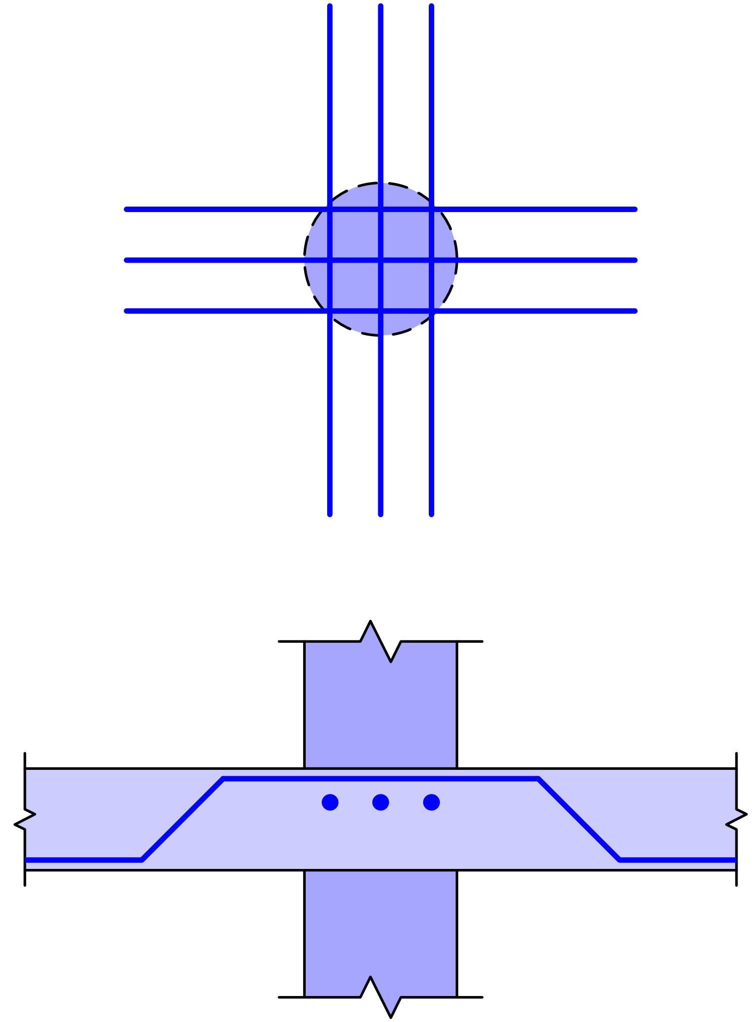
$$= 41.6 \text{ tons} \;\text{———}\; \text{Eq.(c)}$$

The least of (a), (b) and (c) is 39.3 ton $> V_u = 34.5$ tons **OK**

Punching Shear Reinforcement

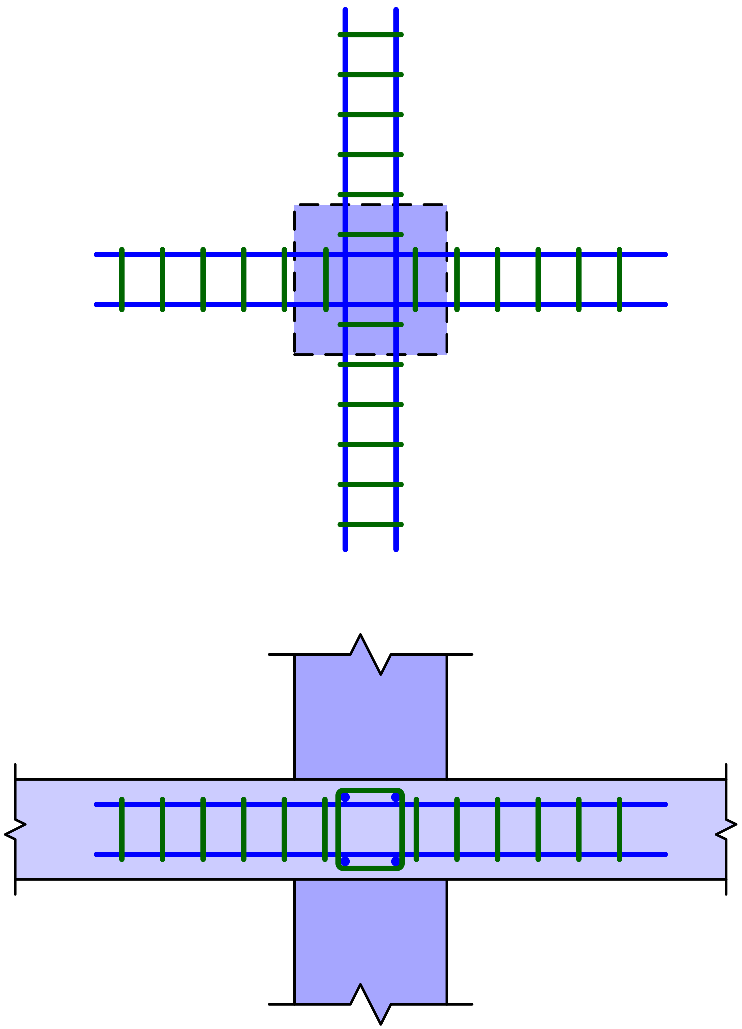


Shear Head

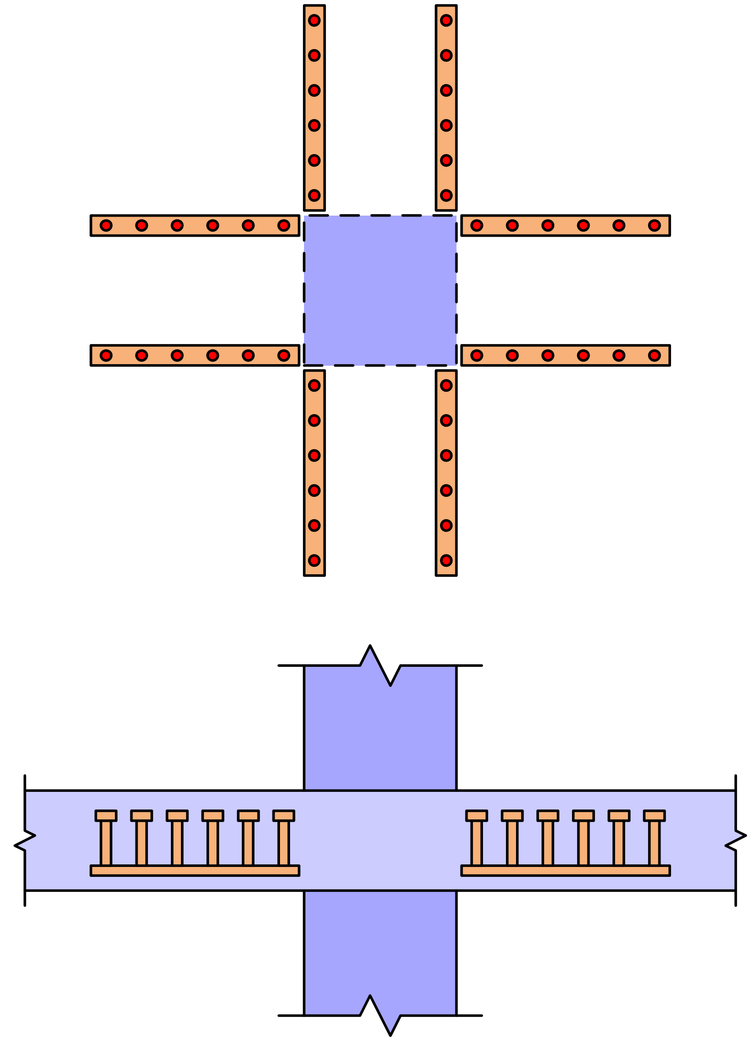


Bent Bar

Punching Shear Reinforcement



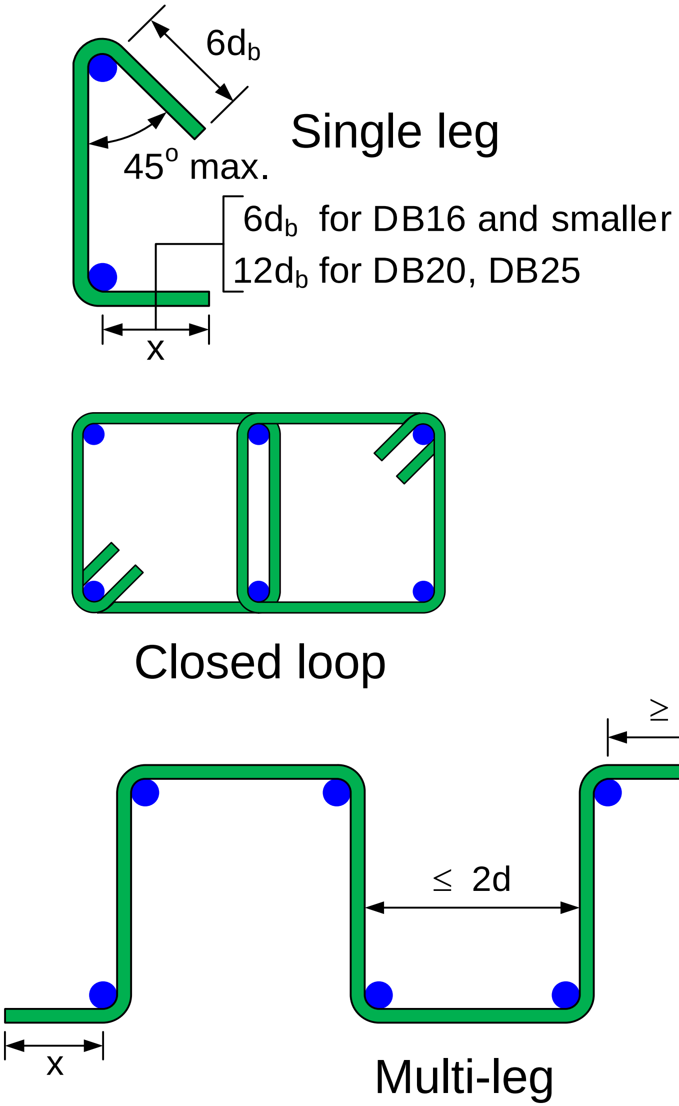
Stirrup

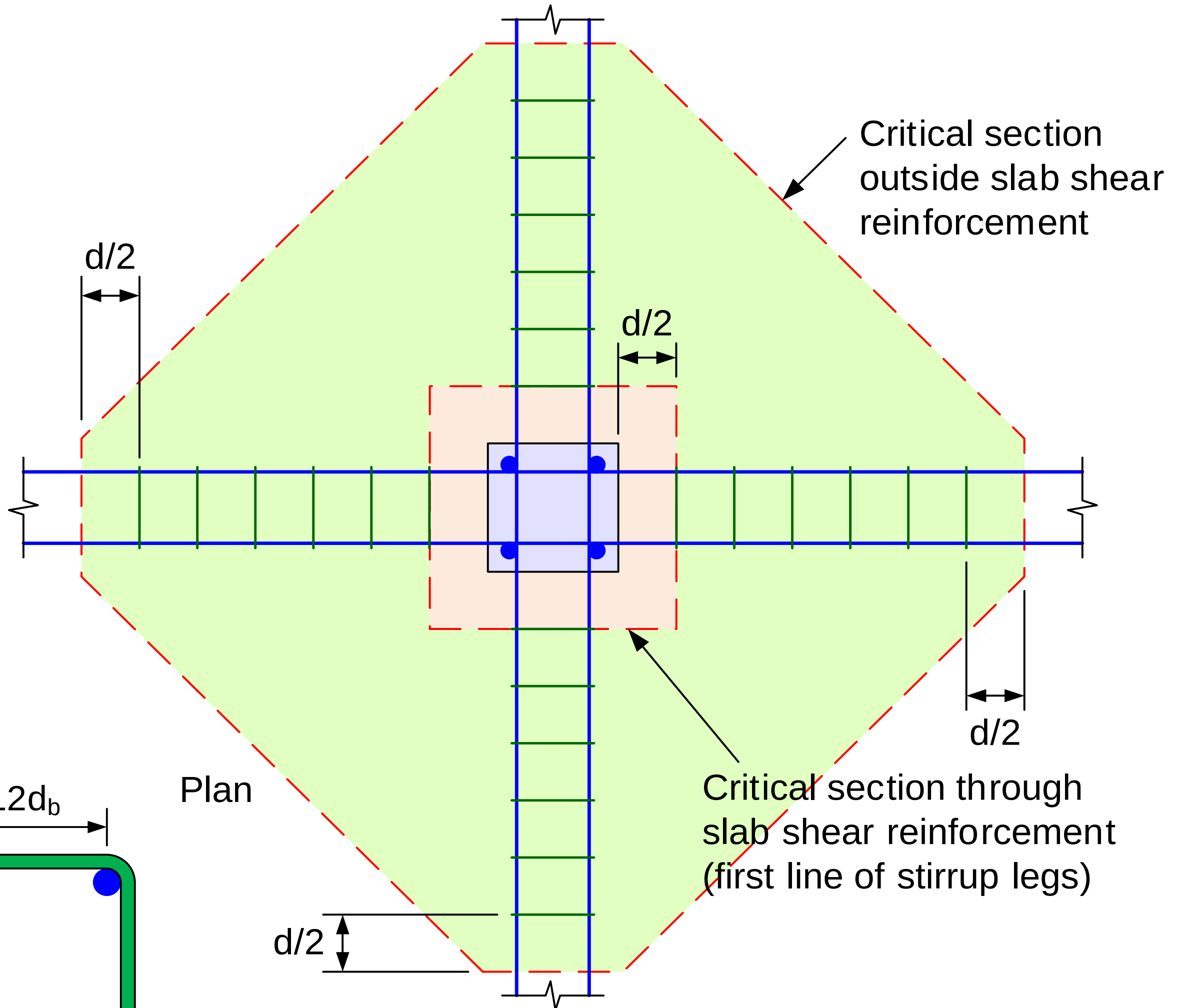


Stud Rail

Shear Reinforcement in Flat Plate Floors

Stirrup



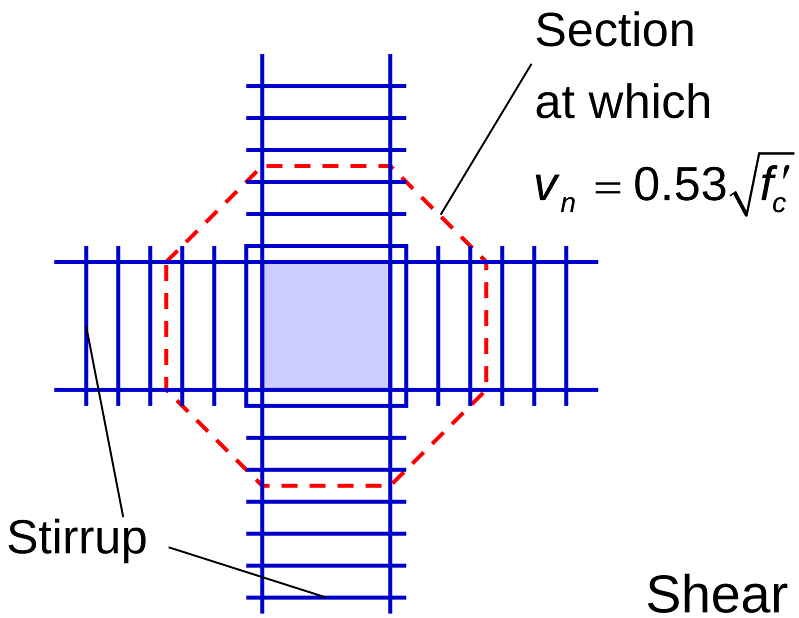


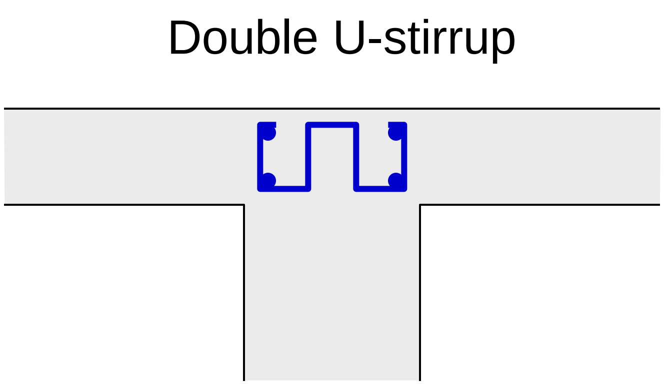
Shear Reinforcement in Flat Plate Floors

where no column capitals and drop panels, shear reinforcement is frequently necessary.

Nominal strength when bar reinforcement is used,

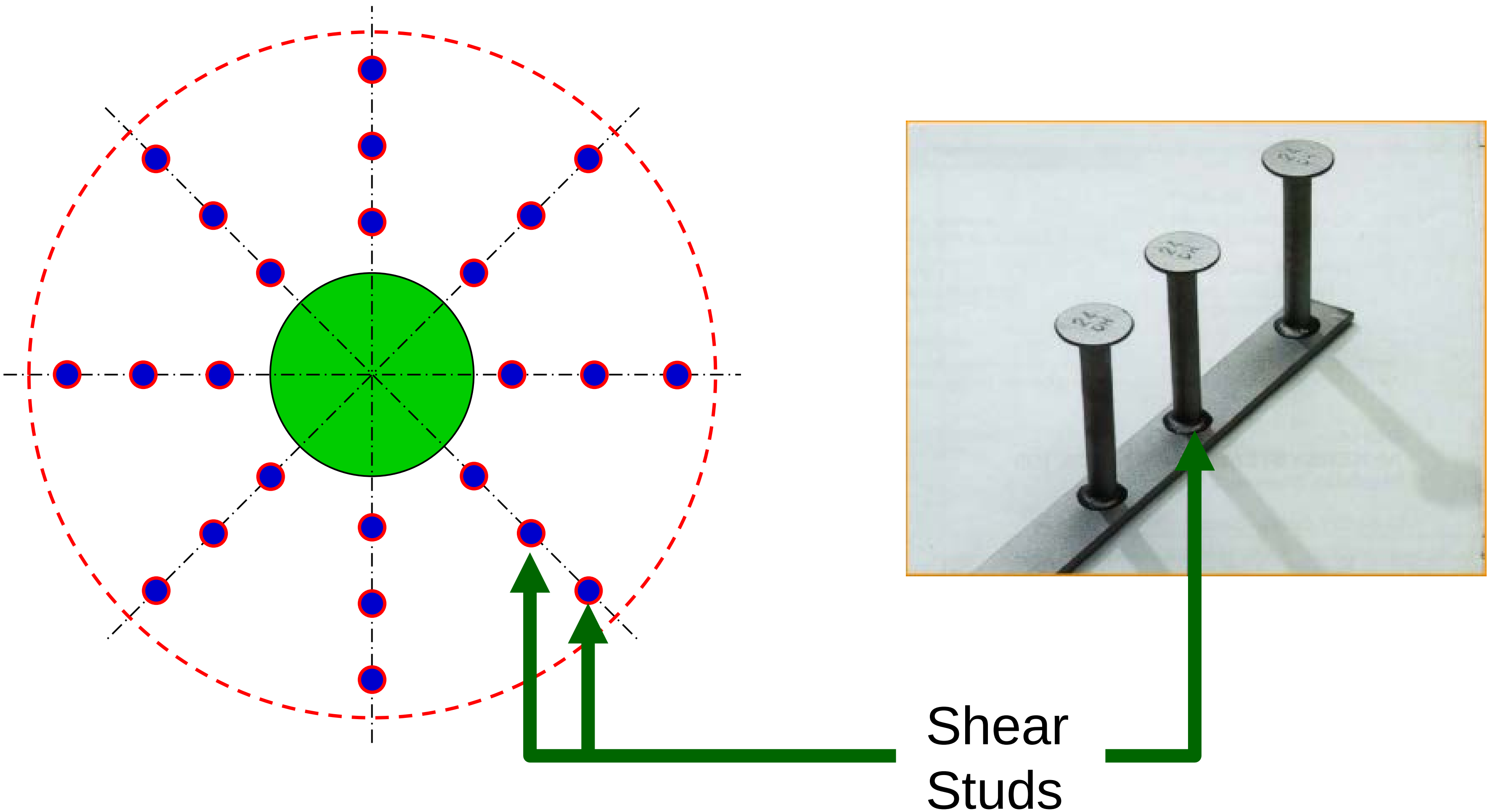
$$V_n = V_c + V_s = 0.53\sqrt{f'_c}\,b_0\,d + \frac{A_v f_y d}{s} \le 1.59\sqrt{f'_c}\,b_0\,d$$



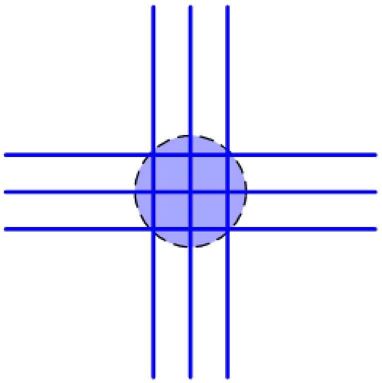


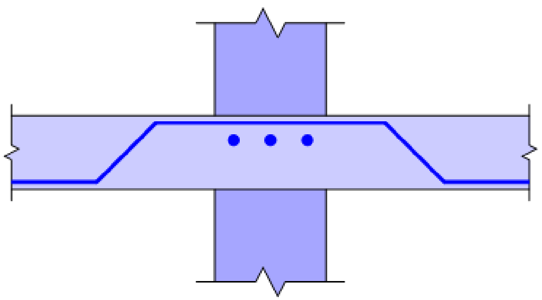
Shear strength from stirrup: $V_s = A_v f_{yt}\dfrac{d}{s}$

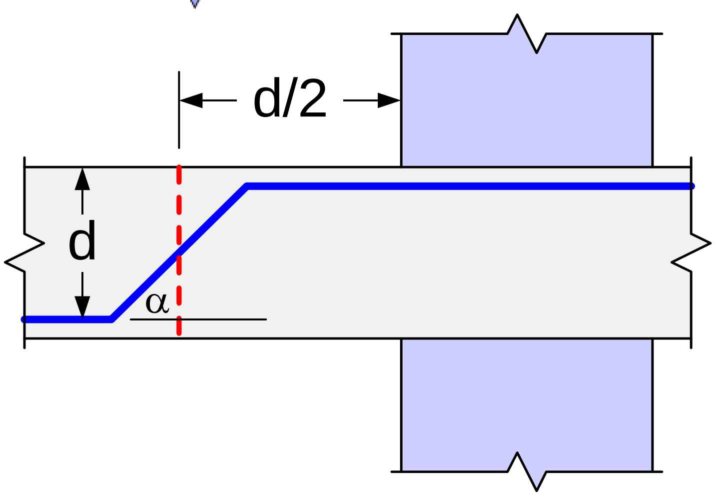
Shear Studs



Design of Bent-Bar Reinforcement







Shear resistance of concrete :

$$V_c = 0.53\sqrt{f'_c}\, b_0 d$$

Provide shear reinforcement :

$$V_s = \frac{V_u}{\phi} - V_c$$

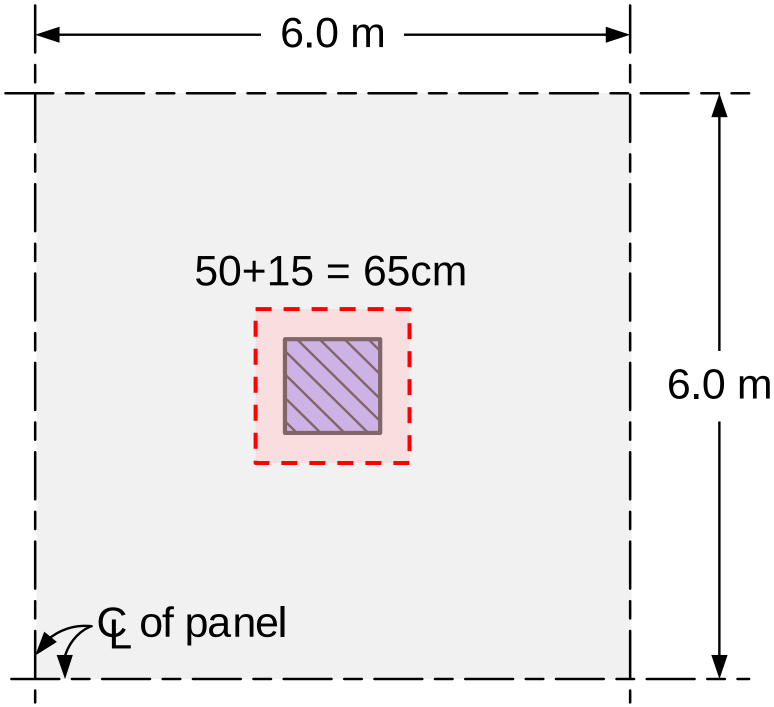
Max. nominal shear strength:

$$V_n = V_c + V_s \le 1.59\sqrt{f'_c}\, b_0\, d$$

Shear strength from reinforcement :

$$V_s = A_v f_y \sin\alpha \le 0.795\sqrt{f'_c}\, b_o d$$

Example 2 : Shear design of flat plate using bent bars



Slab Data:

$f'_c = 280\ kg/cm^2$, $f_y = 4{,}000\ kg/cm^2$

Slab thickness h = 20 cm

Column size = 50 x 50 cm

$w_u = 1{,}700\ kg/m^2$

Solution: Assume $d \approx 15$ cm

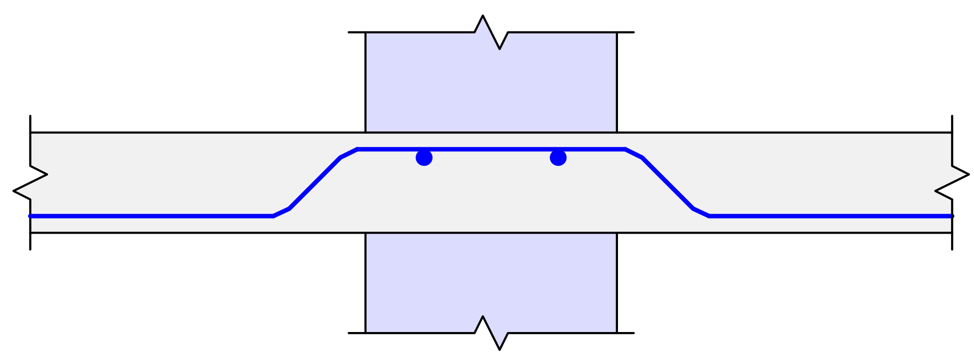
$$b_o = 4(65) = 260\ cm$$

$$V_u = 1.700(6^2 - 0.65^2) = 60.48\ tons$$

If no shear reinforcement, $\phi V_c = 0.85 \times 1.06\sqrt{280} \times 260 \times 15 / 10^3 = 58.80\ tons < V_u$

$\therefore$ shear reinforcement is required

USE Bar bent 45º in 2 directions



Max. nominal shear strength:

$$V_n = V_u / \phi = 60.48/0.85 = 71.16 \text{ tons}$$

$$1.59\sqrt{f'_c}\, b_0\, d = 1.59\sqrt{280} \times 260 \times 15/10^3 = 103.76 \text{ tons} > V_n \quad \textbf{OK}$$

Shear strength of concrete is reduced to,

$$\phi V_c = 0.85 \times 0.53\sqrt{280} \times 260 \times 15/10^3 = 29.40 \text{ tons}$$

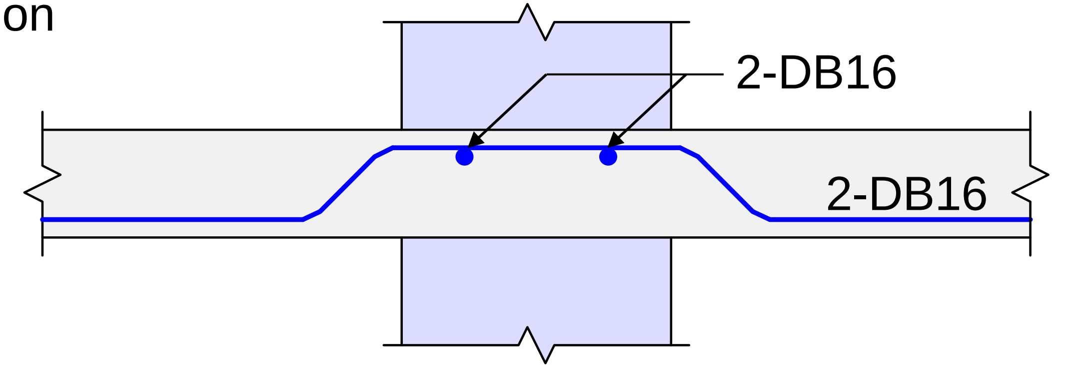
$$V_s = V_u/\phi - V_c = 60.48/0.85 - 29.40 = 41.75 \text{ tons}$$

$$0.795\sqrt{f'_c}\, b_0\, d = 0.795\sqrt{280} \times 260 \times 15/10^3 = 51.88 \text{ tons} > V_s \quad \textbf{OK}$$

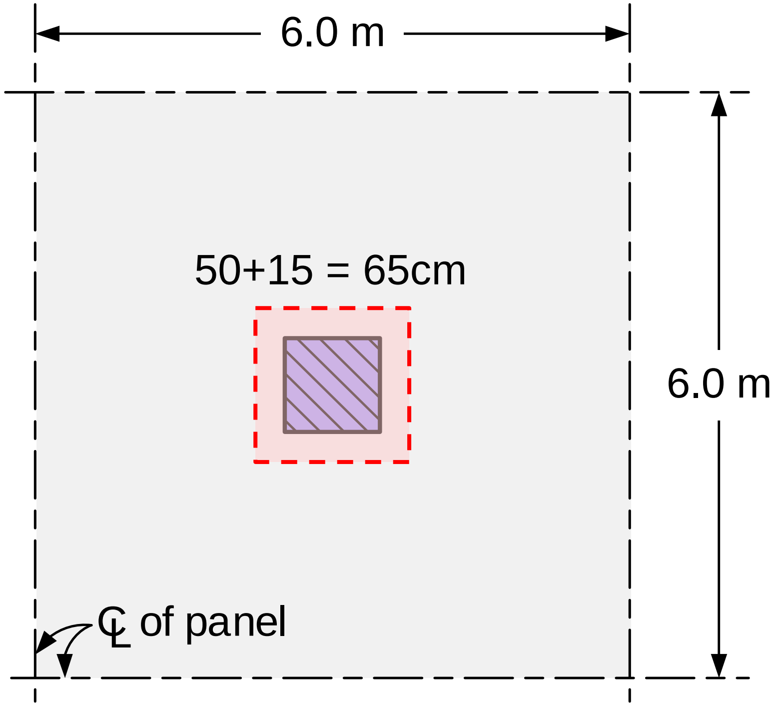
Required bar area: $A_v = \dfrac{V_s}{f_y \sin\alpha} = \dfrac{41.75}{4.0 \times 0.707} = 14.76 \text{ cm}^2$

Use 4-DB16 bars (2 in each direction)
with 8 legs crossing the critical section

8-DB16 : $A_v = 8 \times 2.01 = 16.08 \text{ cm}^2$



Example 3 : Shear design of flat plate using stirrup



Slab Data:

f'_c = 280 kg/cm^2, f_y = 4,000 kg/cm^2

Slab thickness h = 20 cm

Column size = 50 x 50 cm

$w_u = 1{,}700$ kg/m^2

Solution: Assume $d \approx 15$ cm

$$b_o = 4(65) = 260 \text{ cm}$$

$$V_u = 1.700(6^2 - 0.65^2) = 60.48 \text{ tons}$$

If no shear reinforcement, $\phi V_c = 0.85 \times 1.06\sqrt{280} \times 260 \times 15/10^3 = 58.80 \text{ tons} < V_u$

$\therefore$ shear reinforcement is required **USE Stirrup which required min. d = 15 cm**

Max. nominal shear strength:

$$V_n = V_u/\phi = 60.48/0.85 = 71.16 \text{ tons}$$

$$1.59\sqrt{f'_c}\, b_0\, d = 1.59\sqrt{280} \times 260 \times 15/10^3 = 103.76 \text{ tons} > V_n$$ **OK**

Shear strength of concrete is reduced to,

$$\phi V_c = 0.85 \times 0.53\sqrt{280} \times 260 \times 15 / 10^3 = 29.40 \text{ tons}$$

$$V_s = V_u / \phi - V_c = 60.48 / 0.85 - 29.40 = 41.75 \text{ tons}$$

$$0.795\sqrt{f'_c}\, b_0\, d = 0.795\sqrt{280} \times 260 \times 15 / 10^3 = 51.88 \text{ tons} > V_s \text{ OK}$$

Use RB9 stirrup since d must be ≥ 16 times stirrup diameter (16×0.9 = 14.4 cm)

Arrange stirrup along 4 integral beams

$$A_v = 4 \times 2 \times 0.636 = 5.09 \text{ cm}^2$$

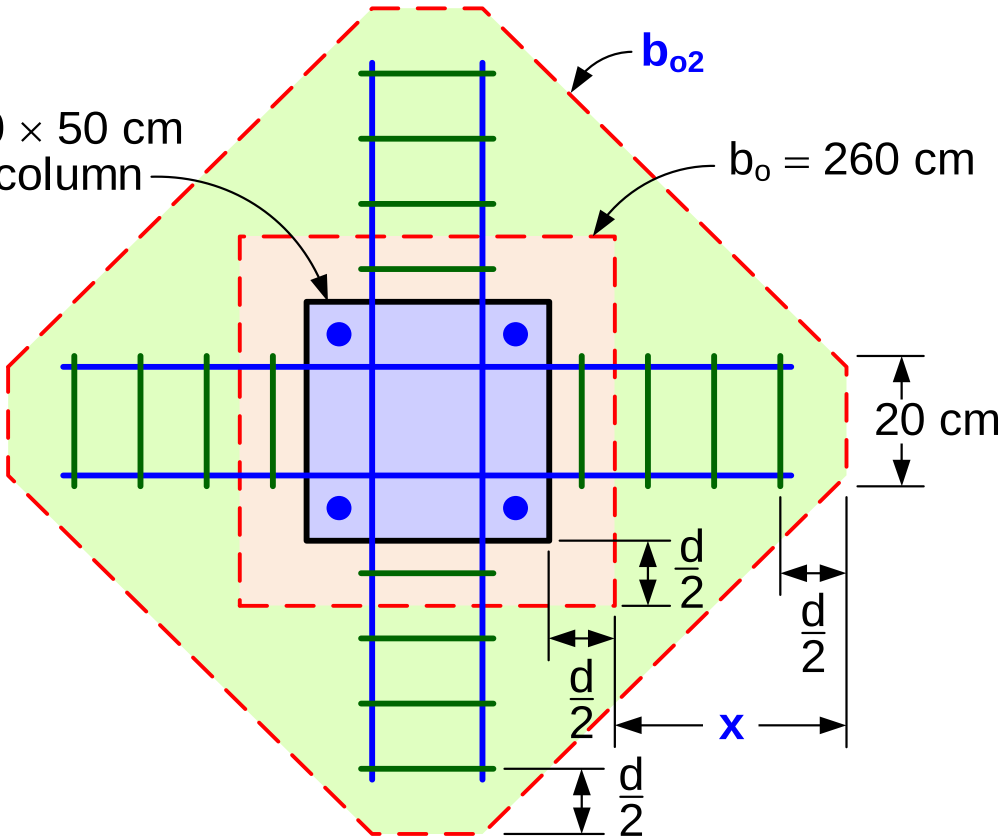
Required stirrup spacing:

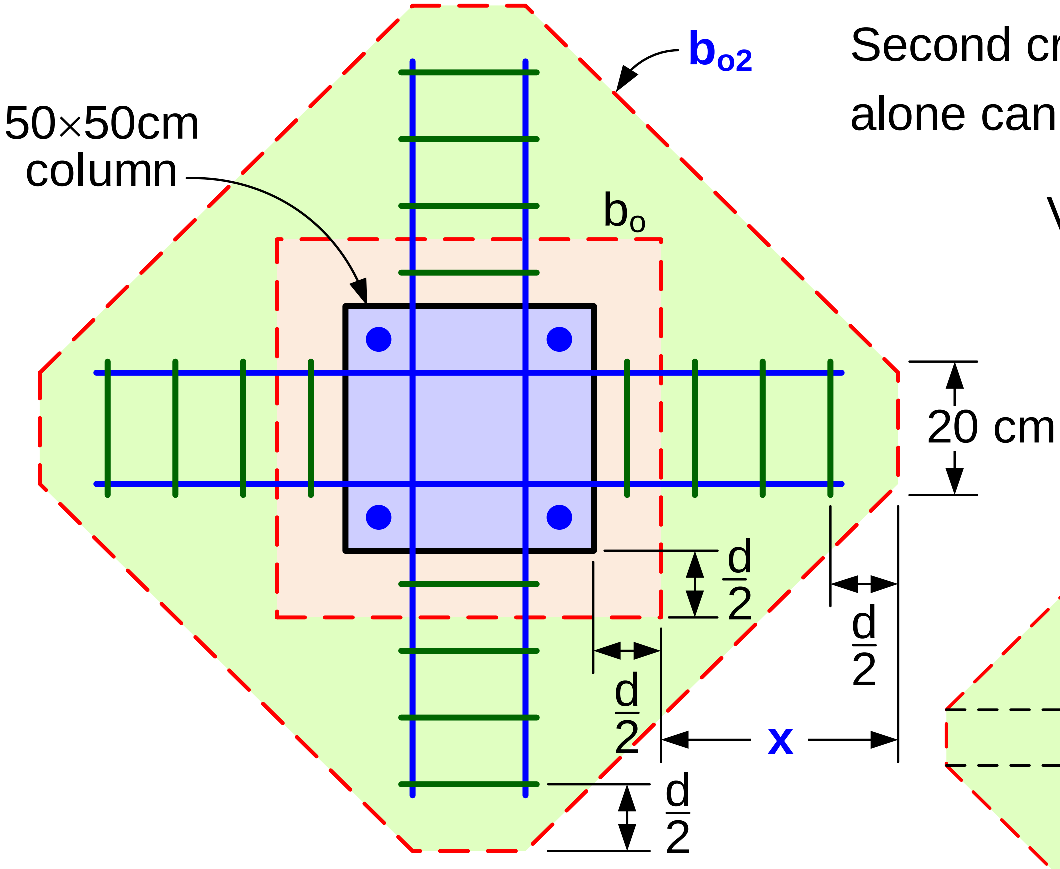
$$s = \frac{A_v f_y d}{V_s}$$

$$= \frac{5.09 \times 4.0 \times 15}{41.75} = 7.32 \text{ cm}$$

$$s_{max} = d/2 = 7.5 \text{ cm}$$

USE Stirrup RB9 @ 7 cm



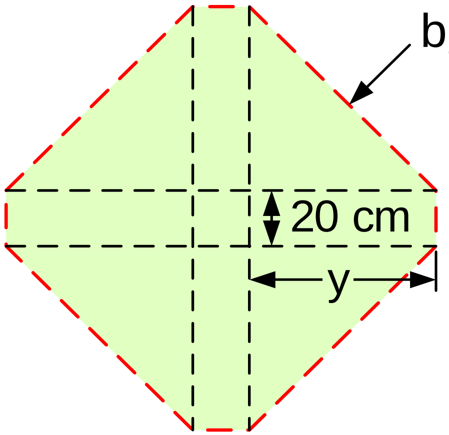


Second critical perimeter b_{o2} at which concrete alone can carry shear:

$$V_c = V_u / \phi = 71.16 \text{ tons}$$

$$= 1.06\sqrt{280} \times b_{o2} \times 15 / 10^3$$

$$\text{Min. } b_{o2} = 267.5 \text{ cm}$$



$$b_{o2} = 4(y\sqrt{2} + 20)$$

$$y = 33.15 \text{ cm}$$

$$x = 33.15 - 15 - 15$$

$$= 3.15 \text{ cm}$$

∴ Using 4 stirrup RB9 @ 7 cm is sufficient.

End of Lecture