

Proof Mixed Exam Questions



*Attempt these exam questions independently showing full and clear solutions.
Check each answer as you go against the exam board mark scheme.*

1. Edexcel A Level Maths Practice Paper 1, Q1

It is suggested that the sequence $a_k = 2^k + 1$, $k \geq 1$ produces only prime numbers.

- (a) Show that a_1 , a_2 and a_4 produce prime numbers.

(2 marks)

- (b) Prove by counter example that the sequence does not always produce a prime number.

(2 marks)

2. OCR AS Maths Specimen Paper 2, Q6

- (a) A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.

For example, 23 has digits 2 and 3 which gives $2^2 + 3^2 = 13$, which is odd.

Show by counter example that this suggestion is false.

[2]

- (b) Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3.

[3]

3. Edexcel AS Maths Sample Paper, Q11

- (a) Prove that for all positive values of x and y

$$\sqrt{xy} \leq \frac{x+y}{2} \quad (2)$$

- (b) Prove by counter example that this is not true when x and y are both negative.

(1)

4. OCR MEI C1, Jun 2007, Q3

Factorise $n^3 + 3n^2 + 2n$. Hence prove that, when n is a positive integer, $n^3 + 3n^2 + 2n$ is always divisible by 6. [3]



5. OCR MEI C1, Jun 2006, Q4

In each of the following cases choose one of the statements

$$P \Rightarrow Q$$

$$P \Leftrightarrow Q$$

$$P \Leftarrow Q$$

to describe the complete relationship between P and Q.

(i) P: $x^2 + x - 2 = 0$
Q: $x = 1$

[1]

(ii) P: $y^3 > 1$
Q: $y > 1$

[1]