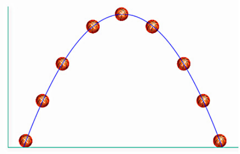
Center of Mass (COM)



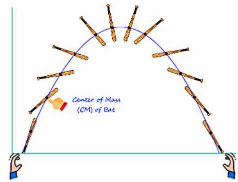
- Finding COM of symmetric objects is quite simple
- ...because it is easy to find the center

Motion of Symmetric Objects



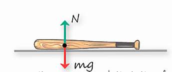
- Even the motion of symmetric objects is quite predictable
- ...They would follow a parabolic path

Motion of Asymmetric Objects



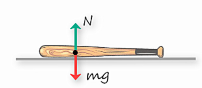
- Motion of asymmetric objects may not be predicatble at first
- But if you observe their COM, you will find it follows a parabolic trajectory

Gravity acts on every particle of the bat.



However it can assumed that the force of gravity for the entire bat acts through the centre of mass

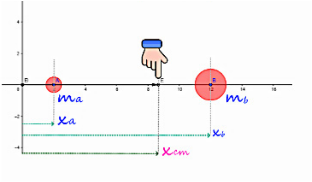
Motion of Asymmetric Objects



- Gravity acts on every particle of the bat
- However, you can assume that the force of gravity for the entire mass acts through the COM

A bat spun on a flat table will have the COM moving in a straight line becasue the net force on it is zero (N cancles mg)

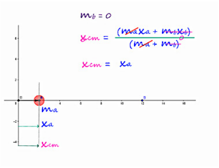
COM of a System of 2 Particles



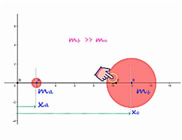
$$x_{cm} = \frac{(m_a x_a + m_b x_b)}{(m_a + m_b)}$$

- COM will be closer to the heavier object

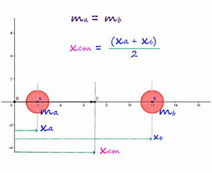
COM of a System of 2 Particles: Various Situations



- When mass b = 0
- COM merges with COM of mass a

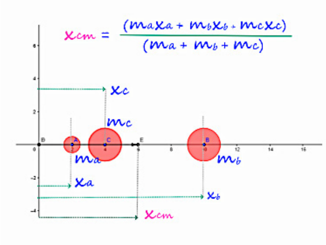


- When b >> a
- COM become much closer to b



- When b = a
- COM is in the middle of a and b

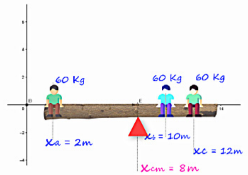
COM of a System of 3 Particles



$$x_{cm} = \frac{(m_a x_a + m_b x_b + m_c x_c)}{(m_a + m_b + m_c)}$$

- You just add $m_c x_c$ in the numerator and m_c in the denominator

COM of 3 Men sitting on a Log



$$x_{cm} = \frac{(m_a x_a + m_b x_b + m_c x_c)}{(m_a + m_b + m_c)}$$

$$x_{cm} = \frac{(60 \times 2 + 60 \times 10 + 60 \times 12)}{(60 + 60 + 60)}$$

$$x_{cm} = 8 \text{ m}$$

- COM falls closer where the concentration of mass is higher, that is b and c

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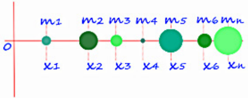
COM of "n" Particles on X axis

$$x_{cm} = \frac{(m_1x_1 + m_2x_2 + m_3x_3 + \dots m_nx_n)}{(m_1 + m_2 + m_3 \dots + m_n)}$$



$$x_{cm} = \frac{(m_1x_1 + m_2x_2 + m_3x_3 + \dots m_nx_n)}{M}$$

$$x_{cm} = 1/M \; \Sigma \; m_ix_i,$$

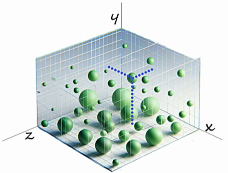
$(i = 1 \text{ to } n)$

- COM of n particles can be found by adding products of mass and distance in the numerator and mass in the denominator
- This formula is useful only when the masses are along the x axis

COM of "n" Particles in 3D Space

- COM of n particles in 3 dimension can be found by establishing the Y and Z cordinates <u>in addition to the X coordinate</u>

$$x_{cm} = \frac{(m_1x_1 + m_2x_2 + m_3x_3 + \ldots\ldots m_nx_n)}{(m_1 + m_2 + m_3 \ldots + m_n)}$$

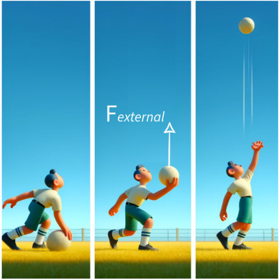


$X_{cm} = 1/M \, \Sigma \, m_ix_i,$ $(i=1 \text{ to } n)$

$y_{cm} = 1/M \, \Sigma \, m_iy_i,$ $(i=1 \text{ to } n)$

$Z_{cm} = 1/M \, \Sigma \, m_iz_i,$ $(i=1 \text{ to } n)$

WORK DONE BY AN EXTERNAL FORCE

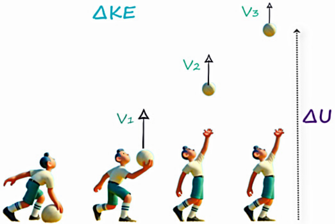


- $F_{applied}$ does work W on the ball
- $F_{applied}$ is a non conservative external force

...that transfers energy

W_{NC} = Work done by applied force F

W_C = Work done by force of gravity



$$W_{NET} = W_{NC} + W_C \quad (1)$$

$$W_{NET} = \Delta K \quad (2)$$

$$W_C = -\Delta U \quad (3)$$

$$\Delta K = W_{NC} - \Delta U$$

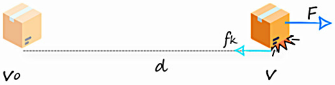
$$W_{NC} = \Delta K + \Delta U$$

$$W_{NC} = \Delta E_{mech}$$

$F - f_k = ma$

$v^2 = v_0^2 + 2ad$

$a = (v^2 - v_0^2)/2d$



$Fd = 1/2(mv^2) - 1/2(mv_0^2) + f_k d$

$Fd = \Delta K + f_k d$

$Fd = \Delta K + \Delta E_{th}$

$W_{NC} = \Delta K + \Delta E_{th}$

$K = K_0 + W_{NC} - f_k d$

$W_{NC} = \Delta K + \Delta U + \Delta E_{th}$

$$W_{NC} = \Delta E_{mech} + \Delta E_{th}$$

LAW OF CONSERVATION OF ENERGY

The change in total energy E of a system

=

Magnitudes of energy transfer to or from the system

$$W_{NC} = \Delta E$$

$$W_{NC} = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}$$