NOTES:

# 2: Linear Inequalities

**Simple Inequalities:**

1. Mathematical sentences containing symbols <, >, ≤ or ≥ are called **inequalities**. The solution of inequalities usually involves a range of values of the variable concerned.

2. Properties of inequalities are shown in the table below.

| Operation | Property | Illustrations |
|---|---|---|
| Multiplied by a **positive** number. | If $x > y$, then $ax > ay$ | - If 5 > 3, then 5 x 2 > 3 x 2<br>- If -3 > -5, then -3 x 2 > -5 x 2 |
| Divided by a **positive** number. | If $x > y$, then $\frac{x}{a} > \frac{y}{a}$ | - If 5 > 3, then $\frac{5}{2} > \frac{3}{2}$<br>- If -3 > -5, then $\frac{-3}{2} > \frac{-5}{2}$ |
| Multiplied by a **negative** number. **(Change sign)** | If $x > y$, then $-ax < -ay$ | - If 5 > 3, then 5 x (-2) < 3 x (-2)<br>- If -3 > -5, then -3 x (-2) < -5 x (-2) |
| Divided by a **negative** number. **(Change sign)** | If $x > y$, then $\frac{x}{-a} < \frac{y}{-a}$ | - If 5 > 3, then $\frac{5}{-2} < \frac{3}{-2}$<br>- If -3 > -5, then $\frac{-3}{-2} < \frac{-5}{-2}$ |

3. To solve an inequality, we can
 • **multiply** or **divide** both sides of an inequality by a **positive** number **without changing** the inequality sign.
 • **multiply** or **divide** both sides of an inequality by a **negative** number, **changing** the inequality sign.
 • **add** or **subtract** both sides of an inequality by a **positive**/**negative** number **without** changing the inequality sign.

**Solving Inequalities Using Cross Multiplication:**
4. When encountering a fractional inequality, do note that it can be confusing when flipping signs. We'll walk you through an example:

When $\frac{a}{b} > \frac{c}{d}$, do note $ad > bc$, and **not** $bc > ad$ (vice versa). Always know that the numerator will stay on their side (e.g. $a$ will always be on the LHS, even after cross multiplication, $ad$, is on the LHS)

5. However, do note that $bd > 0$ (i.e. **both** $b$ and $d$ **must** be **positive** or **negative**.) for said inequality to hold. When one of them is negative while the other one is positive, the inequality **will not** hold. In this case, we can always move the negative to the numerator.

For example:

[Wrong]
$\frac{5}{-1} > \frac{x}{2}$ ← (both denominators aren't positive/negative) →
$10 > -x$ (cross-multiply)
$-x < 10$ (switch so $x$ is at LHS)
$x > -10$ (divide both sides by -1, sign flips)

[Correct]
$\frac{5}{-1} > \frac{x}{2}$
$\frac{-5}{1} > \frac{x}{2}$ (move '-' to numerator)
$-10 > x$ (cross-multiply)
$x < -10$ (switch so $x$ is at LHS)

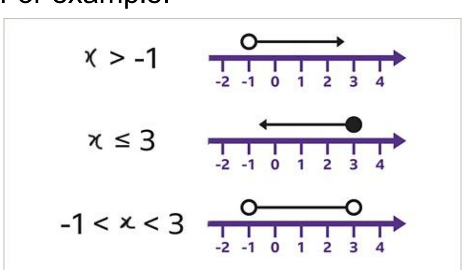
**Solutions of Inequalities:**

6. A solution of an inequality is **any** value of $x$ which **satisfies** an inequality (e.g. $x \leq 5$)
Some integer solutions of the inequality $x \leq 5$ are $x = 5, 4, 3, 2, 1,$ ... To represent all solutions of an inequality, we need to provide a **number line** alongside our solved inequality.

7. To draw a number line, we draw a scale of numbers with the number in the inequality being in the middle/right. Do note the number at the very **left** side of the scale is **numerically smaller** than the number on the very **right** of the scale. After that:

- If the inequality contains **<** or **>**: we draw a ◯ (hollow circle) on top of the number in the inequality and draw an arrow to the left or right depending on the inequality sign.
- If the inequality contains **≤** or **≥**: we draw a ⚫ (shaded circle) on top of the number in the inequality and draw an arrow to the left or right depending on the inequality sign.

For example:



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