Practice Questions

1. **Determine the scope of each quantifier in the wffs below.**
   a. $\forall x[P(x) \wedge Q(y)]$
   b. $P(x) \wedge \forall x[Q(x)]$
   c. $\forall x[P(x) \rightarrow \exists y[D(y) \wedge L(x,y)]]$
   d. $\exists y[D(y) \wedge \forall x[P(x) \rightarrow L(x,y)]]$
   e. $\exists z[\exists y[P(x,y,z) \rightarrow Q(z)] \wedge \forall x[Q(x,y,z)]]$

2. **For each formula, state which variables are free and which variables are bound.**
   a. $\forall x[P(x) \wedge Q(y)]$
   b. $P(x) \wedge \forall x[Q(x)]$
   c. $\forall x[P(x) \rightarrow \exists y[D(y) \wedge L(x,y)]]$
   d. $\exists y[D(y) \wedge \forall x[P(x) \rightarrow L(x,y)]]$
   e. $\exists z[\exists y[P(x,y,z) \rightarrow Q(z)] \wedge \forall x[Q(x,y,z)]]$

Solutions

1. **Determine the scope of each quantifier in the wffs below.**

   a. $\forall x[P(x) \wedge Q(y)]$
   $\forall x$: $P(x) \wedge Q(y)$

   b. $P(x) \wedge \forall x[Q(x)]$
   $\forall x$: $Q(x)$

   c. $\forall x[P(x) \rightarrow \exists y[D(y) \wedge L(x, y)]]$
   $\forall x$: $P(x) \rightarrow \exists y[D(y) \wedge L(x, y)]$
   $\exists y$: $D(y) \wedge L(x, y)$

   d. $\exists y[D(y) \wedge \forall x[P(x) \rightarrow L(x, y)]]$
   $\exists y$: $D(y) \wedge \forall x[P(x) \rightarrow L(x, y)]$
   $\forall x$: $P(x) \rightarrow L(x, y)$

   e. $\exists z[\exists y[P(x, y, z) \rightarrow Q(z)] \wedge \forall x[Q(x, y, z)]]$
   $\exists z$: $\exists y[P(x, y, z) \rightarrow Q(z)] \wedge \forall x[Q(x, y, z)]$
   $\exists y$: $P(x, y, z) \rightarrow Q(z)$
   $\forall x$: $Q(x, y, z)$

2. **For each formula, state which variables are free and which variables are bound.** The underlined variables are free, while the variables with color are bound to the same colored quantifier.

   a. $\forall x[P(x) \wedge Q(\underline{y})]$

   b. $P(\underline{x}) \wedge \forall x[Q(x)]$

   c. $\forall x[P(x) \rightarrow \exists y[D(y) \wedge L(x, y)]]$

   d. $\exists y[D(y) \wedge \forall x[P(x) \rightarrow L(x, y)]]$

   e. $\exists z[\exists y[P(\underline{x}, y, z) \rightarrow Q(z)] \wedge \forall x[Q(x, \underline{y}, z)]]$