QUESTIONS:

1. **What factor determines which variable control chart should be used?**
    - Defects v. defectives
    - The accuracy of the measurement system being used
    - The acceptance sampling plan associated with your product
    - The number of units sampled within each subgroup

2. **Pencil Makers Incorporated uses an x-bar and R chart of n=5 to monitor the length of pencils coming off the production line. The inspector takes two samples, measures the length and plots their values on the X-bar chart as both data points are outside of the upper control limit and decides to stop the process. What does this mean?**
    - Only the process range is out of statistical control
    - Only the process average is out of statistical control
    - Both the average and range are out of statistical control
    - Nothing, the inspector is not executing the control chart appropriately

3. **Using the following data points from these 5 sub-groups, calculate R-bar:**
    - 2
    - 3
    - 4
    - 5

Sub-group	Sample 1	Sample 2	Sample 3
1	12	15	16
2	14	12	13
3	10	13	13
4	14	16	16
5	14	12	16

4. **You manufacture a widget and use an x-bar and R chart to monitor your process, where you sample 3 units in each subgroup, and R-bar = 16.0. Estimate the population standard deviation for this process.**
    - 16.0
    - 9.5
    - 27.1
    - 13.2

5. **You're manufacturing a widget and using an X-bar and R chart to control the critical feature of the product. Your normal process has the following attributes: X-double bar is 225, R-bar is 12, n = 8. Identify the lower control limits for the X-bar chart:**
    - 220.52
    - 229.48
    - 233.14
    - 218.71

6. **You're manufacturing a widget and using an X-bar and R chart to control the critical feature of the product. Your normal process has the following attributes: X-double bar is 225, R-bar is 12, n = 8. Identify the upper control limits for the range chart:**
   - 5.73
   - 18.23
   - 22.37
   - 24.17

7. **You're manufacturing a product to a specification of 5.25" ± 0.25". You control that process with an X-bar and R chart with a sample size of 5, and your X-bar = 5.20 and R-bar = 0.20. What is the Cpk of your process?**
   - 0.775
   - 0.969
   - 1.163
   - 1.243

8. **What type of variation occurs when a process is out of control?**
   - Variable
   - Attribute
   - Common Cause
   - Special Cause

9. **You build 100 units per day, and your process has an average number of defects per day of 4. What is the probability that in a given production day, you will experience 1 defect?**
   - 5.2%
   - 7.3%
   - 9.6%
   - 11.2%

10. **You're performing a hypothesis test for the population mean and the population standard deviation is unknown. You sample 20 units from your population and you'd like to use a 1-sided test at a 5% significance level. What is the rejection criteria for this hypothesis test?**
    - 1.725
    - 1.729
    - 2.086
    - 2.093

SOLUTIONS:

1. What factor determines which variable control chart should be used?

- Defects v. defectives
- The accuracy of the measurement system being used
- The acceptance sampling plan associated with your product
- **The number of units sampled within each subgroup**

2. Pencil Makers Incorporated uses an x-bar and R chart of n=5 to monitor the length of pencils coming off the production line. The inspector takes two samples, measures the length and plots their values on the X-bar chart as both data points are outside of the upper control limit and decides to stop the process. What does this mean?

- Only the process range is out of statistical control
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- Both the average and range are out of statistical control
- **Nothing, the inspector is not executing the control chart appropriately**

The sub-group size for this control chart is five. The inspector stops the inspection after only 2 measurements, which is inappropriate.

The inspector should complete the entire sub-group measurement before making a conclusion about the process.

3. Using the following data points from these 5 sub-groups, calculate R-bar:

- 2
- **3**
- 4
- 5

Sub-group	Sample 1	Sample 2	Sample 3	Range
1	12	15	16	4
2	14	12	13	2
3	10	13	13	3
4	14	16	16	2
5	14	12	16	4
				3 R-Bar

First, we must solve for the Range value for each sub-group, shown in the far-right hand column.

Then we must take the average value of the 5 sub-group ranges to find the average range value of **3** (R-bar).

4. You manufacture a widget and use an x-bar and R chart to monitor your process, where you sample 3 units in each subgroup, and R-bar = 16.0. Estimate the population standard deviation for this process.

- 16.0
- **9.5**
- 27.1
- 13.2

X-Bar and R Chart				
Subgroup Sample Size	X-Bar Factor	Range Factors		Variance Factor
n	A_2	D_3	D_4	d_2
2	1.880	-	3.267	1.128
3	1.023	-	2.575	1.693
4	0.729	-	2.282	2.059
5	0.577	-	2.115	2.326
6	0.483	-	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
15	0.223	0.347	1.653	3.472
20	0.180	0.415	1.585	3.735
25	0.153	0.459	1.541	3.931

We divide R-bar by the factor d_2, which is based on the n=3 sample size.

$$Population\ Standard\ Deviation = \hat{\sigma} = \frac{\bar{R}}{d_2}$$

$$\hat{\boldsymbol{\sigma}} = \frac{\bar{R}}{d_2} = \frac{16}{1.693} = \mathbf{9.5}$$

5. **You're manufacturing a widget and using an X-bar and R chart to control the critical feature of the product. Your normal process has the following attributes: X-double bar is 225, R-bar is 12, n = 8. Identify the lower control limits for the X-bar chart:**
   - **220.52**
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X-Bar and R Chart				
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$$Lower\ Control\ Limit\ for\ \bar{X}:\ LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

The A2 constant for a subgroup sample size of 8 is 0.373.

$$\boldsymbol{LCL_{\bar{X}}} = \bar{\bar{X}} - A_2\bar{R} = 225 - 0.373 * 12 = \mathbf{220.52}$$

6. **You're manufacturing a widget and using an X-bar and R chart to control the critical feature of the product. Your normal process has the following attributes: X-double bar is 225, R-bar is 12, n = 8. Identify the upper control limits for the range chart:**
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X-Bar and R Chart				
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First, we must look up the constants required to calculate the upper control limits for the range chart using the sample size (n=8), and we find $D_4 = 1.864$.

Now we can calculate the upper control limits for the Range control chart:

$$UCL_R = D_4 * \bar{R} = 1.864 * 12 = 22.37$$

7. **You're manufacturing a product to a specification of 5.25" + 0.25". You control that process with an X-bar and R chart with a sample size of 5, and your X-bar = 5.20 and R-bar = 0.20. What is the Cpk of your process?**
   - **0.775**
   - 0.969
   - 1.163
   - 1.243

X-Bar and R Chart				
Subgroup Sample Size	X-Bar Factor	Range Factors		Variance Factor
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The first thing we must do is to convert the average range (R-bar) into an estimate of the population standard deviation using the following equation. Because our sample size is 5, the d2 value is 2.326.

$$Population\ Standard\ Deviation = \hat{\sigma} = \frac{\bar{R}}{d_2}$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.2}{2.326} = 0.086$$

Now, we can solve for the Cpk given the specification limits (5.50" and 5.00"), along with the center of our process (5.20").

$$C_{pk} = Min(C_{p,Lower}, C_{p,Upper}) = Min\left(\frac{USL - \tilde{x}}{3s}, \frac{\tilde{x} - LSL}{3s}\right)$$

$$C_{pk} = Min\left(\frac{5.50 - 5.20}{3 * 0.086}, \frac{5.20 - 5.00}{3 * 0.086}\right) = Min\left(\frac{0.300}{0.258}, \frac{0.200}{0.258}\right) = Min(1.163, 0.775) = \mathbf{0.775}$$

8. **What type of variation occurs when a process is out of control?**
   - Variable
   - Attribute
   - Common Cause
   - **Special Cause**

9. **You build 100 units per day, and your process has an average number of defects per day of 4. What is the probability that in a given production day, you will experience 1 defect?**
    - 5.2%
    - **7.3%**
    - 9.6%
    - 11.2%

    To calculate the probability of occurrence when using the Poisson distribution, we use the following equation:

$$f(x) = P(X = x) = \frac{e^{-\lambda} * \lambda^x}{x!}$$

$$f(1) = P(X = 1) = \frac{e^{-4} * 4^1}{1!} = 0.073 = 7.3\%$$

10. **You're performing a hypothesis test for the population mean and the population standard deviation is unknown. You sample 20 units from your population and you'd like to use a 1-sided test at a 5% significance level. What is the rejection criteria for this hypothesis test?**
    - 1.725
    - **1.729**
    - 2.086
    - 2.093

    Because we do not know the population standard deviation and we're sampling less than 30 units we must use the t distribution for our hypothesis test.

    Based on the 1-sided test, and 5% significance level, we can look up the critical t-value associated with 5% significance and 19 degrees of freedom to be t_{crit} = ~1.729

Critical Values of the Student T's Distribution at Various Degrees of Freedom and Alpha Levels (α)

df (*v*)	α = 0.1	α = 0.05	α = 0.025	α = 0.01	α = 0.005	α = 0.001
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579