1.4: Solving Absolute Value Equations

The ***absolute value*** of a number is its *distance from zero* on a number line. Since distance is always non-negative, absolute values are always non-negative.

Symbol: $|x|$

Another way of understanding it is that the absolute value bars are like a "positivity machine." Any number that enters the positivity machine will come out *positive*. Zero will come out as zero.

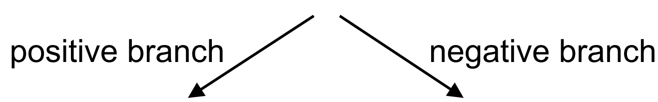
Ex #1: Please evaluate the following if $x = -2$.

a. $|4x+3| - 3\frac{1}{2}$

b. $-2|3-x|+8$

Solving Absolute Value Equations – "BIFURCATE" – meaning, dividing into two branches

$$|x-5| = 3$$

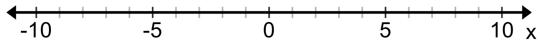


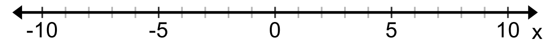
(then solve both branches)

Ex #2: Please solve each equation. Then graph your solution(s) on a number line.

a) $|x+3| = 6$

b) $|x-7| = 4$





No solution?

We know that an absolute value is always equal to a positive number.

Thus, whenever an absolute value equation equals a *negative number*, there is ***no solution***.

Here are some examples of an equation having “no solution” for the variable, ‘a’.

$|a| = -8$ (there is no number that a can be that would make the equation true)

$-2|3a| = 8$ (divide both sides by –2, to see that abs. value = neg.)

Ex #3: **Extraneous Solutions** – When an absolute value expression is set equal to an expression containing a variable, ***extraneous solutions*** may be encountered.

(Hint: first combine like terms. Then isolate the absolute value. Then bifurcate, and solve each.)

$$2|x+1| - x = 3x - 4$$