1. **You're manufacturing a widget and using an X-bar and S chart to control the critical feature of the product. Your normal process has the following attributes: X-double bar is 12.5, S-bar is 1.6, n = 10. Identify the lower control limits for the X-bar chart:**
   - 9.65
   - **10.94**
   - 11.26
   - 14.06

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{s}$$

$$At\ a\ sample\ size\ of\ 10, the\ A_3\ factor = 0.975$$

$$\boldsymbol{LCL_{\bar{X}}} = 12.5 - 0.975 * 1.6 = \mathbf{10.94}$$

X-Bar and S Chart				
Subgroup Sample Size	X-Bar Factor	Standard Deviation Factors		Variance Factor
n	A_3	B_3	B_4	c_4
2	2.659	-	3.267	0.7979
3	1.954	-	2.568	0.8862
4	1.628	-	2.266	0.9213
5	1.427	-	2.089	0.9400
6	1.287	0.030	1.970	0.9515
7	1.182	0.118	1.882	0.9594
8	1.099	0.185	1.815	0.9650
9	1.032	0.239	1.761	0.9693
10	0.975	0.284	1.716	0.9727
15	0.789	0.428	1.572	0.9823
20	0.680	0.510	1.490	0.9869
25	0.606	0.565	1.435	0.9896

2. **You're randomly selecting a card from a 52-card deck. What is the probability of selecting a three, a seven or a King?**
   - 16 in 52
   - **12 in 52**
   - 8 in 52
   - 4 in 52

These events are mutually exclusive and do not contain an intersection.
Thus, we can use the *addition rule* for mutually exclusive events:

$$The\ Probability\ of\ A\ or\ B\ or\ C = P(A\ U\ B\ U\ C) = P(A) + P(B) + P(C)$$

$$The\ Probability\ of\ A\ or\ B\ or\ C = P(three) + P(seven) + P(King)$$

$$\boldsymbol{The\ Probability\ of\ A\ or\ B\ or\ C} = \left(\frac{4}{52}\right) + \left(\frac{4}{52}\right) + \left(\frac{4}{52}\right) = \mathbf{\frac{12}{52}}$$

3. **Calculate P_{pk} for the following parameters: (LSL = 600, T = 625, USL = 655, σ = 5.5, μ = 633)**
   - 0.67
   - 1.0
   - **1.33**
   - 1.67

$$P_{pk} = Min\left(\frac{USL - \tilde{x}}{3s_{p_p}}, \frac{\tilde{x} - LSL}{3s_{p_p}}\right) = Min\left(\frac{655 - 633}{3 * 5.5}, \frac{633 - 600}{3 * 5.5}\right)$$

$$\boldsymbol{P_{pk}} = Min\left(\frac{22}{16.5}, \frac{33}{16.5}\right) = Min(1.33, 2.0) = \mathbf{1.33}$$

4. **Calculate the sample standard deviation of the following data set: 3, 4, 7, 10**
   - 7.50
   - 10.0
   - 2.74
   - **3.16**

$$Sample\ Standard\ Deviation = s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

$$sample\ mean = \bar{x} = \frac{\sum x}{n} = \frac{3 + 4 + 7 + 10}{4} = \frac{24}{4} = 6$$

(x)	$(x - \bar{x})$	$(x - \bar{x})^2$
3	(3 - 6) = -3	9
4	(4 - 6) = -2	4
7	(7 - 6) = 1	1
10	(10 - 6) = 4	16
		$\sum(x - \bar{x})^2 = 30$

$$\boldsymbol{Sample\ Standard\ Deviation = s} = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} = \sqrt{\frac{30}{4 - 1}} = \sqrt{\frac{30}{3}} = \sqrt{10} = \mathbf{3.16}$$

5. **How many treatments would be required for a DOE with 3 factors where a half factorial design is chosen?**
   - 3
   - 8
   - **4**
   - 2

$$\boldsymbol{Half\ Factorial\ Design}:\ \ Number\ of\ Treatments = \frac{Levels^{Factors}}{2} = 2^{F-1} = 2^{3-1} = 2^2 = \mathbf{4}$$

6. **The one-way ANOVA Analysis below has 19 treatment groups with the total degrees of freedom of 37. Complete this ANOVA table and calculate the F-value.**
   - 3.66
   - 1.95
   - **6.51**
   - 0.42

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	370			
Error (Within)				
Total	430			

First, we can solve for the error sum of squares by simply subtracting 370 from 430, to get an error sum of Square of 60.

Then we must solve for the degrees of freedom, according to the following equations:

$DF_{total} = DF_{error} + DF_{treatment}$

$DF_{total} = N - 1$

$DF_{treatment} = a - 1$

$DF_{error} = a(n-1)$

Note that *a* is the number of treatments, so a = 19. Hence:

$$DF_{treatment} = a - 1 = 19 - 1 = 18$$

The total degrees of freedom is said in the question to be 37. So:

$$DF_{error} = DF_{total} - DF_{treatment} = 37 - 18 = 19$$

Then we can calculate the mean squares as the sum of squares, divided by the degrees of freedom.

Finally, the F value is calculated by dividing the mean square values.

Variation Source	Sum of Squares (SS)	Degrees of freedom (DF)	Mean Squares (MS)	F-Value
Treatment (Between)	370	= 19 - 1 = 18	= 370 / 18 = 20.56	= 20.56 / 3.16 **= 6.51**
Error (Within)	= 430 - 370 = 60	= 37 - 18 = 19	= 60 / 19 = 3.16	
Total	430	37		

7. **A control chart showed a data point outside the control limit however no action was taken. What is this an example of?**
   - Common cause variation
   - The re-sampling fallacy
   - **Under Adjustment**
   - Over Adjustment

8. According to a recent survey, 35% of households in the U.S. have a pet. If you were to randomly select 10 houses, what is the likelihood that two of them have a pet?

- 20.0%
- 57.1%
- 35.0%
- **17.6%**

Ok, so the first thing that you must realize is that we're talking about discrete data not continuous data. Houses either have a pet or they don't. So, we can use the binomial equation to solve this problem.

$$P(x = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Where:
n = 10 random houses sampled
k = 2 houses found with a pet in the random sample of 10 houses
p = 35%

$$\mathrm{P}(\mathrm{x} = 2) = \binom{10}{2} * 0.35^2 * (1 - 0.35)^{10-2}$$

Let's first solve for $\binom{10}{2}$:

$$\binom{10}{2} = \frac{10!}{2! * (10-2)!} = \frac{10 * 9 * 8!}{2! * 8!} = \frac{10 * 9}{2 * 1} = 45$$

Therefore:

$$\mathbf{P}(\mathbf{x} = \mathbf{2}) = (45) * (0.1225) * (0.65)^8 = 0.176 = \mathbf{17.6\%}$$

9. You're creating a linear regression model for your data and you've calculated the following values. What is the predicted value of Y when X = 12? (S_{yy} = 192, S_{xy} = 32, S_{xx} = 96, β_0 = 9)

- 111.0
- 45.0
- **13.0**
- 108.3

$$\beta_1 = \frac{S_{xy}}{S_{xx}} = \frac{32}{96} = 0.33$$

$$Y(x) = \beta_0 + \beta_1 * x$$

$$\boldsymbol{Y}(\mathbf{12}) = 9 + 0.33 * 12 = \mathbf{13.0}$$

10. **You performed a full factorial DOE to improve the yield of a process with two factors at two levels and have measured the following response values. What is the estimated effect of Factor A?**

- **-9.5**
- -21.5
- 11
- -8

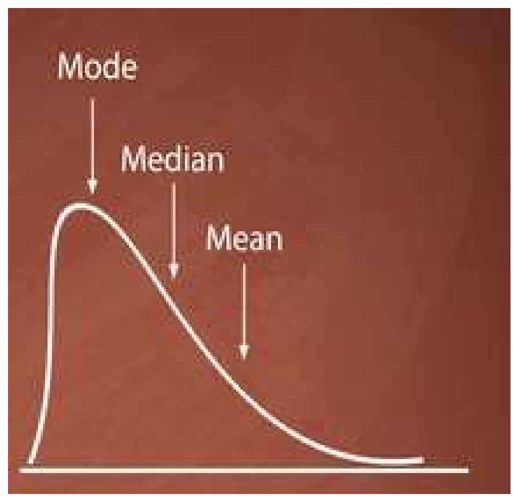
Treatments	Factors		Response
	A	B	% Yield
1	+	+	64
2	-	+	75
3	+	-	87
4	-	-	95

$$Estimated\ Effect = Average\ at\ High - Average\ at\ Low$$

$$\boldsymbol{Factor\ A\ Estimated\ Effect} = \frac{64+87}{2} - \frac{75+95}{2} = \mathbf{-9.5}$$

11. **Which characteristic of this distribution is located at point A on the image below?**

- Skewness
- Median
- **Mode**
- Mean



12. **Calculate C_r for the following parameters: (USL = 630, T = 590, LSL = 570, μ = 610, σ = 10)**

- 0.60
- 0.95
- **1.00**
- 1.20

$$C_r = \frac{1}{C_p} = \frac{6\sigma}{USL - LSL} = \frac{6 * 10}{630 - 570} = \frac{60}{60} = \mathbf{1.00}$$

13. You're running a series of experiments, and you determine that the results of the first experiment changes the probabilities of the potential outcomes in the second experiment. How would you describe these experiments?

- Independent
- **Dependent**
- Mutually Inclusive
- Mutually Exclusive

When the outcome of one event impacts (or changes) the probability of the next experiment, these two outcomes are said to be **dependent**.

14. You've sampled 20 units from the last production lot and found that 3 of them are non-conforming. Find the 95% confidence interval for the true population proportion of defective products.

- -0.070 < p < 0.229
- **0.000 < p < 0.306**
- -0.006 < p < 0.306
- 0.018 < p < 0.282

First, we can calculate the sample proportion, p using n = 20, and the number of non-conformances (3):

$$\mathit{Sample\ Proportion}:\ p = \frac{3}{20} = 0.150$$

Then we can look up our Z-score at the 5% alpha risk: $Z_{\frac{\alpha}{2}} = Z_{\frac{0.05}{2}} = Z_{.025} = 1.96$

$$\mathit{Confidence\ Interval\ (Proportion)}:\ p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{p*(1-p)}{n}}$$

$$\mathit{Confidence\ Interval}:\ 0.150 \pm 1.96 * \sqrt{\frac{0.150*(1-0.150)}{20}}$$

$$\mathit{Confidence\ Interval}:\ 0.150 \pm 1.96 * \sqrt{0.0064} = 0.150 \pm 0.156$$

$$\boldsymbol{Confidence\ Interval\ for\ Population\ Proportion}:\ \mathbf{0.000 < p < 0.306}$$

NOTE: *The negative value for the lower side of the confidence interval is adjusted to zero as it is impossible to have a negative proportion of defects.*

15. You're comparing two population variances against each other using the F-distribution. You take 6 samples from each population, and you're performing a right-handed hypothesis test at 10% significance level. What is the critical value of this test?

- 3.055
- **3.453**
- 3.405
- 3.108

To solve for this, we should first solve for the degrees of freedom for each sample.

v_1= n – 1 = 6 – 1 = 5.

v_2 = n – 1 = 6 – 1 = 5.

Now we can find the intersection of those two degrees of freedom at 10% alpha risk, which is **3.453**.

Upper Critical Values of the F Distribution at 10% Significance Level											
		Numerator Degrees of Freedom (v_1)									
		1	2	3	4	5	6	7	8	9	10
Denominator Degrees of Freedom (v_2)	1	39.860	49.500	53.593	55.833	57.240	58.204	58.906	59.439	59.858	60.195
	2	8.526	9.000	9.162	9.243	9.293	9.326	9.349	9.367	9.381	9.392
	3	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.240	5.230
	4	4.545	4.325	4.191	4.107	4.051	4.010	3.979	3.955	3.936	3.920
	5	4.060	3.780	3.619	3.520	3.453	3.405	3.368	3.339	3.316	3.297
	6	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.958	2.937
	7	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.725	2.703
	8	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.561	2.538
	9	3.360	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.440	2.416
	10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.347	2.323

16. You're constructing an NP chart, where you've sampled from 18 subgroups, each with 30 samples, and found a total of 169 defective units. Calculate the lower control limit for this process.

- 0
- 6.85
- **1.77**
- 17.0

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\bar{p} = \frac{\sum np}{\sum n} = \frac{Sum\ of\ All\ Defectives}{Sum\ of\ Subgroup\ Quantity} = \frac{169}{18 * 30} = 0.313$$

$$n\bar{p}\ Centerline = \frac{\sum np}{k} = \frac{Sum\ of\ All\ Defectives}{\#\ of\ subgroups} = \frac{169}{18} = 9.39$$

$$UCL_{np} = n\bar{p} - 3 * \sqrt{n\bar{p} * (1-\bar{p})}$$

$$\boldsymbol{LCL_{np}} = 9.39 - 3 * \sqrt{9.39 * (1-0.313)} = 9.39 - 7.62 = \mathbf{1.77}$$

Start Time: __________ Number Correct: ___________

Stop Time: __________

Total Time: __________

Available Time: 30 Minutes Target Time: 25 Minutes

Question #	Chapter	Topic
1	6	Design of Experiments (DOE)
2	2	Probability Distributions
3	7	Statistical Process Control (SPC)
4	1	Statistical Decision Making
5	8	Process and Performance Capability
6	4	Quantitative Concepts (Probability)
7	6	Relationship Between Variables
8	3	Collecting and Summarizing Data
9	5	Statistical Process Control (SPC)
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16	6	Collecting and Summarizing Data

Chapter	Title	Questions		
1	Collecting and Summarizing Data	4	11	
2	Quantitative Concepts (Probability)	2	13	
3	Probability Distributions	8	15	
4	Statistical Decision Making	6	14	
5	Relationship Between Variables	9		
6	Statistical Process Control (SPC)	1	7	16
7	Process and Performance Capability	3	12	
8	Design of Experiments (DOE)	5	10	