STUDY GUIDE

ROTATIONAL DYNAMICS

Net Torque and Rotational Dynamics

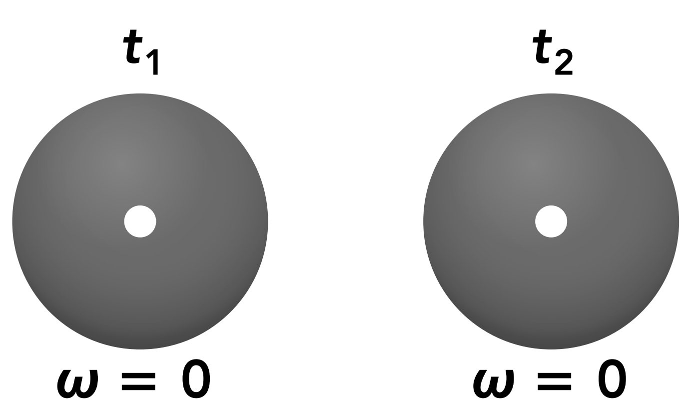
- Newton's laws of motion described how objects move and the relationship between linear forces and linear acceleration. Newton's 1st and 2nd laws of motion can also be applied to torques and rotational motion.
- When working with rotational dynamics, it will help to review the material on rotational kinematics.
- The rotational version of a **force** is a **torque**.
- The rotational version of **acceleration** is **angular acceleration**.
- The rotational version of **mass** is **rotational inertia**, also referred to as the **moment of inertia**.

Variables		SI Unit
τ	torque	$N \cdot m$
I	rotational inertia	$kg \cdot m^2$
α	angular acceleration	$\frac{rad}{s^2}$
m	mass	kg
r	distance from rotation axis	m

- **Newton's 1st law of motion (applied to rotation):** An object at rest (with no angular velocity) will remain at rest and a rotating object will maintain its angular velocity unless there is a net torque acting on the object (the sum of all the torques acting on the object is not zero).
- When we see a rotating or spinning object slow down, there must be a net torque acting on the object caused by forces such as friction or air resistance. In the absence of a net torque a rotating object will rotate forever.
- If an object is not rotating (or if it's rotating at a constant angular velocity) that doesn't mean there are no torques acting on the object, only that the net torque is zero (the torques balance each other in opposite directions).

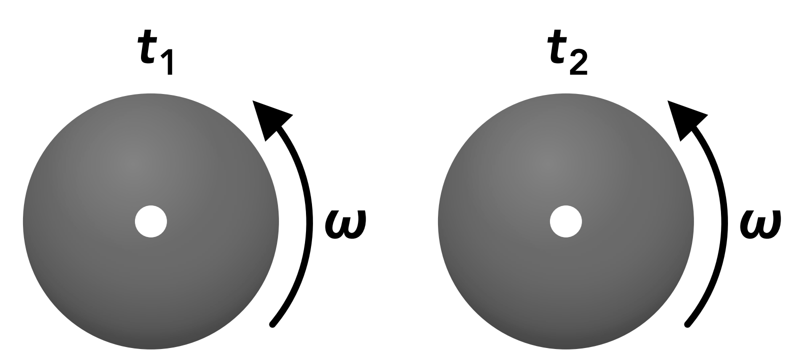
An object at rest (with zero angular velocity) will remain at rest if there is no net torque acting on it

$$\tau_{net} = 0$$
$$\alpha = 0$$



A rotating object will maintain its angular velocity if there is no net torque acting on it

$$\tau_{net} = 0$$
$$\alpha = 0$$

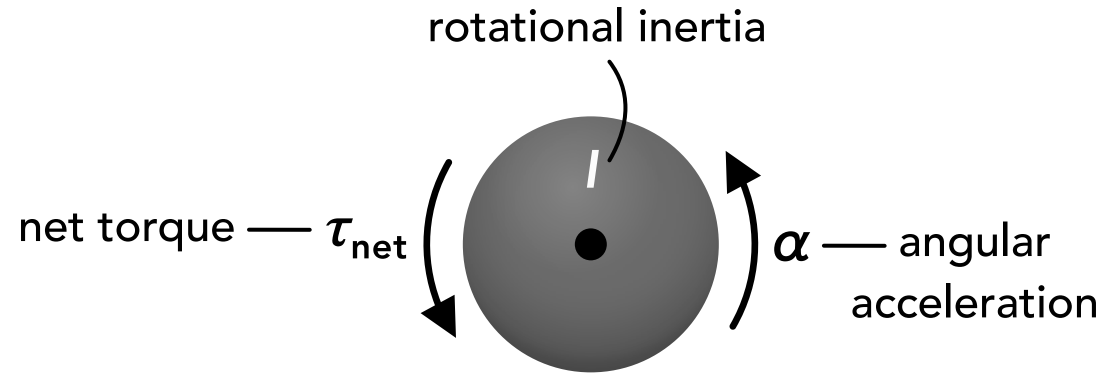


- **Newton's 2nd law of motion (applied to rotation):** A net torque τ_{net} acting on an object with a rotational inertia I will cause an angular acceleration α in the same direction as the net torque, and the net torque is equal to the rotational inertia multiplied by the angular acceleration: $\tau_{net} = I\alpha$
- The **rotational inertia** or the **moment of inertia** is covered in another section, but it's a value that represents the mass of an object and how far that mass is distributed from the axis of rotation.

Newton's 2nd law of motion applied to rotation

$\tau_{net} = I\alpha$ or $\sum\tau = I\alpha$

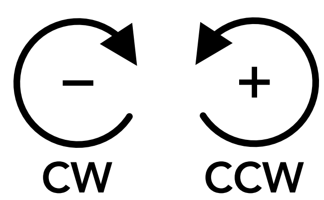
$\sum$: the sum of __

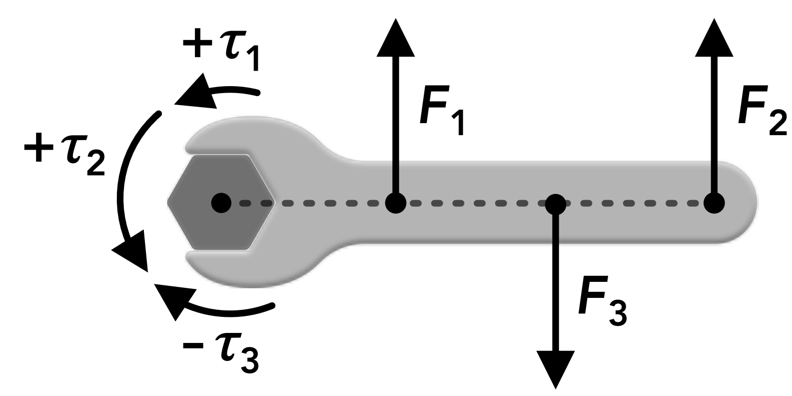


The net torque is the sum of all of the torques acting on an object

$$\tau_{net} = \sum\tau = \tau_1 + \tau_2 - \tau_3$$

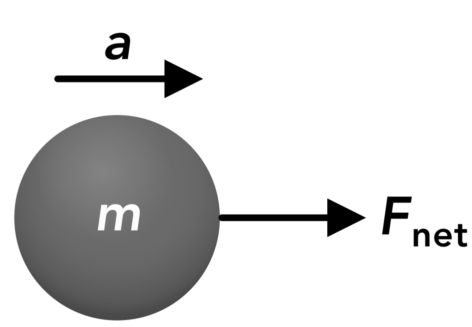
counterclockwise torque is positive
clockwise torque is negative



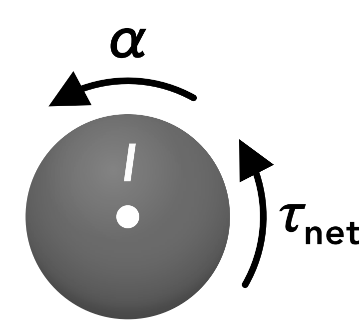


Newton's 2nd law of motion for linear motion and rotational motion

Linear



Rotational



$\sum F = ma$

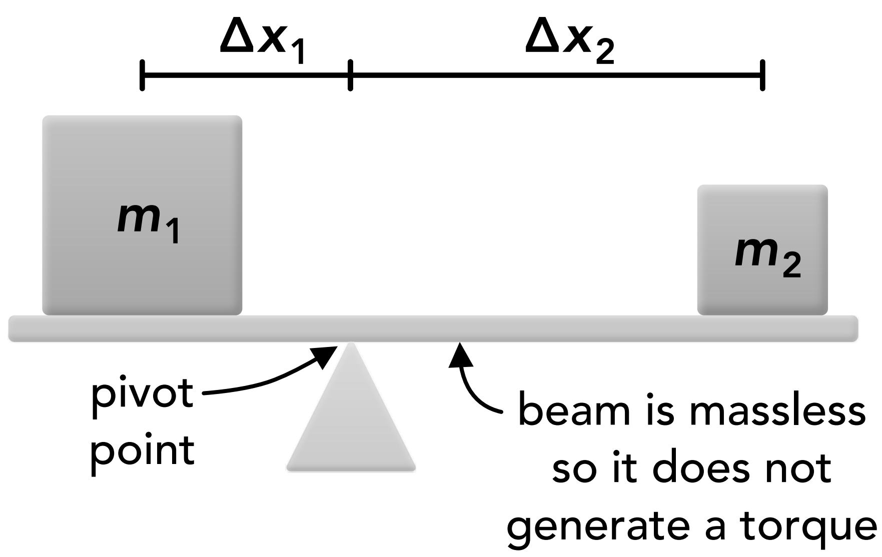
$\sum\tau = I\alpha$

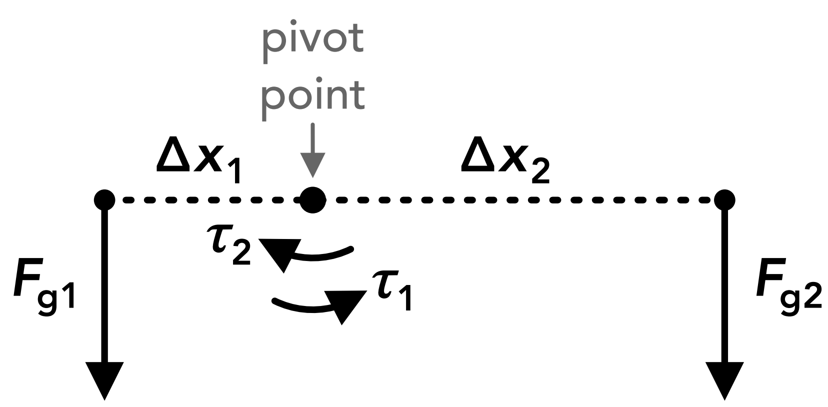
F: force (N)	⟷	τ: torque (N·m)
m: mass (kg)	⟷	I: rotational inertia (kg·m²)
a: acceleration (m/s²)	⟷	α: angular acceleration (rad/s²)

- If the net torque acting on an object or system is zero, the angular acceleration is zero and we say the object or system is in a state of **rotational equilibrium**.
- If an object or system is not rotating (or is rotating at a constant angular velocity), the net torque acting about **any point** on the object or system is zero, not just about the object's pivot point or center of mass. We can use this to analyze the forces and torques acting on an object or system.

Two blocks sit on a massless beam on a pivot point and the system is in rotational equilibrium

$$\alpha = 0$$
$$\tau_{net} = 0$$



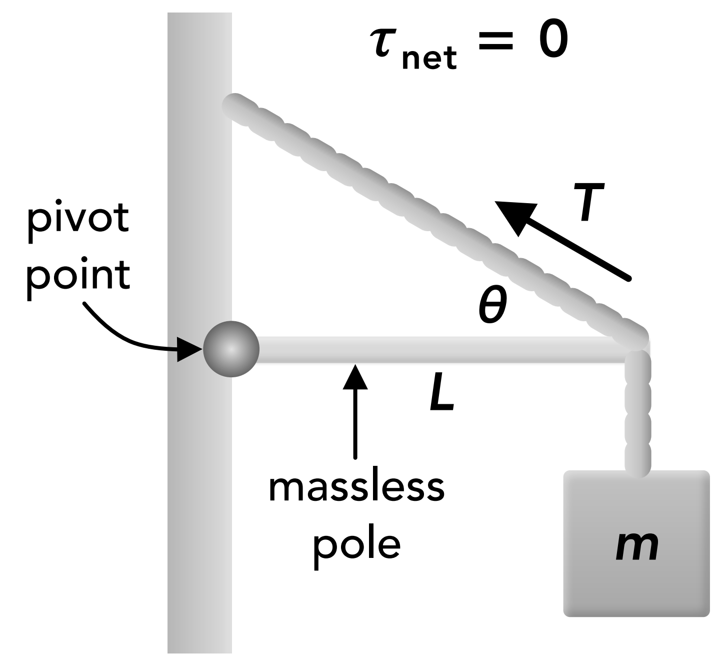


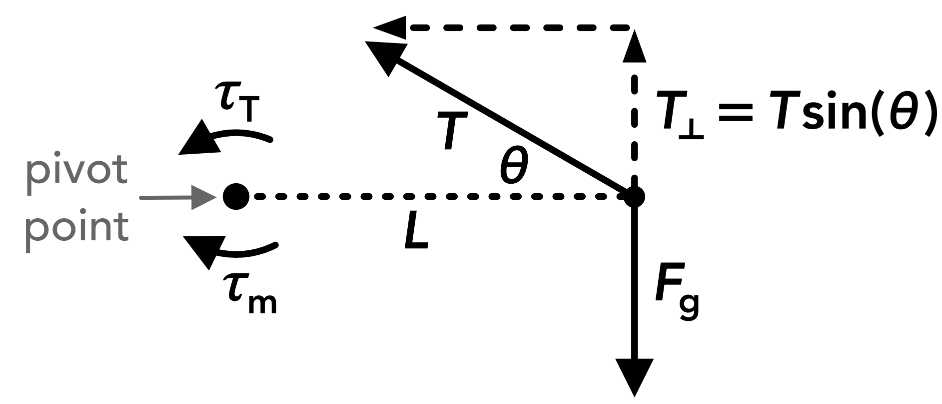
$$\sum \tau = I\alpha$$
$$\tau_1 - \tau_2 = I(0)$$
$$(r_1 F_1) - (r_2 F_2) = 0$$
$$(\Delta x_1 m_1 g) - (\Delta x_2 m_2 g) = 0$$

Any variable can be solved for if the other variables are known

A mass hangs from the end of a massless pole which is supported by an upper rope at an angle, and the system is in rotational equilibrium

$$\alpha = 0$$
$$\tau_{net} = 0$$



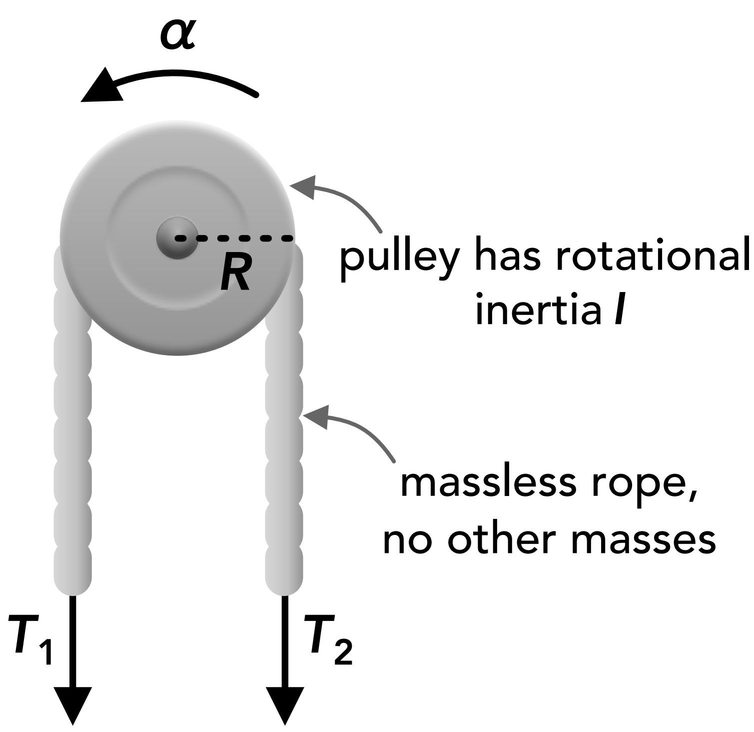


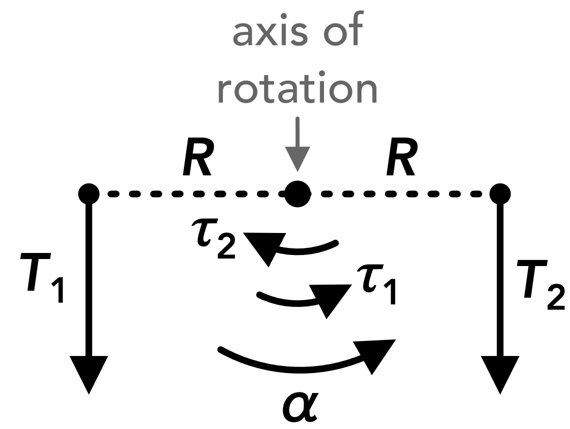
$$\sum \tau = I\alpha$$
$$\tau_T - \tau_m = I(0)$$
$$(r_T F_T) - (r_m F_m) = 0$$
$$(LT\sin(\theta)) - (Lmg) = 0$$

Any variable can be solved for if the other variables are known

- If the net torque acting on an object is not zero, the object is not in rotational equilibrium and it will rotate with an angular acceleration.

A rope is wrapped around a pulley that has mass and rotational inertia, so the tensions in the sections of rope on each side of the pulley are not equal. The ropes are massless and there are no other masses involved. The different tension forces cause the pulley to rotate with an angular acceleration.





$$\sum \tau = I\alpha$$
$$\tau_1 - \tau_2 = I\alpha$$
$$(r_1F_1) - (r_2F_2) = I\alpha$$
$$(RT_1) - (RT_2) = I\alpha$$

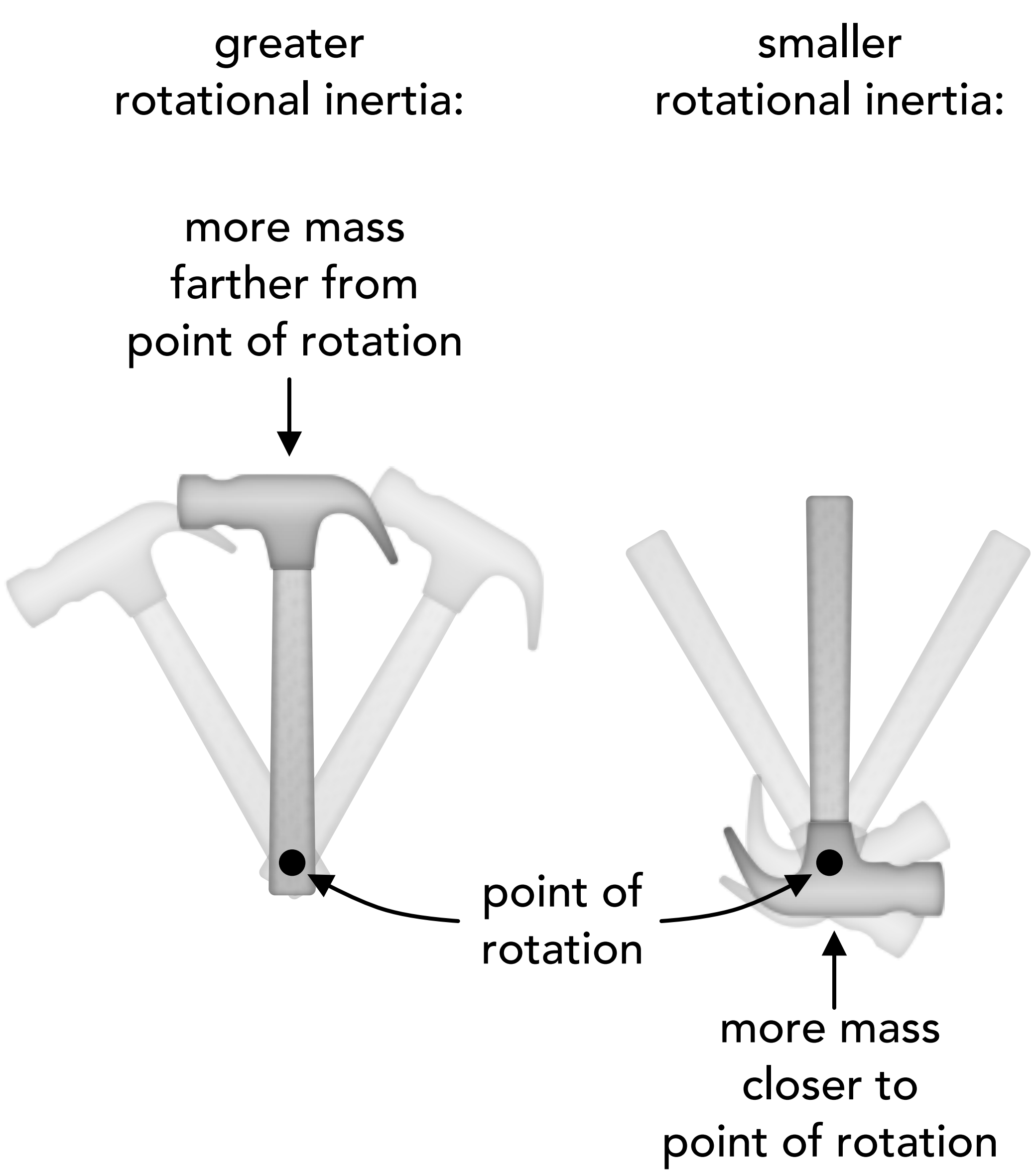
Any variable can be solved for if the other variables are known

Note: If there were other objects attached to the ropes they would also have mass and their own inertia, and the angular acceleration of the pulley would also depend on the linear dynamics of those masses and the rope.

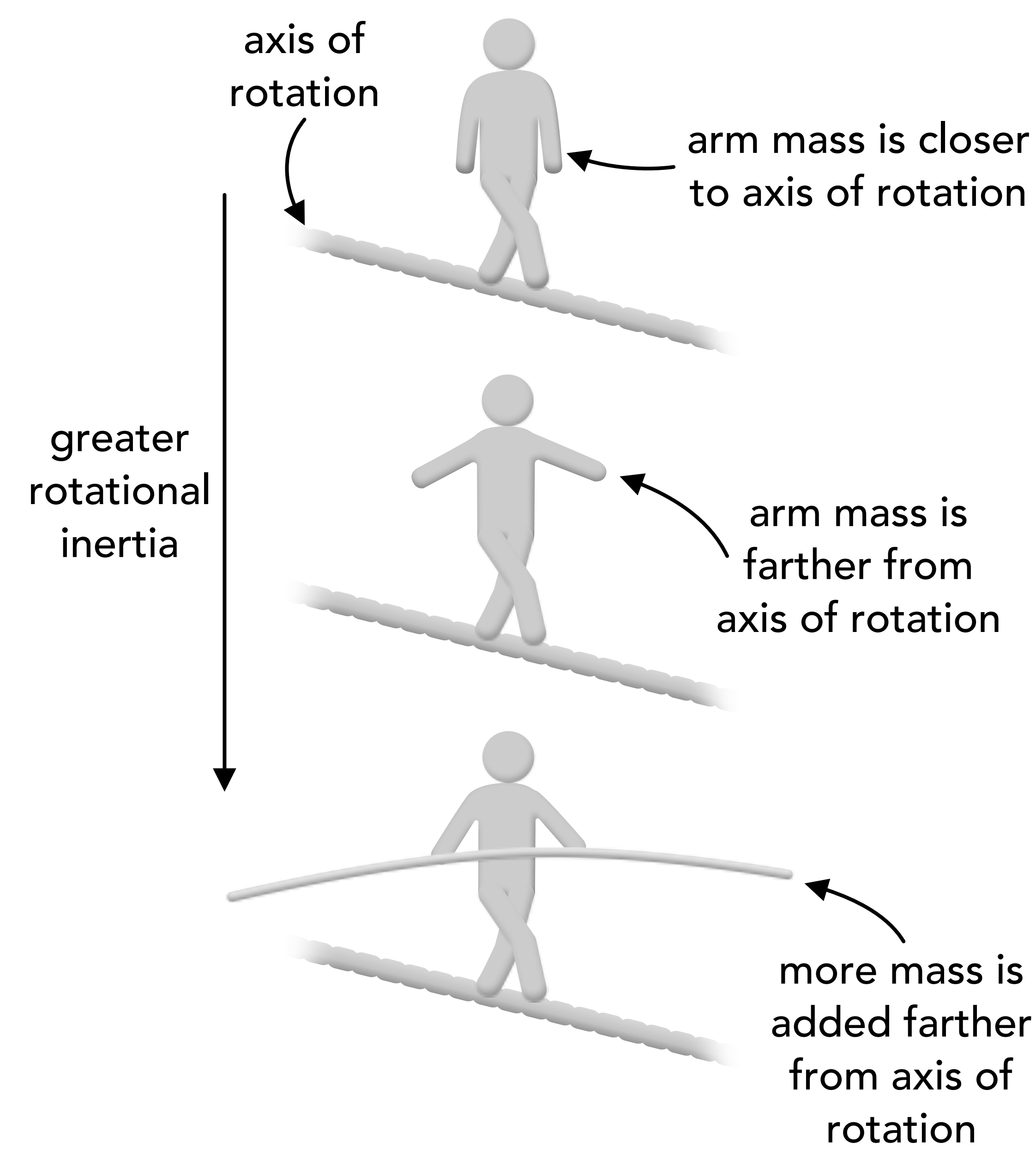
Rotational Inertia (Moment of Inertia)

- An object's **rotational inertia** I, also referred to as the **moment of inertia**, is the object's resistance to angular acceleration. The greater the rotational inertia, the more an object will resist a change to its state of rotation.
- The word "moment" has nothing to do with time and "rotational inertia" may be easier to remember, but the term "moment of inertia" is still widely used.
- The rotational inertia can be thought of as the **position-weighted sum of its mass** or its **mass distribution**.
- The more mass an object has and the farther that mass is distributed from the axis of rotation, the greater the object's rotational inertia.

A hammer is easier to rotate quickly (an angular acceleration) when held and rotated about the end with more mass



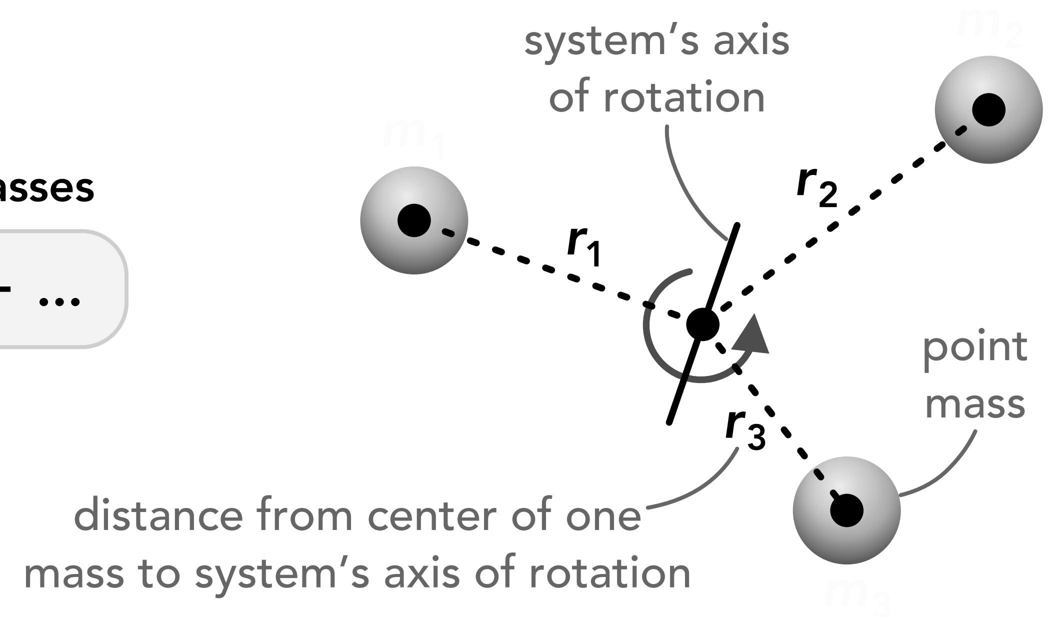
You automatically stick your arms out when trying to balance because it increases your rotational inertia and your resistance to rotation (falling sideways)



- The rotational inertia for a system of point masses (or a group of objects) can be calculated using the equation below, which is the sum of each mass ***m*** multiplied by the square of the distance between its own center and the axis of rotation r^2.

Rotational inertia for a system of point masses

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$$

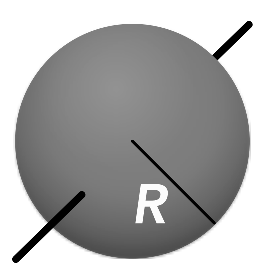


- Any rigid body (an object that does not change shape) can be modeled as a system of many individual point masses or particles (small sections of the object, molecules, atoms or even subatomic particles). If an object has a complex shape the rotational inertia will usually be given if needed.
- Many objects can be modeled as one of the shapes shown below.

Rotational inertia for some common shapes, where ***m*** is the total mass of the object, ***r*** is the radius, ***L*** is the total length of the object, and the axis of rotation is either through the center or one end

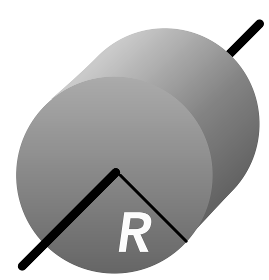
Solid sphere (center)

$$I = \frac{2}{5} mR^2$$



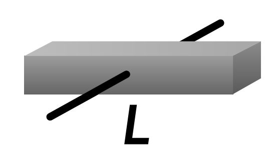
Solid cylinder (center)

$$I = \frac{1}{2} mR^2$$



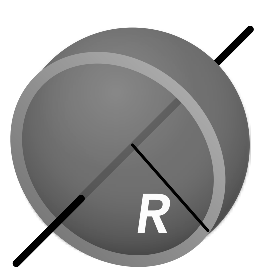
Solid rod (center)

$$I = \frac{1}{12} mL^2$$



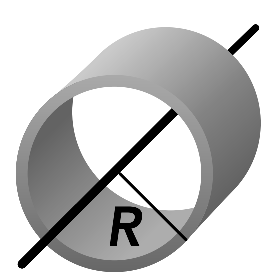
Sphere shell (center)

$$I = \frac{2}{3} mR^2$$



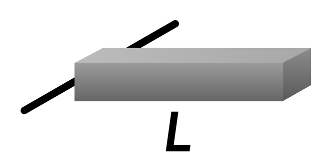
Cylinder shell (center)

$$I = mR^2$$



Solid rod (end)

$$I = \frac{1}{3} mL^2$$



- Since the rotational inertia of an object or system depends on how far the mass is from the axis of rotation, the rotational inertia will change if we move the axis of rotation.
- The **parallel axis theorem** can be used to calculate the rotational inertia of an object or system about any axis that is parallel to an axis passing through the object's or system's center of mass.

Parallel axis theorem

$$I' = I_{cm} + Md^2$$

I_{cm}: rotational inertia about an axis passing through the center of mass

I': rotational inertia about a different, parallel axis

M: mass of the object or system

d: distance between the two axes