Impulse & Momentum

Impulse is a measure of the change in momentum of an object when a force is applied over a short period of time

- $J = \int F(t)\, dt \quad (t_{initial} \text{ to } t_{final})$

  (when force is known as a function of time)

- $J = \Delta p$

  Δp = change in momentum

Impulse Momentum Theorem

$F = dp/dt$

$dp = F(t)\ dt$

$\int dp = \int F(t)\ dt$

$\Delta p = J$ (Impulse-Momentum Theorem)

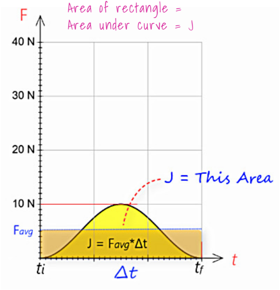
Force changes between the time the ball connects and then disconnects

Impulse and change in momentum always have the same direction

REMEMBER

"The impulse experienced by an object is equal to the change in its momentum"

Average Force & Impulse

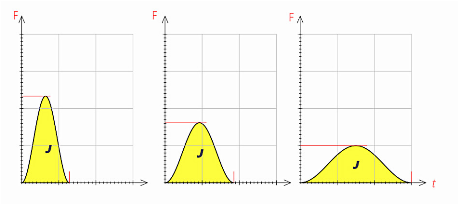


If the force is constant over a time interval Δt, the impulse can be calculated as:

$$J = F_{avg} \, \Delta t$$

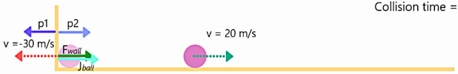
Newton's 3rd law tells us that the bat experiences the same force as the ball but in the opposite direction. The impulse on the bat therefore is also of the same magnitude but in the opposite direction

Force vs. Time Graphs



- The area under a force-time (F-t) graph represents the impulse.
- Area under each of the 3 graphs is equal and therefore the impulse is also equal J

Impulse Momentum (Solved Example)



- A 0.40 kg ball is thrown against a wall at 30 m/s and rebounds at 20 m/s. Collision duration is 0.01 s. Find:

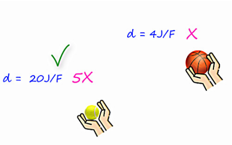
  (a) Impulse on the ball.
  (b) Average horizontal force exerted by the wall

- 1. Initial Momentum: $p_i = mv_1 = 0.40\ kg\ (-30\ m/s) = -12\ kg{\cdot}m/s$
  2. Final Momentum: $p_f = mv_2 = 0.40\ kg\ (+20\ m/s) = +8\ kg{\cdot}m/s$
  3. Impulse: $J = p_f - p_i = 8\ kg{\cdot}m/s - (-12\ kg{\cdot}m/s) = 20\ kg{\cdot}m/s$
  4. Average Force: $F_{avg} = J / \Delta t = 20\ N{\cdot}s / 0.01\ s = 2000\ N$

Comparing Momentum and Kinetic Energy

- Impulse J is a result of change in particle's momentum, which depends on the time over which the net force acts ($J = \int F(t)\, dt$)
- Work-energy theorem gives us the change in kinetic energy. Total work done depends on the distance over which the net force acts and has no dependence on time it took. ($W_{tot} = Fd$)

An Example to Compare



Catching a 0.10-kg ball at 20 m/s vs. a 0.50-kg ball at 4 m/s. Which is easier to catch?

1. Both have the same momentum ($p = 2$ kg·m/s).
2. But, KE differs: KE of the small ball is 20 J, and the larger ball is 4 J.

Although momentum and impulse are the same, more work is needed to stop the smaller ball due to its higher KE ($W_{tot} = K_2 - K_1$)

Summary of Tables

Formula	When to Use	Caution/Keep in Mind
$F = dp/dt$	When determining the relationship between force and momentum	Ensure F is the net external force
$\int dp = \int F\,dt$ from t_i to t_f $\Delta p = \int F\,dt$	To calculate total change in momentum over a time interval	Properly set the integration limits from t_i to t_f
$J = \int F\,dt$ from t_i to t_f	To find the impulse when force varies over time	Accurately integrate over the time interval
$\Delta p = J$	To relate change in momentum to impulse	Ensure consistency in the direction of vectors
$J = F_{av}\,\Delta t$	When the force is constant over the time interval Δt	Use average force F_{av}
$p = m\,v$	To calculate linear momentum of an object	Momentum p is a vector, direction is important