Test 2

Part A: Knowledge and Understanding (15 marks)

1) $Determine\ the\ derivative\ of\ f(x) = \sqrt{x^2 + 1}\ from\ first\ principles.\ (5\ marks)$

2) $If\ y = 2u^3 + 3u^2\ and\ u = x + \sqrt{x},\ find\ \frac{dy}{dx}\ when\ x = 1.\quad (4\ marks)$

3) $If\ f(x) = ax^3 + bx^2 + cx + d,\ where\ a, b, c\ and\ d\ are\ constants,\ find\ a\ so\ that\ f'''(x) = 9.\quad (4\ marks)$

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4) $Determine\ \frac{dy}{dx}.\ Do\ not\ simplify.\quad (3\ marks)$

$$y = \sqrt[5]{x^2} + \frac{2x+1}{x^2+1}$$

Part B: Application (14 marks)

1) $A\ particle\ has\ a\ displacement\ of\ d(t) = 20 - \frac{30}{\sqrt{9+2t}}\ metres\ after\ t\ minutes.\ When\ is\ the\ particle's\ speed\ 0.24\ m/min?\quad (4\ marks)$

2) $Determine\ the\ equation\ of\ the\ tangent\ to\ the\ curve\ y = \frac{x}{\sqrt{2x+3}} at\ x = 3.$
$(5\ marks)$

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3) $If\ d(x) = f(g(h(x)))$, $find\ d'(3)\ if\ h(3) = 2$, $h'(3) = 3$, $g(x) = x^2 + 2x$, and $f(x) = x^3 - 2x$. $(5\ marks)$

Part C: Thinking (22 marks)

1) $The\ function\ f(x) = x^4 - 18x^2 + 3\ has\ a\ tangent\ at\ the\ point\ (-3, -78)$. $The\ same\ tangent\ line\ is\ tangent\ to\ another\ point(s)\ on\ f(x)$. $Find\ the\ point\ (s)$. $(4\ marks)$

2) $For\ which\ value(s)\ of\ x\ are\ the\ slopes\ of\ the\ tangents\ to\ f(x) = \frac{x+1}{x-1} and\ g(x) = \frac{x+2}{x-2}$ $equal?$ $(5\ marks)$

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3) *Find the points on* $y = \frac{6x}{x-3}$ *where the tangent is perpendicular to the line* $2x - 9y - 3 = 0$. $(4\ marks)$

4) *If* $f(4) = 3$ *and* $f'(4) = -5$, *find the exact value of* $g'(4)$ *where* $g(x) = \sqrt{x}\, f(x)$ $(4\ marks)$

5) *Let* (a, b) *be any point on the graph* $f(x) = \frac{1}{x}$. *Prove the triangle area formed by the tangent through* (a, b) *and the coordinate axes is* 2. $(5\ marks)$

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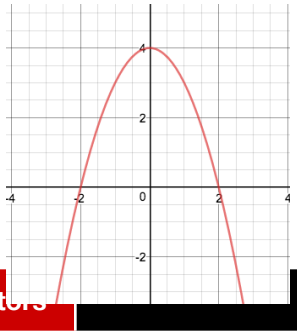
Part D: Communication (7 marks)

1) *Jane has incorrectly differentiated the function* $f(x) = 4(2x^2 + 5)^3$ *as* $f'(x) = 12(4x)^2$. *Explain what her error was and what rule she should be using to correctly differentiate* $f(x)$. (2 *marks*)

2) *What does it mean if the derivative of a function is always negative? Give an example of such a function.* (2 *marks*)

3) *Draw a potential function* $f(x)$ *if the graph of* $f'(x)$ *is below.* (3 *marks*)



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