Integration Practice – 2

No calculator allowed on these 12 exercises. [**answers** on next page]

content includes: definite & indefinite integrals, integration by substitution, area under a curve, and anti-differentiation with a boundary condition to determine the constant of integration (Q #10)

1. Find an expression for y given that $\dfrac{dy}{dx} = \dfrac{2}{3x-5}$.

2. Find $\displaystyle\int \frac{x^4 + 3x^2 - 6}{x^2}\,dx$.

3. Evaluate $\displaystyle\int_{\pi/2}^{2\pi} \sin\left(\frac{x}{2}\right) dx$.

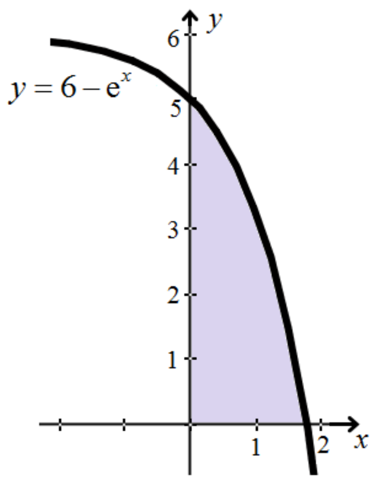
4. Find $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$.

5. (a) Using an appropriate substitution, show that $\displaystyle\int \sin x \cos x\,dx = \frac{1}{2}\sin^2 x + C$

(b) Apply the trigonometric identity $\sin 2x = 2\sin x \cos x$ to use an alternative substitution to find a different result for $\displaystyle\int \sin x \cos x\,dx$.

(c) Explain why the results for $\displaystyle\int \sin x \cos x\,dx$ in (a) and (b) are equivalent.

6. Find the **exact** area of the region bounded by the curve $y = 6 - e^x$, the x-axis and the y-axis, as shown in the diagram.



7. Evaluate $\displaystyle\int_0^1 x\sqrt{4-3x^2}\,dx$.

8. Without performing any computation or integration, explain why $\displaystyle\int_{-c}^{c} \sin x\,dx = 0$ for any real number c.

9. For real constants a and b, show that $\displaystyle\int x\left(ax^2+b\right)^4 dx = \frac{\left(ax^2+b\right)^5}{10a} + C$

10. The graph of the function f passes through the point $(2, 4)$. Given that $\dfrac{dy}{dx} = \dfrac{x}{x^2-3}$, find a specific expression for $f(x)$.

11. Using the substitution $u = 3x - 5$, find $\displaystyle\int \frac{x}{3x-5}\,dx$.

12. Find the **exact** value of $k > 0$ such that the area under the graph of $y = e^{3x}$ from $x = 0$ to $x = k$ is 5 square units.

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ANSWERS

1. $y = \frac{2}{3}\ln|3x-5| + C$

2. $\frac{x^3}{3} + 3x + \frac{6}{x} + C$

3. $2+\sqrt{2}$

4. $\frac{1}{2}\ln\left(1+e^{2x}\right)+C$

5. (b) $-\frac{1}{4}\cos 2x + C$

(c) the two constants in the results $\frac{1}{2}\sin^2 x + C$ and $-\frac{1}{4}\cos 2x + C$ are not necessarily equal; let them be C_1 and C_2 respectively; substituting trig identity $\cos 2x = 1 - 2\sin^2 x$:

$$-\frac{1}{4}\cos 2x + C_2 = -\frac{1}{4}\left(1-2\sin^2 x\right) + C_1 = -\frac{1}{4} + \frac{1}{2}\sin^2 x + C_1 = \frac{1}{2}\sin^2 x + C_1 - \frac{1}{4}$$

Since C_2 and $C_1 - \frac{1}{4}$ are both constants – which may be equal; if they are then

$$-\frac{1}{4}\cos 2x + C_2 = \frac{1}{2}\sin^2 x + C_1 - \frac{1}{4} \qquad \textbf{\textit{Q.E.D.}}$$

6. $6\ln 6 - 5$

7. $\frac{7}{9}$

8. The graph of $y = \sin x$ is an odd function (rotation symmetry about the origin) such that any region bounded by $y = \sin x$ and the x-axis to the left of the origin will be matched by an equivalent region on the right side of the origin – except that one region will be above the x-axis (positive definite integral) and the other region on other side of origin will be below the x-axis (negative definite integral) so the result for the definite over the entire interval from $-c$ to c will always be zero.

9. (answer given)

10. $f(x) = \frac{1}{2}\ln\left|x^2-3\right| + 4$

11. $\frac{5}{9}\ln|3x-5| + \frac{x}{3} + C$

12. $\frac{\ln 16}{3}$ $\left[\text{OR } \frac{4\ln 2}{3}\right]$