

EQUATION SHEET

Circle Geometry

Diameter

$$d = 2r$$

Circumference

$$C = 2\pi r = \pi d$$

Area

$$A = \pi r^2$$

$$1 \text{ revolution} = 360^\circ = 2\pi \text{ radians}$$

Triangle Geometry

Area

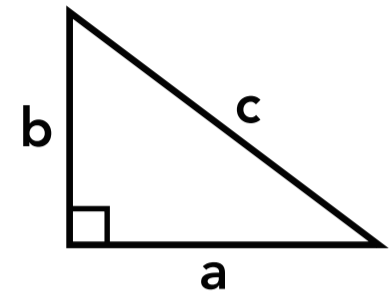
$$A = \frac{1}{2}bh$$

Angles

$$\theta_1 + \theta_2 + \theta_3 = 180^\circ$$

Pythagorean theorem

$$c^2 = a^2 + b^2$$



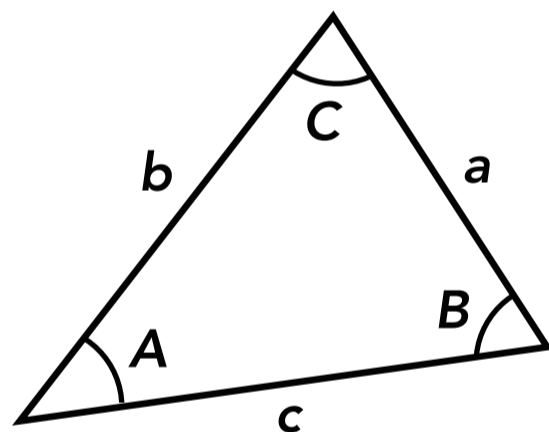
Trig identities:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

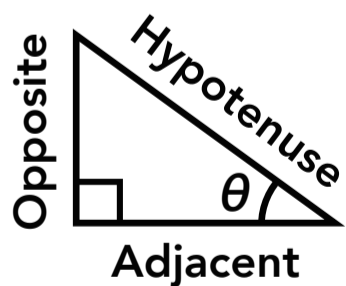


Law of sines

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

$$\theta = \sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)$$

$$\theta = \cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)$$

$$\theta = \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$$

Quadratic Formula

Quadratic formula for

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1D and 2D Kinematics

Average speed

$$v_{\text{avg}} = \frac{\text{total distance}}{\text{total time}}$$

delta

$$\Delta = \text{final} - \text{initial}$$

$$\Delta x = x_f - x_i$$

or

$$\Delta x = x - x_0$$

Variables		SI Unit
t	time	s
x	horizontal position	m
y	vertical position	m
v	velocity	$\frac{\text{m}}{\text{s}}$
a	acceleration	$\frac{\text{m}}{\text{s}^2}$

Horizontal motion:

Displacement:

$$\Delta x = x_f - x_i$$

Velocity:

$$v_x = \frac{\Delta x}{\Delta t}$$

Velocity
(rearranged):

$$x_f = x_i + v_x \Delta t$$

Acceleration:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

Acceleration
(rearranged):

$$v_{xf} = v_{xi} + a_x \Delta t$$

Kinematic equations for
constant acceleration:

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2 a_x (x_f - x_i)$$

Vertical motion:

$$\Delta y = y_f - y_i$$

$$v_y = \frac{\Delta y}{\Delta t}$$

$$y_f = y_i + v_y \Delta t$$

$$a_y = \frac{\Delta v_y}{\Delta t}$$

$$v_{yf} = v_{yi} + a_y \Delta t$$

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{yf}^2 = v_{yi}^2 + 2 a_y (y_f - y_i)$$

Subscripts

i	0	initial
f	_	final
x		horizontal
y		vertical

Projectile Motion

Range

$$\Delta x = v_i \cos(\theta) \frac{v_i \sin(\theta) + \sqrt{(v_i \sin(\theta))^2 + 2 g y_i}}{g}$$

Range

(if $y_i = y_f$)

$$\Delta x = \frac{v_i^2 \sin(2\theta)}{g}$$

Circular and Rotational Motion

Variables	SI Unit
s	tangential position m
Δs	tangential displacement m
v_t	tangential velocity $\frac{m}{s}$
a_t	tangential acceleration $\frac{m}{s^2}$

Variables	SI Unit
θ	angular position rad
$\Delta\theta$	angular displacement rad
ω	angular velocity $\frac{rad}{s}$
α	angular acceleration $\frac{rad}{s^2}$

Conversion
(Angular variable must use radians)



Circular motion
(tangential description)

Rotational motion
(angular description)

Position:

$$s \text{ m}$$

$$s = r\theta$$

$$\theta \text{ rad}$$

Displacement:

$$\Delta s = s_f - s_i \text{ m}$$

$$\Delta s = r \Delta\theta$$

$$\Delta\theta = \theta_f - \theta_i \text{ rad}$$

Velocity:

$$v_t = \frac{\Delta s}{\Delta t} \frac{m}{s}$$

$$v_t = r\omega$$

$$\omega = \frac{\Delta\theta}{\Delta t} \frac{rad}{s}$$

Acceleration:

$$a_t = \frac{\Delta v_t}{\Delta t} \frac{m}{s^2}$$

$$a_t = r\alpha$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \frac{rad}{s^2}$$

Kinematic equations
with acceleration:

$$s_f = s_i + v_{ti}t + \frac{1}{2}a_t t^2$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$v_{tf}^2 = v_{ti}^2 + 2a_t(s_f - s_i)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

Newton's 2nd Law of Motion

Newton's 2nd law of motion

$$\vec{F}_{net} = m\vec{a}$$

or

$$\sum \vec{F} = m\vec{a}$$

Σ : the sum of ___

Variables	SI Unit
F	force $N = \frac{kg \cdot m}{s^2}$
m	mass kg
a	acceleration $\frac{m}{s^2}$
v	velocity $\frac{m}{s}$

Gravitational Force & Weight

Newton's Law of Universal Gravitation
(gravitational force)

$$F_g = \frac{Gm_1m_2}{r^2} = F_{1 \text{ on } 2} = F_{2 \text{ on } 1}$$

Gravitational field strength
or acceleration due to gravity

$$g = \frac{GM}{r^2}$$

Gravitational force on mass
in gravitational field

$$F_g = mg = F_g = \frac{GMm}{r^2}$$

Constants

Constants	Unit	Name
G	$6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$	gravitational constant

Variables

Variables	SI Unit
F_g	gravitational force N
w	weight force N
m	mass kg
M	mass producing a field kg
r	distance between centers m
g	gravitational acceleration $\frac{\text{m}}{\text{s}^2}$

Weight force

$$F_g = mg \text{ or } w = mg$$

Friction

Maximum static friction force

$$f_{s \text{ max}} = \mu_s F_n$$

μ_s : coefficient of static friction

Kinetic friction force

$$f_k = \mu_k F_n$$

μ_k : coefficient of kinetic friction

Rolling friction force

$$f_r = \mu_r F_n$$

μ_r : coefficient of kinetic friction

Variables

Variables	SI Unit
f_s	static friction force N
f_k	kinetic friction force N
f_r	rolling friction force N
μ_s	coefficient of static friction
μ_k	coefficient of kinetic friction
μ_r	coefficient of rolling friction
F_n	normal force N

Spring Force

Spring force
(Hooke's Law)

$$F_{sp} = k \Delta x$$

Equivalent spring constant
for springs in series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$$

Equivalent spring constant
for springs in parallel

$$k_{eq} = k_1 + k_2 + \dots$$

Variables		SI Unit
F_{sp}	spring force	N
Δx	displacement	m
k	spring constant	$\frac{N}{m}$

Elasticity of Materials

"Spring constant"
for a material

$$k = \frac{YA}{L}$$

Elastic Force

$$F = \frac{YA}{L} \Delta L$$

Stress

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

Variables		SI Unit
F	force	N
k	spring constant	$\frac{N}{m}$
Y, E	Young's modulus	$\frac{N}{m^2}$
A	cross-sectional area	m^2
L	length	m

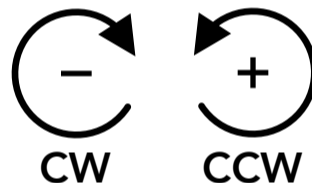
Torque

Torque

$$\tau = rF_{\perp}$$

or

$$\tau = r_{\perp} F$$



Variables		SI Unit
τ	torque	$N \cdot m$
F	force	N
r	distance from rotation axis	m

Rotational Dynamics

Newton's 2nd law of motion
applied to rotation

$$\tau_{net} = I\alpha$$

or

$$\sum \tau = I\alpha$$

Σ : the sum of _

Rotational inertia for a system of masses

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

Variables		SI Unit
τ	torque	$N \cdot m$
I	rotational inertia	$kg \cdot m^2$
α	angular acceleration	$\frac{rad}{s^2}$
m	mass	kg
r	distance from rotation axis	m

Rotational inertia for common shapes:

Solid sphere
(center)

$$I = \frac{2}{5} mR^2$$

Sphere shell
(center)

$$I = \frac{2}{3} mR^2$$

Solid cylinder
(center)

$$I = \frac{1}{2} mR^2$$

Cylinder shell
(center)

$$I = mR^2$$

Solid rod
(center)

$$I = \frac{1}{12} mL^2$$

Solid rod
(end)

$$I = \frac{1}{3} mL^2$$

Center of Mass

x coordinate of center of mass of a system

$$x_{\text{COM}} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

y coordinate of center of mass of a system

$$y_{\text{COM}} = \frac{m_1 y_1 + m_2 y_2 + \dots}{m_1 + m_2 + \dots}$$

Variables		SI Unit
x	x position	m
y	y position	m
m	mass	kg

Uniform Circular Motion

Frequency

$$f = \frac{1}{T}$$

Tangential velocity

$$v = \frac{2\pi r}{T}$$

$$v = 2\pi r f$$

Variables		SI Unit
v	velocity	$\frac{\text{m}}{\text{s}}$
r	radius	m
T	period	s
f	frequency	$\text{Hz} = \frac{\text{cycles}}{\text{s}}$
ω	angular velocity	$\frac{\text{rad}}{\text{s}}$

Centripetal Acceleration and Force

Centripetal acceleration

$$\vec{a}_c = \frac{v^2}{r} \text{ (towards center of circle)}$$

v : tangential speed (m/s)

r : radius of circular path (m)

Centripetal acceleration
(other variables substituted for speed)

$$a_c = \frac{v^2}{r} = \omega^2 r = (2\pi f)^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

ω : angular speed (rad/s)

f : frequency (Hz = rev/s)

T : period (s)

Variables		SI Unit
a_c	centripetal acceleration	$\frac{\text{m}}{\text{s}^2}$
a	acceleration	$\frac{\text{m}}{\text{s}^2}$
v	velocity	$\frac{\text{m}}{\text{s}}$
r	radius	m
t	time	s

Centripetal force

$$\vec{F}_c = m \frac{v^2}{r} \text{ (towards center of circle)}$$

Orbital Motion

Constants	Unit	Name
G	6.67×10^{-11}	$\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
		gravitational constant

Orbital velocity

$$v = \sqrt{\frac{GM}{r}}$$

Orbital period

$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

Orbital period for elliptical orbit

$$T = 2\pi \sqrt{\frac{a^3}{G(M+m)}}$$

Orbital period for elliptical orbit
(assuming M is much larger than m)

$$T = 2\pi \sqrt{\frac{a^3}{GM}}$$

Kinetic energy of object
in a circular orbit

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

Gravitational potential
energy of two-mass system

$$U_g = -\frac{GMm}{r}$$

Total energy of object
in a circular orbit

$$E = K + U_g = -\frac{GMm}{2r}$$

Variables	SI Unit
M	planet mass
m	object mass
R	planet radius
r	orbital radius
v	orbital speed
T	orbital period
F_g	gravitational force
F_c	centripetal force

Variables	SI Unit
E	total energy
K	kinetic energy
U_g	potential energy

Kinetic Energy

Kinetic energy

$$K = \frac{1}{2}mv^2$$

Rotational
kinetic energy

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

Variables	SI Unit
K	kinetic energy
m	mass
v	speed

Variables	SI Unit
K_{rot}	rotational kinetic energy
I	rotational inertia
ω	angular speed

Gravitational Potential Energy

Gravitational potential energy of a two-mass system

$$U_g = -\frac{GMm}{r}$$

$$U_g = 0 \text{ at } r = \infty$$

Change in gravitational potential energy of an object-earth system

$$\Delta U_g = mg\Delta y$$

Gravitational potential energy of an object-earth system
*relative to a reference point

$$U_g = mgy \quad U_g = 0 \text{ at } y = 0$$

Constants

Constants	Unit	Name
G	$6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$	gravitational constant

Variables

Variables	SI Unit
U_g	gravitational potential energy J
M	planet mass kg
m	object mass kg
r	distance between centers m
y	height m
g	gravitational acceleration $\frac{\text{m}}{\text{s}^2}$

Spring Potential Energy

Spring potential energy

$$U_{\text{sp}} = \frac{1}{2}k\Delta x^2$$

Δx or Δy

Variables

Variables	SI Unit
U_{sp}	spring potential energy J
k	spring constant $\frac{\text{N}}{\text{m}}$
Δx	displacement m

Conservation of Energy

Conservation of energy
(universe and isolated systems)

$$\Delta E_{\text{total}} = 0, \quad E_{\text{total i}} = E_{\text{total f}}$$

Variables

Variables	SI Unit
E	energy J
K	kinetic energy J
U_g	gravitational potential energy J
U_{sp}	spring potential energy J

Work

Work

$$\Delta E_{\text{system}} = W$$

Work

$$W = F_{\parallel} d$$

F_{\parallel} : component of force parallel to d

*F is an external force

d : displacement of the system

Variables

Variables	SI Unit
W	work J = N · m
E	energy J
F	force N
d	displacement m

Power

Power

$$P = \frac{\Delta E}{\Delta t}$$

Power

$$P = \frac{W}{\Delta t} = F_{\parallel} v$$

F_{\parallel} : component of force parallel to v
 *F is an external force
 v : velocity of the system

Variables	SI Unit
P	power $W = \frac{J}{s}$
E	energy J
W	work J
F	force N
v	velocity $\frac{m}{s}$

Momentum

Momentum

$$\vec{p} = m\vec{v}$$

Momentum vector components

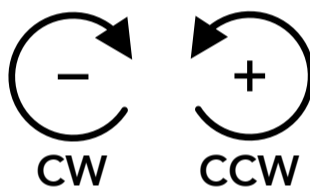
$$p_x = mv_x$$

$$p_y = mv_y$$

Variables	SI Unit
p	momentum $\frac{kg \cdot m}{s}$
m	mass kg
v	velocity $\frac{m}{s}$

Angular momentum

$$L = I\omega$$



Variables	SI Unit
L	angular momentum $\frac{kg \cdot m^2}{s}$
I	rotational inertia $kg \cdot m^2$
ω	angular velocity $\frac{rad}{s}$

Impulse

Impulse

$$\vec{J} = \Delta\vec{p} = \vec{F}_{avg}\Delta t$$

F_{avg} : average force over time

Variables	SI Unit
J	impulse $\frac{kg \cdot m}{s} = N \cdot s$
p	momentum $\frac{kg \cdot m}{s}$
F	force N
t	time s

Rotational impulse

$$\Delta L = \tau_{avg} \Delta t$$

τ_{avg} : average torque over time

Variables	SI Unit
τ	torque N · m
L	angular momentum $\frac{kg \cdot m^2}{s}$
I	rotational inertia $kg \cdot m^2$
ω	angular velocity $\frac{rad}{s}$

Conservation of Momentum

Law of conservation of momentum
(universe and isolated systems)

$$\Delta \vec{p}_{\text{total}} = 0, \quad \vec{p}_{\text{total } i} = \vec{p}_{\text{total } f}$$

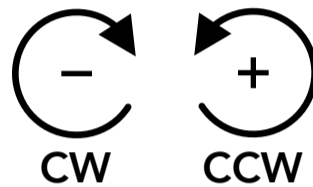
$$\Delta p_{x \text{ total}} = 0, \quad p_{xi \text{ total}} = p_{xf \text{ total}}$$

$$\Delta p_{y \text{ total}} = 0, \quad p_{yi \text{ total}} = p_{yf \text{ total}}$$

Variables		SI Unit
p	momentum	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$
m	mass	kg
v	velocity	$\frac{\text{m}}{\text{s}}$
J	impulse	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$
F	force	N
t	time	s

Law of conservation of angular momentum
(universe and isolated systems)

$$\Delta \vec{L}_{\text{total}} = 0, \quad \vec{L}_{\text{total } i} = \vec{L}_{\text{total } f}$$



Variables		SI Unit
L	angular momentum	$\frac{\text{kg} \cdot \text{m}^2}{\text{s}}$
I	rotational inertia	$\text{kg} \cdot \text{m}^2$
ω	angular velocity	$\frac{\text{rad}}{\text{s}}$

Simple Harmonic Motion

Period of a
mass-spring oscillation

$$T_{\text{sp}} = 2\pi \sqrt{\frac{m}{k}}$$

Frequency of a
mass-spring oscillation

$$f_{\text{sp}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Maximum velocity of a
mass-spring oscillation

$$v_{\text{max}} = A \sqrt{\frac{k}{m}}$$

Period of a
pendulum oscillation

$$T_p = 2\pi \sqrt{\frac{L}{g}}$$

Frequency of a
pendulum oscillation

$$f_p = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

Maximum velocity of a
pendulum oscillation

$$v_{\text{max}} = \theta_{\text{max}} \sqrt{gL}$$

Variables		SI Unit
T	period	s
f	frequency	$\text{Hz} = \frac{\text{cycles}}{\text{s}}$
A	amplitude	m
m	mass	kg
k	spring constant	$\frac{\text{N}}{\text{s}}$
U_{sp}	spring potential energy	J
K	kinetic energy	J

Variables		SI Unit
T	period	s
f	frequency	$\text{Hz} = \frac{\text{cycles}}{\text{s}}$
θ	angle	rad
L	length	m
g	grav. acceleration	$\frac{\text{m}}{\text{s}^2}$
U_g	grav. potential energy	J
K	kinetic energy	J

Waves

Wave speed

$$v = \lambda f = \frac{\lambda}{T}$$

Linear density

$$\mu = \frac{m}{L}$$

Speed of a wave
in a string

$$v_{\text{string}} = \sqrt{\frac{T_s}{\mu}}$$

Variables	SI Unit
λ	wavelength m
T	period s
f	frequency Hz = $\frac{\text{cycles}}{\text{s}}$
A	amplitude m, ...
v	velocity $\frac{\text{m}}{\text{s}}$

Sound

Constants	Unit	Name
I_0	1×10^{-12}	$\frac{\text{W}}{\text{m}^2}$ threshold of hearing

Constants	Unit	Name
R	8.3145	$\frac{\text{J}}{\text{mol} \cdot \text{K}}$ ideal gas constant

Speed of sound in a gas

$$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$$

Variables	SI Unit
v	velocity $\frac{\text{m}}{\text{s}}$
γ	adiabatic index
T	temperature K
M	molar mass $\frac{\text{kg}}{\text{mol}}$

Sound intensity

$$I = \frac{P_{\text{source}}}{4\pi r^2}$$

Sound intensity level

$$\beta = (10 \text{ dB}) \log_{10} \left(\frac{I}{I_0} \right)$$

Variables	SI Unit
I	sound intensity $\frac{\text{W}}{\text{m}^2}$
P	power $\frac{\text{J}}{\text{s}}$
r	distance from source m
β	sound intensity level dB

Observed frequency,
receding sound source

$$f_o = \frac{f_s}{1 + (v_s/v)}$$

Observed frequency,
approaching sound source

$$f_o = \frac{f_s}{1 - (v_s/v)}$$

Observed frequency,
receding observer

$$f_o = \left(1 - \frac{v_o}{v} \right) f_s$$

Observed frequency,
approaching observer

$$f_o = \left(1 + \frac{v_o}{v} \right) f_s$$

Variables	SI Unit
f_s	source frequency Hz
f_o	observed frequency Hz
v_s	source speed $\frac{\text{m}}{\text{s}}$
v_o	observer speed $\frac{\text{m}}{\text{s}}$
v	speed of sound $\frac{\text{m}}{\text{s}}$

Wave Interference

Beat frequency

$$f_{\text{beat}} = |f_1 - f_2|$$

Variables	SI Unit
d	in-line path length m
r	radial path length m
λ	wavelength m
m	number of wavelengths

In-line interference:

Constructive interference

$$\Delta d = m\lambda \quad m = 0, 1, 2, \dots$$

Destructive interference

$$\Delta d = \left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2, \dots$$

Radial interference:

Constructive interference (point C)

$$\Delta r = m\lambda \quad m = 0, 1, 2, \dots$$

Destructive interference (point D)

$$\Delta r = \left(m + \frac{1}{2}\right)\lambda \quad m = 0, 1, 2, \dots$$

Standing Waves

Both ends are either nodes or antinodes:

Wavelengths

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, \dots$$

Frequencies

$$f_m = \frac{v}{\lambda_m} = m \left(\frac{v}{2L} \right) \quad m = 1, 2, 3, \dots$$

One end is a node, one end is an antinode:

Wavelengths

$$\lambda_m = \frac{4L}{m} \quad m = 1, 3, 5, \dots$$

Frequencies

$$f_m = \frac{v}{\lambda_m} = m \left(\frac{v}{4L} \right) \quad m = 1, 3, 5, \dots$$

Variables	SI Unit
λ	wavelength m
f	frequency Hz
L	length m
v	velocity $\frac{\text{m}}{\text{s}}$
m	mode