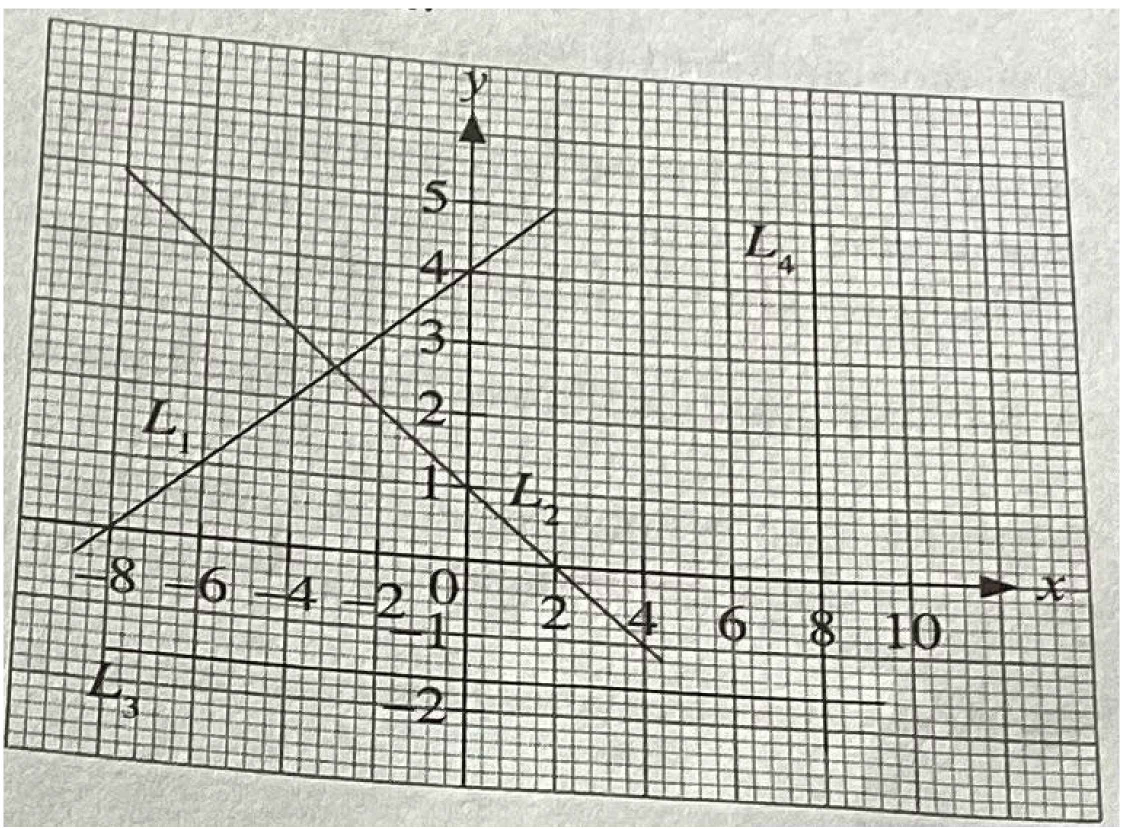
1: Linear Graphs and Simultaneous Linear Equations

Graphs of Linear Equations:

1. The general linear equation of a line is in the form $ax + by = k$.

2. There are 4 different forms of straight lines:
   (a) "Uphill" line, e.g. L_1
   (b) "Downhill" line, e.g. L_2
   (c) Horizontal line, e.g. L_3
   (d) Vertical line, e.g. L_4



3. For the "uphill" line (positive gradient) such as L_1, and the "downhill" line (negative gradient) such as L_2, the values of a and b in their equation of the form $ax + by = k$ are non-zero values.

4. In the case of horizontal and vertical lines,
   a. when $a = 0$,
      $by = k \rightarrow y = \frac{k}{b}$ where $\frac{k}{b}$ is a constant.
      $y = \frac{k}{b}$ will be the equation of a horizontal line.

   b. where $b = 0$,
      $ax = k \rightarrow x = \frac{k}{a}$ where $\frac{k}{a}$ is also a constant.
      $x = \frac{k}{a}$ will be the equation of a vertical line.

Simultaneous Linear Equations:

5. In general, it requires a pair of simultaneous equations to solve for two unknowns. A pair of simultaneous linear equations is of the form:
   $ax + by = c$,
   $dx + ey = f$, where a , b , c , d , e are fare constants.

6. When solving a pair of simultaneous equations, we are finding the common value of the two unknown variables x and y which will satisfy the two linear equations $ax + by = c$ and $dx + ey = f$ simultaneously. The common value is known as the solution of the simultaneous equations.

For example:
$x = 0$ and $y = 2$ is the solution of the simultaneous equations $3x + y = 2$ and $x + 2y = 4$ since only this particular pair, $x = 0$ and $y = 2$, satisfies the two equations simultaneously.

7. There are three ways of solving a pair of simultaneous linear equations in two variables:
   (a) Graphical Method
   (b) Elimination Method
   (c) Substitution Method

Solving Simultaneous Linear Equations by Graphical Method:

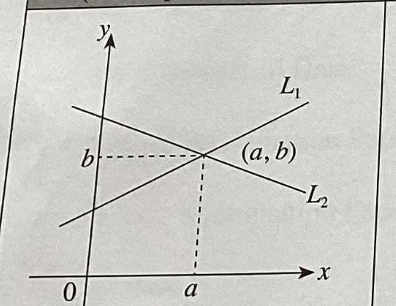
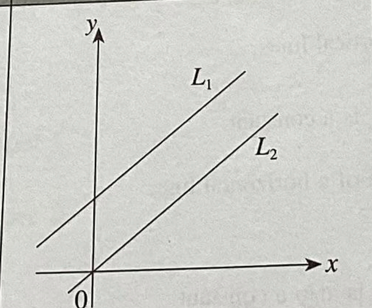
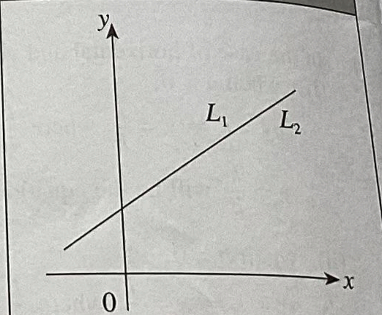
8. In this method, the solution of a pair of simultaneous equations lies at the point of intersection of their graphs.

9. When plotting the graphs of the two simultaneous equations,
   (a) first draw a table of values of x and y for each linear graph,
   (b) then plot both graphs on the same axes.

Note: For each linear graph, select at least 3 points that spread out for plotting.

10. The point where the two graphs meet is the common point of the two simultaneous equations. Hence, the graphical solution is the x-coordinate and the y-coordinate of the point of intersection of the two graphs.

11. The following shows the possible types of solutions for two linear graphs L_1 and L_2:

Unique solution (i.e. only one solution)	No solution	Infinitely many solutions
		
In this case, L_1 and L_2 are **intersecting lines** with different gradients (i.e. $m_1 \neq m_2$).	In this case, L_1 and L_2 are **parallel lines** with the same gradient but different y-intercepts (i.e. $m_1 = m_2$ but $c_1 \neq c_2$).	In this case, L_1 and L_2 are the **same line** with the same gradient and the same y-intercept (i.e. $m_1 = m_2$ and $c_1 = c_2$).
E.g. $y = x + 4$ and $2y + x = 3$.	E.g. $y = x - 3$ and $2y = 2x + 4$.	E.g. $y - 2x = 5$ and $3y = 6x + 15$.

Solving Simultaneous Linear Equations by Elimination Method:

12. In this method, we first eliminate one of the two unknowns. Either one of the unknowns can be eliminated first. If the unknown variable x is eliminated, the pair of simultaneous equations in x and y will be reduced to one equation in one unknown y and the unknown variable y can be found. The corresponding value of x can be obtained by substituting the value of y into either one of the given simultaneous equations.

13. The steps of the elimination method are shown below.

Step 1: Compare the coefficients of the unknown we want to eliminate.
(Look out for the coefficients of x and y that are 1 or -1.)

Step 2: If the numerical values of the coefficients of one unknown are the same in both the equations, we can eliminate the unknown either by adding or subtracting the two equations.
(For **same** signs of the coefficients, we **subtract**; for **different** signs of the coefficients, we **add**)

Note: If the numerical values of the coefficients of the chosen variable to be eliminated are not the

same in both equations, it is necessary to manipulate the equations first.
Consider the LCM of the coefficients to either:

- multiply one equation by a number or
- multiply both equations by some numbers,

so as to obtain a new/new equation(s) where the numerical values of the coefficients of the variable are the same.

Step 3: Solve the resulting linear equation in one unknown.

Step 4: Substitute the value obtained for the unknown into either one of the original equations to solve for the other unknown.

Step 5: Check the solution by substituting the values of the 2 unknowns which you have found into the two original equations. (Ensure LHS = RHS for each equation.)

Example Question:

$3x + 2y = 7$ —-- (1)
$5x - 3y = 37$ —-- (2)

(1) $\times$ 3: $9x + 6y = 21$ —-- (3)
(2) $\times$ 2: $10x - 6y = 74$ —-- (4)

(3) + (4): $(9x + 6y) + (10x - 6y) = 21 + 74$

$$19x = 95$$
$$x = 5 \text{ —- } (5)$$

Substitute (5) into (1): $3(5) + 2y = 7$

$$15 + 2y = 7$$
$$2y = -8$$
$$y = -4$$

$\therefore x = 5,\ y = -4$

Continue on the next page.

Solving Simultaneous Linear Equations by Substitution Method:

14. In this method, we use one equation from a pair of simultaneous equations to express one unknown in terms of the other unknown, e.g. variable y in terms of x or vice versa. Next, we substitute this equation into the other original equation to obtain an equation in only one unknown, e.g. x.

15. The steps of the substitution method are shown below.

Step 1: Choose one of the two equations and rearrange it to express one unknown in terms of the other, i.e. make the chosen variable the subject of the equation. (Look out for the coefficient of the variable that is already 1 or -1 and use this equation.)

Step 2: Substitute the expression (found in step 1) into the other equation to obtain an equation with only one unknown. Solve the equation to find the value of this unknown.

Step 3: Substitute the solution found in step 2 into the expression (found in step 1) to find the value of the other unknown.

Step 4: Check the solution by substituting the values of the 2 unknowns which you have found into the two original equations. (Ensure LHS = RHS for each equation.)

Example Question:

$x + 3y = 11$ —-- (1)
$4x - 7y = 6$ —-- (2)

From (1), $x = 11 - 3y$ —-- (3)

Substitute (3) into (2):
$$\begin{aligned} 4(11 - 3y) - 7y &= 6 \\ 44 - 12y - 7y &= 6 \\ 44 - 19y &= 6 \\ 19y &= 38 \\ y &= 2 \text{ —-- (4)} \end{aligned}$$

Substitute (4) into (3):
$$\begin{aligned} x &= 11 - 3(2) \\ x &= 11 - 6 \\ x &= 5 \end{aligned}$$

$\therefore x = 5,\ y = 2$

Version 2