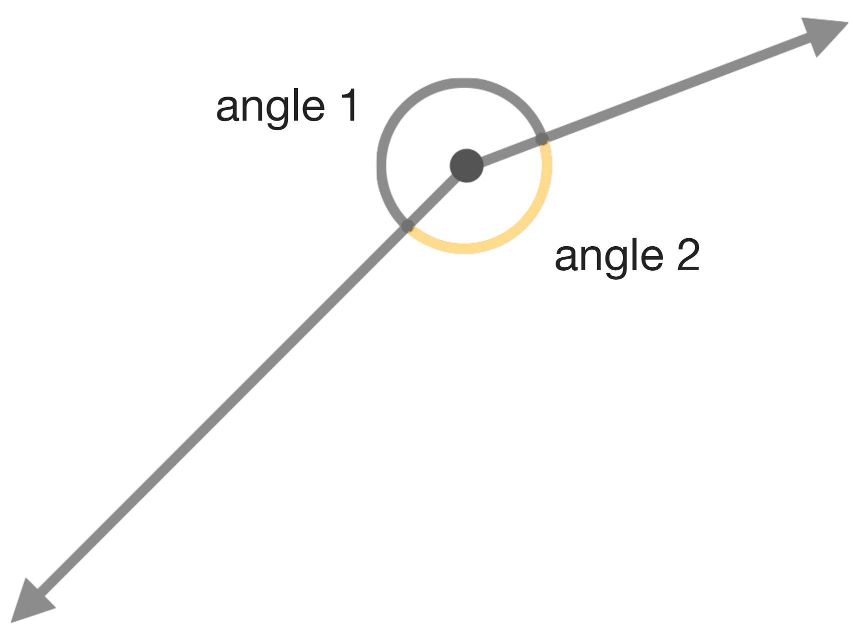
Complementary and supplementary angles

You can think of an angle as a wedge. Technically, an angle is a figure that's defined by its boundaries: a pair of rays, called the sides of the angle. A ray is a line that's infinitely long in only one direction, so a ray has just one endpoint. The two rays that make up the sides of an angle must have the same endpoint, which is called the vertex of the angle.

An angle actually includes not only the vertex and the sides but also the region (of 2-dimensional space) between its sides; that region is called the interior of the angle. Whenever you have one angle, there is always another angle: the angle that has the same vertex and same sides as the first angle, but its interior is the other, “opposite”, region between the sides. When you draw an angle and you want to show which of these two angles you mean, you can also draw an arc (of a circle with its center at the vertex of the angle) that passes through the interior of your angle and has its endpoints on the sides of your angle. An example of two angles with the same vertex and the same pair of sides is shown below; the angles are labeled angle 1 and angle 2.



Even though a ray extends to infinity (in one direction from its endpoint), we often think of the rays that form the sides of an angle as having only finite length, especially when we refer to angles whose sides are (infinite) extensions of sides of a triangle (a 3-sided figure), a quadrilateral (a 4-sided figure), or a figure with more than four sides.

Besides having a vertex and two sides, every angle has measure. If you think of an angle, and of any circle that has the vertex of that angle at its center, the measure of that angle is proportional to the fraction of the circumference of the circle that's contained in the angle.

A Greek letter (especially θ) is often used to represent both an angle and its measure; however, you could use any Greek letter (such as α, β, or γ) or any other variable name. You can measure an angle θ in units of degrees or in units of radians.

First, let's talk about degree measure.

1. An angle θ that contains the entire circumference of any circle with center at its vertex has a measure of 360 degrees (i.e., $\theta = 360^\circ$).

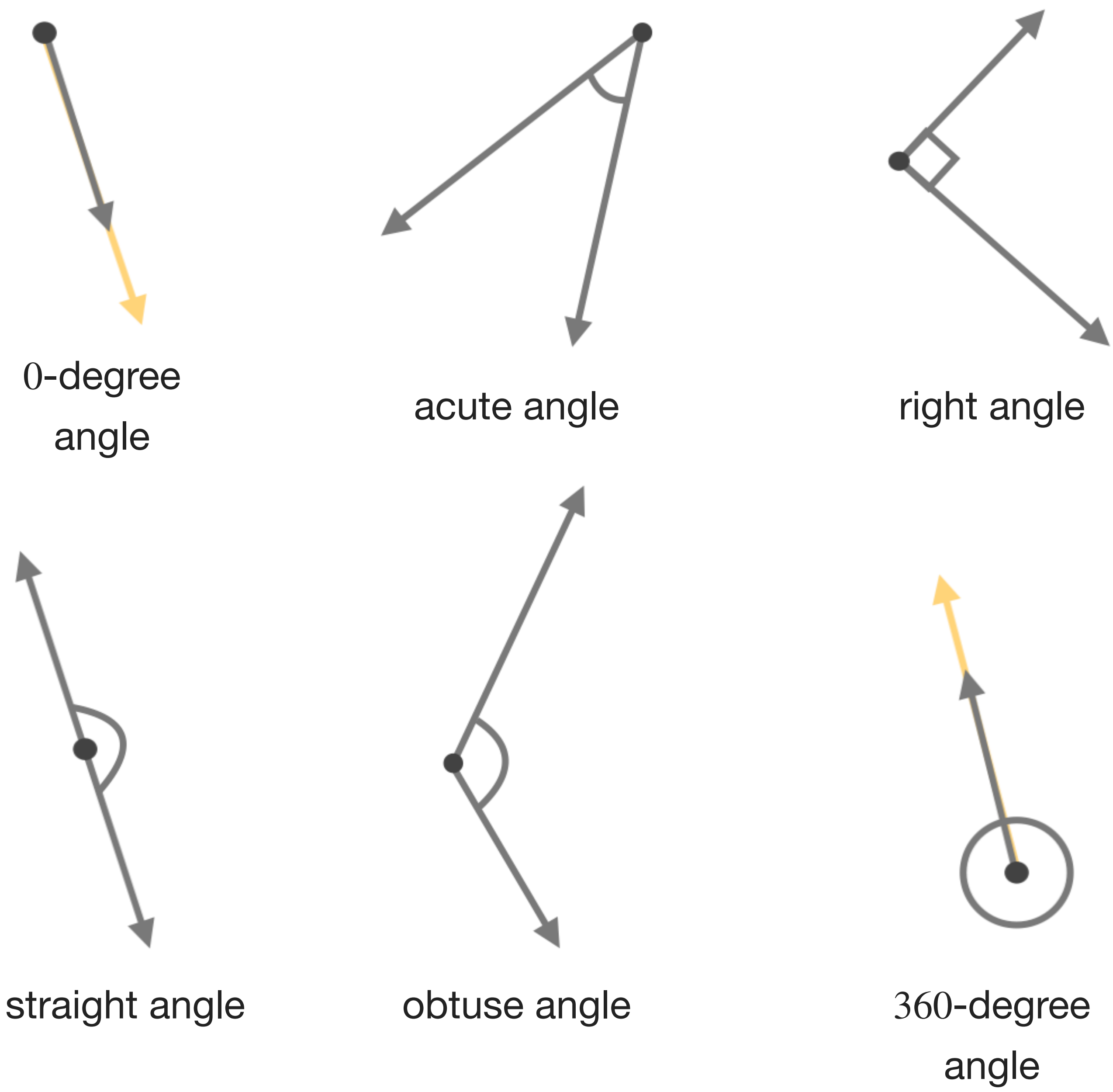
2. An angle θ that contains only one point of the circumference of any circle with center at its vertex has a measure of 0 degrees (i.e., $\theta = 0^\circ$). In this case, the two rays of the angle actually coincide (i.e., they're one and the same ray).

3. An angle θ that contains exactly $1/4$ of any circle with center at its vertex has a measure of $(1/4)(360) = 90$ degrees (i.e., $\theta = 90^\circ$). Such an angle is called a right angle, and we say that its sides are “at right angles” to each other, or that they're “perpendicular” to each other. To indicate that an angle is a right angle, you can draw a small square (instead of an arc of a circle). Two of the sides of your square would lie along the sides of the right angle, the other two sides would be parallel to the sides of the right angle, and one corner of the square would be at the vertex of the angle.

4. An angle θ that contains exactly $1/2$ of any circle with center at its vertex has a measure of $(1/2)(360) = 180$ degrees (i.e., $\theta = 180^\circ$). Such an angle is called a straight angle. Note that the two rays that form the sides of a straight angle actually make up an entire straight line, since it extends from the vertex of the angle to infinity in both directions.

5. An angle θ with $0^\circ < \theta < 90^\circ$ is called an acute angle.

6. An angle θ with $90^\circ < \theta < 180^\circ$ is called an obtuse angle.



Next, let's consider measuring angles in units of radians. We don't have a symbol for radians (like the symbol " $\circ$ " for degrees). Sometimes, we write out the word radians or use the abbreviation rad. But when we express the measure of an angle in radians, we often don't write either radians or rad; in a case like that, it's understood that we mean radians. For example, $\theta = 0.456$ is understood to mean $\theta = 0.456$ radians.

In trigonometry, it's often convenient to express the measure of an angle (in units of radians) in the form $c\pi$, that is, as the product of a constant c and the number π. For example, we may want to refer to an angle θ with a measure of $\pi/3$ (if $c = 1/3$), π (if $c = 1$), or 1.6π (if $c = 1.6$).

1. An angle θ that contains the entire circumference of any circle with center at its vertex has a measure of 2π radians (i.e., $\theta = 2\pi$).

2. An angle θ that contains only one point of (the circumference of) any circle with center at its vertex has a measure of 0 radians (i.e., $\theta = 0$).

3. A right angle θ has a measure of $(1/4)(2\pi) = \pi/2$ radians (i.e., $\theta = \pi/2$).

4. A straight angle θ has a measure of $(1/2)(2\pi) = \pi$ radians (i.e., $\theta = \pi$).

5. The measure of an acute angle θ is in the range $0 < \theta < \pi/2$.

6. The measure of an obtuse angle θ is in the range $\pi/2 < \theta < \pi$.

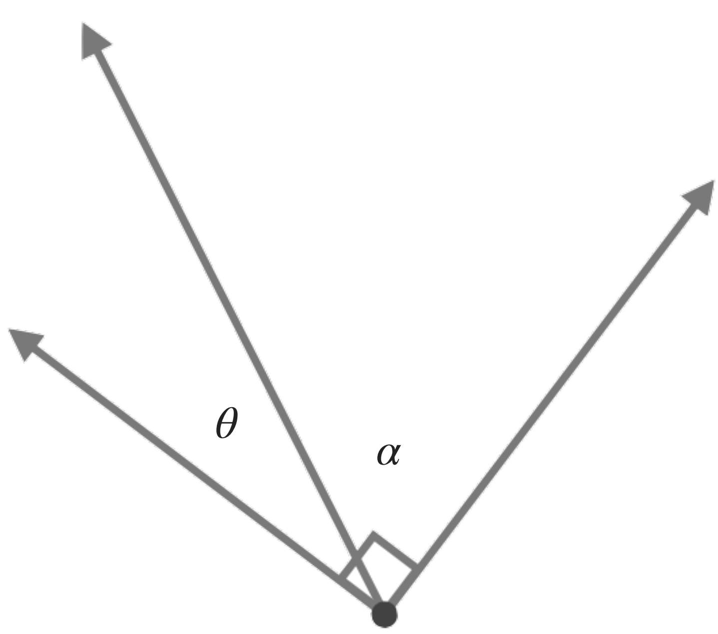
In trigonometry, if you're given an acute angle α, you will often find it useful to refer to the acute angle θ which, when added to α, produces a right angle, that is,

$$\alpha + \theta = 90^\circ$$

In radians,

$$\alpha + \theta = \frac{\pi}{2}$$

Acute angles α, θ that satisfy this condition are said to be complementary (to each other). We can draw a pair of complementary angles adjacent to each other, so that they have one side in common and their interiors do not intersect.



Example

Find the angle θ that's complementary to a 37° angle.

Since $37^\circ + \theta = 90^\circ$, we see that

$$\theta = 90^\circ - 37^\circ$$

$$\theta = 53^\circ$$

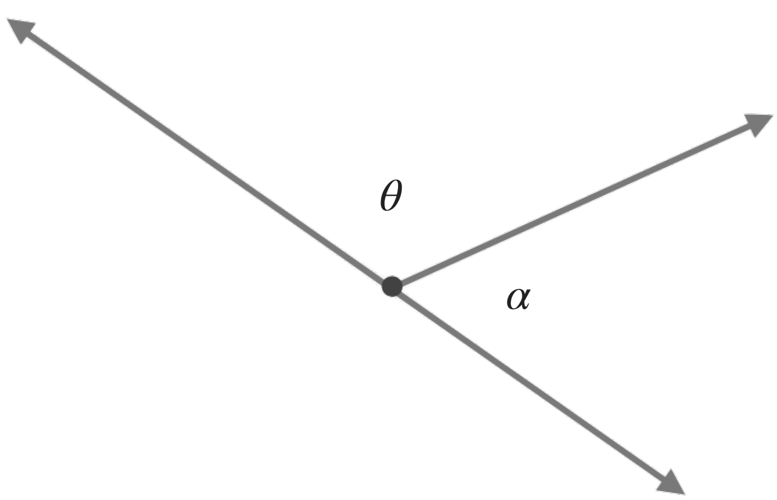
Similarly, if you're given an angle α with $0^\circ < \alpha < 180^\circ$, you will often want to refer to the angle θ with $0^\circ < \theta < 180^\circ$ which, when added to α, gives a straight angle, that is,

$$\alpha + \theta = 180^\circ$$

In radians,

$$\alpha + \theta = \pi$$

Angles α, θ that satisfy this condition are said to be supplementary (to each other). We can draw a pair of supplementary angles adjacent to each other, so that they have one side in common and their interiors do not intersect.



Example

Find the angle θ that's supplementary to an angle of $\pi/4$ radians.

Since $\pi/4 + \theta = \pi$, we have

$$\theta = \pi - \frac{\pi}{4}$$

$$\theta = \pi \left(1 - \frac{1}{4}\right)$$

$$\theta = \pi \left(\frac{3}{4}\right)$$

You should always do a careful reading of problem statements about complementary and supplementary angles. You may actually be asked to determine the measure of some angle that's related in some way (but isn't necessarily equal) to an angle that's complementary (or supplementary) to a certain angle.

Example

Find the angle θ whose measure is twice that of the angle which is supplementary to $\pi/3$ radians.

The angle that's supplementary to $\pi/3$ radians, which we will call α, is $\pi - (\pi/3)$.

$$\alpha = \pi - \frac{\pi}{3}$$

$$\alpha = \pi \left(1 - \frac{1}{3}\right)$$

$$\alpha = \pi \left(\frac{2}{3}\right)$$

Note that we're not yet finished! We still need to find the angle $\theta = 2\alpha$.

$$\theta = 2\alpha$$

$$\theta = 2\left[\pi\left(\frac{2}{3}\right)\right]$$

$$\theta = \pi\left(\frac{4}{3}\right)$$
