ROTATIONAL KINETIC ENERGY

Rotational Kinetic Energy

1. Which of the following objects has rotational kinetic energy? (Select all that apply)
   - A A moving block attached to a spring
   - B A block sliding down an incline
   - C A ball rolling on the ground
   - D A cylinder rolling down an incline

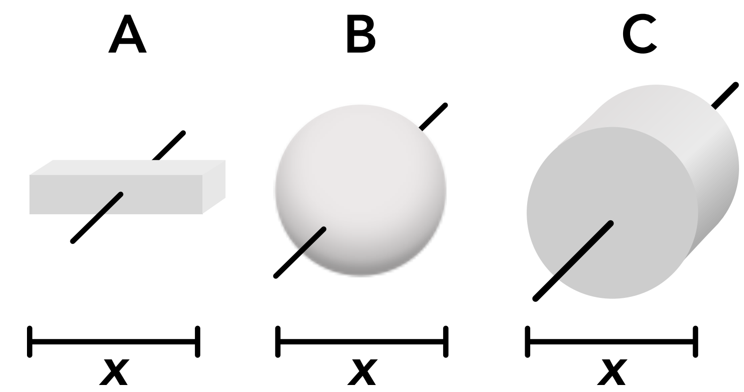
2. What is the SI unit for rotational kinetic energy?
   - A m/s
   - B rad/s
   - C $kg \cdot m^2$
   - D J

3. A solid sphere and a hollow sphere with the same mass and radius are rotating with the same angular speed. Which object has the greater rotational kinetic energy?
   - A The solid sphere
   - B The hollow sphere
   - C They have the same rotational kinetic energy
   - D Cannot be determined

4. Object A and object B have the same rotational inertia. Object B has an angular velocity of ω_B and object A has an angular velocity of $3\omega_B$. What is the rotational kinetic energy of object A in terms of the rotational kinetic energy of object B, $K_{rot\ B}$?
   - A $K_{rot\ B}/9$
   - B $3K_{rot\ B}$
   - C $9K_{rot\ B}$
   - D $K_{rot\ B}/3$

5. A solid cylinder with a mass of m and a radius of R is rolling down an incline with an angular speed of ω. The cylinder has rotational kinetic energy K_{rot}. Which of the following is equal to the cylinder's rotational inertia?
   - A $\frac{1}{2}mR^2$
   - B $\frac{1}{2}m\omega^2$
   - C $\frac{K_{rot}}{m}$
   - D $\frac{2K_{rot}}{\omega^2}$

6. Three solid objects are shown on the right. The objects have the same mass and the same width as shown. Object A is rotating at 2 rad/s, object B is rotating at 4 rad/s, and object C is rotating at 3 rad/s. Rank the rotational kinetic energies of the three objects.



   - A $K_{\text{rot A}} = K_{\text{rot B}} = K_{\text{rot C}}$
   - B $K_{\text{rot B}} < K_{\text{rot C}} < K_{\text{rot A}}$
   - C $K_{\text{rot A}} < K_{\text{rot B}} < K_{\text{rot C}}$
   - D $K_{\text{rot A}} < K_{\text{rot C}} < K_{\text{rot B}}$

7. A ball is at rest at position A, rolls without slipping down an incline and compresses a spring until it momentarily comes to a stop (position C) as shown on the right. If we define the system as the ball, the spring and the earth, and the ground is the zero reference for gravitational potential energy, what types of energy exist in the system when the ball is at position B? (Select all that apply)

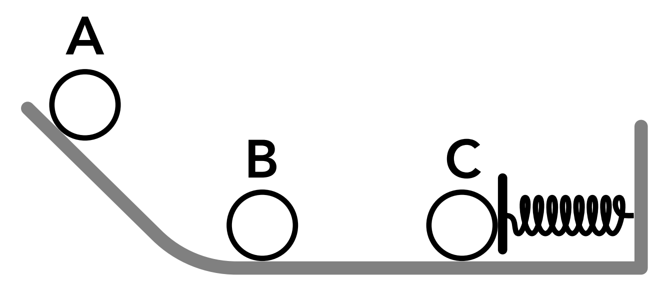


Figure 1

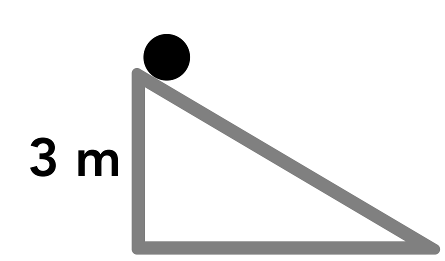
   - A Kinetic energy
   - B Spring potential energy
   - C Rotational kinetic energy
   - D Gravitational potential energy

8. For the same scenario described for Figure 1, what types of energy exist in the system when the ball is at position C? (Select all that apply)
   - A Kinetic energy
   - B Spring potential energy
   - C Rotational kinetic energy
   - D Gravitational potential energy

9. A wheel with a rotational inertia of 500 kg·m² has a rotational kinetic energy of 250 J. What is the angular speed of the wheel?

10. A solid cylinder with a radius of 10 cm and a mass of 8 kg rolls down an incline. When it reaches the bottom is has a linear speed of 2 m/s. What is the rotational kinetic energy of the cylinder at the bottom of the incline?

11. A solid cylinder with a radius of 4 cm is released from rest at the top of an incline as shown on the right. The cylinder rolls without slipping down the incline. What is the linear speed of the cylinder when it reaches the bottom of the incline?



Answers

1. C, D
2. D
3. B
4. C
5. A, D
6. D
7. A, C
8. B
9. 1 rad/s
10. 8 J
11. 6.26 m/s

Answers - Rotational Kinetic Energy

1. **Answer: C, D**
   An object has rotational kinetic energy when it is in rotational motion and has some angular velocity. The block attached to a spring and the block sliding down an incline are only translating and not rotating.

2. **Answer: D**
   The SI unit for rotational kinetic energy, and any type of energy, is a joule (J).

3. **Answer: B**
   The equation for rotational kinetic energy is given below. If the objects have the same angular speed then the object with a greater rotational inertia will have a greater rotational kinetic energy. If they spheres have the same mass and radius, the hollow sphere will have a greater rotational inertia.

   $K_{rot} = \frac{1}{2} I\omega^2$ solid sphere: $I = \frac{2}{5} mR^2$ hollow sphere: $I = \frac{2}{3} mR^2$

4. **Answer: C**
   Rotational kinetic energy is directly proportional to the square of the angular velocity. If the angular velocity is multiplied by 3, the rotational kinetic energy is multiplied by 9.

   $K_{rot} = \frac{1}{2} I\omega^2$

5. **Answer: A, D**

   The equation for the rotational inertia of a solid cylinder is: $I = \frac{1}{2} mR^2$

   The equation for rotational kinetic energy is: $K_{rot} = \frac{1}{2} I\omega^2$ $I = \frac{2K_{rot}}{\omega^2}$

6. **Answer: D**
   Each object has a different rotational inertia due to its shape. The radius of objects B and C is $x/2$.

   Object A: $I = \frac{1}{12} mL^2 = \frac{1}{12} mx^2$ $K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{1}{12} mx^2 (2 \text{ rad/s})^2 = \frac{1}{6} mx^2$

   Object B: $I = \frac{2}{5} mR^2 = \frac{2}{5} m\left(\frac{x}{2}\right)^2 = \frac{1}{10} mx^2$ $K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{1}{10} mx^2 (4 \text{ rad/s})^2 = \frac{4}{5} mx^2$

   Object C: $I = \frac{1}{2} mR^2 = \frac{1}{2} m\left(\frac{x}{2}\right)^2 = \frac{1}{8} mx^2$ $K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2}\frac{1}{8} mx^2 (3 \text{ rad/s})^2 = \frac{9}{16} mx^2$

7. **Answer: A, C**
When the ball is at position B it is moving to the right and it's rolling so it has kinetic energy and rotational kinetic energy. The ball is at the height which is defined as the zero reference for gravitational potential energy so the system does not have gravitational potential energy. The spring is not compressed so the system does not have spring potential energy.

8. **Answer: B**
When the ball is at position C it is momentarily at rest so it does not have kinetic energy or rotational kinetic energy. The ball is compressing the spring so the system does have spring potential energy. The ball is at the height which is defined as the zero reference for gravitational potential energy so the system does not have gravitational potential energy.

9. **Answer: 1 rad/s**

$$K_{rot} = \frac{1}{2}I\omega^2 \qquad (250\text{ J}) = \frac{1}{2}(500\text{ kg·m}^2)\omega^2 \qquad \omega = 1\text{ rad/s}$$

10. **Answer: 8 J**

The equation for the rotational inertia of a solid cylinder is: $I = \frac{1}{2}mR^2$

The angular speed of the cylinder is related to its linear speed using this equation: $v = r\omega$. When an object is rolling without slipping, the point on the object in contact with the ground is not moving relative to the ground, so the tangential velocity of points on the perimeter of the cylinder are moving at the same speed as the cylinder itself (the cylinder's center of mass) is moving relative to the ground, which is its linear speed.

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}(8\text{ kg})(0.1\text{ m})^2\left(\frac{2\text{ m/s}}{0.1\text{ m}}\right)^2 = 8\text{ J}$$

11. **Answer: 6.26 m/s**
The total amount of energy is conserved in the cylinder-earth system. This includes gravitational potential energy, kinetic energy (from its linear speed) and rotational kinetic energy (from its angular speed). The linear speed is always related to the angular speed using this equation: $v = r\omega$. We can say $y = 0$ at the bottom of the incline.

$$E_i = E_f \qquad K_i + K_{rot\ i} + U_{g\ i} = K_f + K_{rot\ f} + U_{g\ f}$$

$$\frac{1}{2}m(0)^2 + \frac{1}{2}I(0)^2 + mg(3\text{ m}) = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mg(0) \qquad mg(3\text{ m}) = \frac{1}{2}mv_f^2 + \frac{1}{2}\frac{1}{2}m(0.04\text{ m})^2\left(\frac{v_f}{0.04\text{ m}}\right)^2$$

$v_f = 6.26$ m/s